Part V

Astronomical Equipment and Methods

Statistical Photometry of Stars : Concept and Methods

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Abstract

A new approach is presented to search for microvariability and high-speed phenomena in variable stars. Statistical photometry stands on three pillars: the digital filtering technique, the Mandel equation based on photon counting statistics and integral transforms of the light curves. These techniques, entering the practically unexplored territories permit to investigate extreme low-amplitude and rapid variability in stars.

Introduction

Variability seen with the naked eye belongs to ancient epoch of stellar photometry. The standard light curve technique fails to reveal fine-scale variability due to low signal to noise ratio (S/N). Modern observation of microvariability may be considered in the context of statistical photometry which is based on the theory of detection and estimation of a signal hidden behind the atmospheric and photon noises. In statistical photometry we apply three techniques : 1) the digital filtering which enables an appreciable increase in photometric quality of the light curve by reduction of noises; 2) the Mandel relation for the factorial moments based on photon counting statistics to evaluate the temporal changes in fluctuation strength; 3) integral transforms of the light curve which convert the time series photometric data into significance levels for each of photometer reading. These techniques make possible the detection of photometric events comparable to the limit of the counting statistic.

1 Digital filtering technique for detection of low-amplitude stellar variability

A digital filtering is not finding ever–widening application in stellar photometry. One of a reason is that it requires either a continuous registration of data or taking readings at regular interval. At the same time the filtering technique

(FT) enables an appreciable increase in photometric quality of the light curve by suppression of the atmospheric and photon noises. Grauer & Bond [5] used the filtered comparison star observations with a two–channel photometer. The filtering was done with a digital nine–point low–pass filter in the time interval 18–90 s. Then the channel–one data were divided by the smoothed comparison star data. In this case a high precision could be attained even in "nonphotometric" observation conditions. Nather & Robinson [9] smoothed the light curve by differentiating and integrating filtering procedures when they studied coherent oscillations in UX UMa with a period of about 30 s and an amplitude of about 0.02 mag. The authors point out that only an extensive data filtering allowed them to detect a small–amplitude signal. Zhilyaev et al. [15] argued that prefiltering of the data obtained with high time resolution and subsequent integration increases the S/N ratio in the light curve by the factor 1.3–1.7 as compared to a simple data integration. The questions of an accuracy and a distortion of the data spectral structure produced by the filter action are often the open questions.

In this section we will briefly review some aspects of the theory of linear filtering to select the valid low–amplitude signal in the series of stellar brightness measurements. We will obtain the standard error and covariance in the postfiltering light curve for arbitrary low–pass filter with a finite pulse response characteristic. Application given in this section to test the technique proposed shows that FT may lead to substantial possibilities for detecting of extreme small–scale stellar variability. By the example of EV Lac it will be demonstrated that the FT makes possible the detection of flare events with an amplitude in several times lower then is generally appreciated. However, we must keep in mind that the FT is of use if only the S/N ratio is somewhat greater than one. Otherwise we must apply to other tools.

1.1 Mathematical formalism

Let there is a series of measurements $n(t) = n_0 + \xi(t)$, where n_0 is the mean intensity, $\xi(t)$ is a band–limited stationary white Gaussian noise in the frequency bandwidth f_0 and with the variance σ_0^2. Then the counts $n(t_k) = n(k)$ at the moments $t_k = k/(2f_0)$, (k = 1, 2, 3, ...) are statistically independent, the correlation function is (Goryainov et al. [4])

$$R(t) = \sigma_0^2 \sin(2\pi f_0 \tau)/(2\pi f_0 \tau) \tag{1}$$

and vanishes at the points $\tau = k/(2f_0)$. A photon counter can be considered as an ideal integrator and the assumption that the spectrum of a series is band–limited is justified. The assumption that the noise spectrum is white holds exact for the photon (Poisson) noises in the frequency range $>> 3mHz$ (Fossat [2], Young et al. [13]). Thus, the above model is quite adequate to actual measurements with a stellar photometer on a good photometric night.

We consider the filtering of a measurement data stream by an low–pass filter (LPF) to detect small–scale stellar variability. Frequency filtering can be made by convolving the series counts $n(i)$ with the filter pulse response characteristic coefficients $h(i)$:

$$n_f(k) = \sum_{i=-\infty}^{\infty} h(i)\, n(k-i) \tag{2}$$

The pulse response characteristic is the filter response to a signal in the form of a delta function $\delta(k-i)$. It determines the time resolution in the filtered data series. The spectra of the filtered and initial series $G(f)$ and $S(f)$ are related by the expression

$$G(f) = S(f) \cdot H(f)$$

The frequency and pulse response characteristics, $H(f)$ and $h(i)$ are related by the reciprocal Fourier transformation

$$H(\nu) = h(0) + 2\sum_{k=1}^{\infty} h(k)\, \cos \pi\nu k \tag{3}$$

$$\nu = f/f_0, \quad 0 \le \nu \le 1$$

The simplest LPF is the moving–average filter. The pulse response characteristic coefficients of this filter are constant: $h(i) = 1/L$, where $L = 2l+1$ is the filter length. The frequency characteristic of the filter is

$$\begin{aligned} H(\nu) &= \frac{1}{2l+1}(1 + 2\sum_{k=1}^{l} \cos \pi k\, \nu) = \\ &\frac{1}{2l+1}\frac{\sin 2\,\pi(l+1/2)\,\nu}{\sin \pi\nu} \end{aligned} \tag{4}$$

The filter passband is $\nu_c = 1/(2l+1)$. The postfiltering covariance function of a stationary series with an uniform spectral density is (Jenkins et al. [6])

$$R_f(k) = \sigma_0^2 \sum_{i=-\infty}^{\infty} h(i)\, h(i+k) \tag{5}$$

In the case of the moving–average filter the covariance function is

$$R_f(k) = \begin{cases} \sigma_0^2 \frac{2l+1-k}{(2l+1)^2}, & |k| \le 2l \\ 0, & |k| > 2l \end{cases} \tag{6}$$

In view of (5) the variance of the filtered data is

$$\sigma_f^2 = R_f(0) = \sigma_0^2 \sum_{i=-\infty}^{\infty} h^2(i) \tag{7}$$

The filtering results in a decrease in the series variance, and the counts become correlated. The postfiltering correlation function is

$$r_f(k) = \frac{\sum\limits_{i=-\infty}^{\infty} h(i)\, h(i+k)}{\sum\limits_{i=-\infty}^{\infty} h^2(i)} \tag{8}$$

Let define an interval of correlation of a filtered series as

$$\tau_{corr} = \sum_{i=-\infty}^{\infty} |r_f(i)| \tag{9}$$

It is easy to see from (1) that τ_{corr} for a band–limited stationary white Gaussian noise is equal to unity. In the case of the moving–average filter that is

$$\tau_{corr} = \sum_{k=-2l}^{2l} |r_f(k)| = \sum_{k=-2l}^{2l} \left| 1 - \frac{k}{2l+1} \right| = 2l+1 \tag{10}$$

The postfiltering variance in the last case is

$$\sigma_f^2 = \sigma_0^2 \frac{1}{2l+1} \tag{11}$$

As a consequence, we have very important analytic invariant

$$\sigma_f^2\, \tau_{corr} = \sigma_0^2 \tag{12}$$

This equation relates the dependent values of σ_f^2 and τ_{corr} to an independent value of σ_0^2 .

1.2 Standard errors in the postfiltering light curve

To identify small–amplitude variations, standard error of the filtered data is needed. The postfiltering variance σ_f^2 may be taken from (7). The actual value of σ_0^2 can be determined from check star data. Naturally, a check star has been claimed to be unvariable. The variance has the physical meaning of the power of fluctuations. It consists of two components, caused by Poisson noise and scintillations:

$$\sigma^2 = \sigma_p^2 + \sigma_{sc}^2 \tag{13}$$

We drop subscripts zero herein. The first term is equal to the mean count rate of star and background.

$$\sigma_p^2 = \bar{n} = \bar{n}_* + \bar{n}_b \tag{14}$$

The second term measures the contribution of atmospheric scintillations to the total variance. It may be defined as

$$\sigma_{sc}^2 = \frac{1}{T} \int_0^T [f(t)\, n_*(t)]^2 dt = \varepsilon_{sc}\, \bar{n}_*^2 \tag{15}$$

where $f(t)$ is the light–wave envelope in a turbulent atmosphere. The quantity ε_{sc} defined as

$$\varepsilon_{sc} = \frac{\sigma_{sc}^2}{\bar{n}_*^2} = \frac{\sigma^2 - \bar{n}}{\bar{n}_*^2} \tag{16}$$

specifies the power of scintillations in the operating frequency range $1/T - 1/2\triangle t$, where $\triangle t$ is a sampling time. The quantity ε_{sc} depends on atmospheric conditions as well as on the aperture of telescope. It is independent of the intensity $\bar{n}_*$ (Tatarsky [11]). When evaluated from the check star data, the quantity ε_{sc} gives

$$\sigma_0^2 = \bar{n}_* + \bar{n}_b + \varepsilon_{sc}\,\bar{n}_*^2 \tag{17}$$

For good atmospheric conditions the last term can be neglected and the actual value of σ_0^2 is caused by counting statistics only. The error bar in the light curve is based on the postfiltering value of the standard deviation

$$\sigma_f = \sigma_0 \left(\sum_{i=-\infty}^{\infty} h^2(i) \right)^{1/2} \tag{18}$$

The time resolution bar is determined by τ_{corr} derived from (9). The procedure for detecting of small–amplitude variations in an observation series becomes, therefore, completely formalized and easy software–programmable.

To demonstrate the FT opportunities examine some the above results for detecting of extreme low–amplitude flare event of EV Lac. The FT allows to detect event amplitudes which are comparable or even below to that caused by counting statistics. To recognize the flare event in the light curve limiting amplitude reported by many observers is most often of order of 0.1 mag (Gershberg [3], Ventura et al. [12], Konstantinova–Antova et al. [8]). Lower amplitudes makes it impossible to draw some conclusions about such basic parameters of the observed flare as the duration, the rise and decay times, the energy of event etc.

Observations were obtained with a high–speed two–star photometer with 0.2 s initial integration time at the 60 cm telescope (Zhilyaev et al. [14]) at the Mount Terskol Observing Station in North Caucasus (3100m above sea level). Monitoring was carried out on 1995, August 1, for about half an hour. Simultaneous measurements of a reference star were made in order to control and estimate atmospheric noise. In Figure 1 we plot the light curve based on photometric B–data with final 8 s time resolution. It was subject to the low-pass moving–average 5 point filter. The accuracy of the photometry wich is derived from the raw reference star data equals $\sigma = \pm 0.005 mag$. When filtered the reference star remains constant within 0.002 mag on run time scale lasting about of 1600 s. Deep low-pass filtering shows that it trends of 0.001 mag only. The final postfiltering standard errors $\sigma_s = 0.006$, $\sigma_{r.s.} = 0.003$ mag as well as the correlation intervals are shown for the variable and reference stars, respectively. Most remarkable features of EV Lac's flare can be mentioned as they are.

1. Small–scale flare is present with a duration of about 600 s and with an amplitude of $\Delta B = 0.061 \pm 0.006$ mag. The rise and decay times can be

estimated as about of 16 and of 160 s, respectively.

2. The flare event can be treated as consisting of a classical low–amplitude flare superimposed on an intensity fading of longer duration with mean amplitude of +0.02 mag. The total fading duration is of 12 minutes. Preflare dip can be treated as an integral part of common fading. The preflare dip amplitude and the duration are of 0.04 mag and about of 100 s, respectively. Sudden onset of a preflare dip can be seen on time scale of about 30 s. That is the same order of magnitude as the flare rise time.

3. The possibility of low–amplitude pulsations during the course of the flare and fading must not be ruled out.

In the end, the filtering technique allows one to provide high–precision photometry of rapid stellar variations. Our results suggest it will be possible to reach an accuracy wich are comparable or even below to that caused by counting statistics at the cost of some diminishing in time resolution. In many cases this causes no complication, since the errors and time resolution can be easily

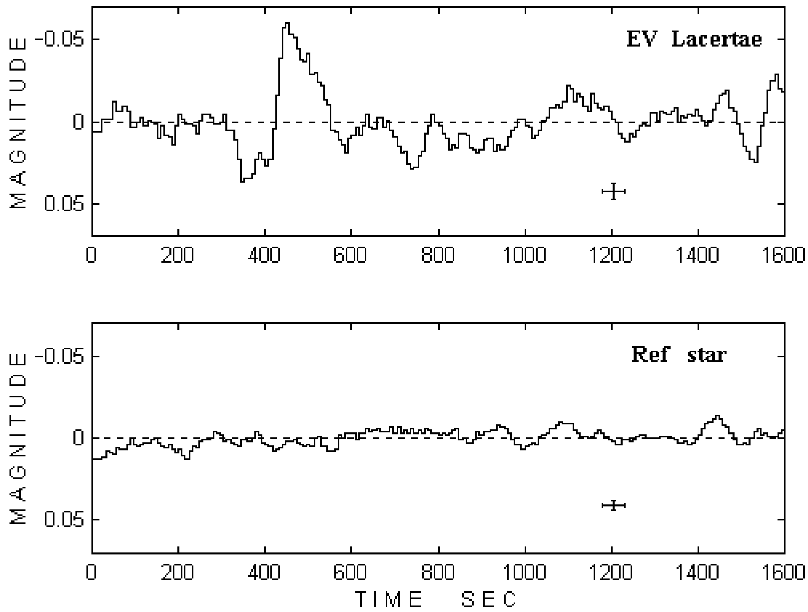

Figure 1: The B light curves of EV Lac and reference star on August 1, 1995, 23:00:16 UT , with 8 s time resolution. The error bars shown are based on the standard deviation derived from equation 18. The time–resolution bar is determined by τ_{corr} from equation (9)

estimated when the filter pulse response characteristic is known. For practical use, we may recommend the moving–average filter seemed comparatively simple for most users. However, this filter has many disadvantages, among which is the aliasing effect, whereby some fraction of the signal power may percolate through the filter side lobes. To a smaller degree this refers to a low-pass filter with a Gaussian window

$$h(i) = \frac{\sqrt{2}}{m}\exp[-2\pi(\frac{i}{m})^2], \tag{19}$$

where m is expressed in units of the sampling time Δt. According to (9), the correlation interval of the filtered series is then $\tau = 2m\Delta t$.

This difficulty is easily resolved by the use of the more refined near-ideal Kaiser filter (Rabiner and Gold [10]). The ideal low-frequency filter with a cutoff frequency $\nu_c < 1$ with the frequency and pulse characteristics

$$H(\nu) = \begin{cases} 1, & \nu \le \nu_c \\ 0, & \nu > \nu_c \end{cases} \tag{20}$$

$$h(k) = \frac{\sin(\pi\nu_c k)}{\pi\ k} \tag{21}$$

cannot be practically realized, as the number of the terms l must be infinite in order for the condition (20) to be fulfilled. The Kaiser filter coefficients are

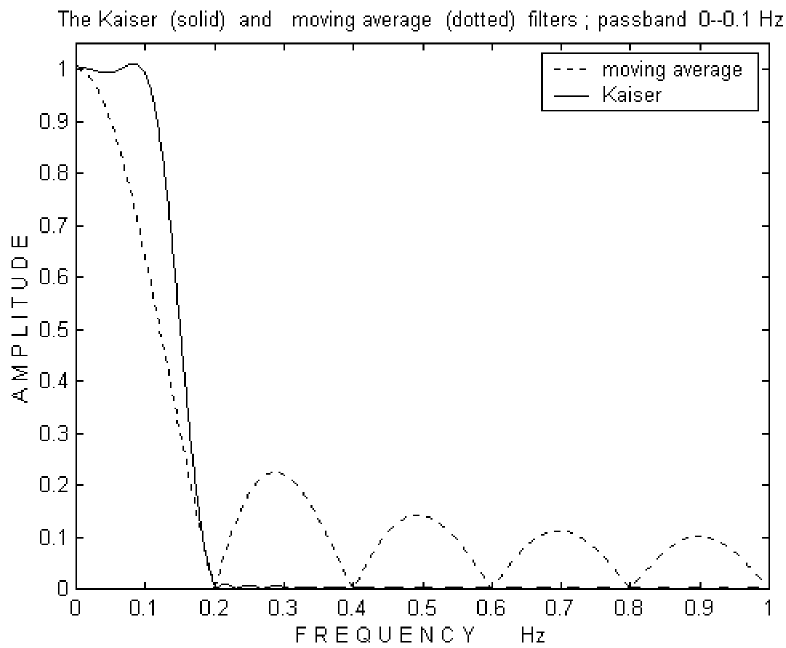

Figure 2: The low-pass Kaiser and moving-average filters equal in width of passband

$$h(k) = h(-k) = \frac{\sin(\pi\nu_c k)}{\pi\ k} \cdot \frac{I_0(\eta\sqrt{1-(k/l)^2})}{I_0(\eta)}, \tag{22}$$

where $I(x)$ is the modified Bessel function of the zero order, η the parameter that enters the filter model, l the number of filter coefficient pairs. Three basic input parameters, i.e. (1) the filter passband ν_c, (2) the width of transition band and (3) the stopband loss in the decibel scale, completely determine quantities η, l and the filter as a whole (for more details see Kaiser and Reed [7]). Thus, using the Kaiser filter we can set limits on signals that would have been aliased through side lobes into the stopband frequency domain.

Figure 2 shows the amplitude-frequency characteristic of low-pass Kaiser filter (solid line) with passband from 0 to 0.1 Hz and stopband from 0.1 to 0.2 Hz with passband ripple of 5% and attenuation of 40 dB. The dotted line represents a moving-average filter with cutoff frequency of 0.2 Hz.

Recall the FT is of use if only the S/N ratio is of the order of unity or greater. Otherwise we must apply to more powerful methods and tools.

2 Statistical approach to stellar photometry

Light curves are of little use if we operate on short time scale or are dealing with faint stars. However, what is impossible with light curves is enabled through a statistical analysis of various function of the photon flux.

It seems, a situation when some unknown signal is dissolved in noises and invisible by the naked eye should be confirmed as hopeless. Fortunately, it is not the case. Any signal possesses of some energy and may be detected and estimated even in the conditions of *a priori* uncertainties. But this lies beyond classical photometry.

Classical physics treats radiation in macroscopic terms of *intensity, spectrum, polarization,* etc. Quantum physics treats radiation as a microscopic *photon gas* with non-classical properties. One of the best known non-classical property of light is the quantum fluctuations. All sources with a constant intensity I exhibit apparent fluctuations in photon counts which are random in their nature. These counts follow the Poisson distribution

$$p(n,\tau) = \frac{n_0^n}{n!} \cdot e^{-n_0} \tag{23}$$

Here n is the number of photon detected for the sampling time τ , $n_0 = qI\tau$ its time average, where $I = const$ is a constant intensity, q the quantum efficiency.

In the case of variable intensity $I \neq const$ we have in reality an integrated intensity

$$U(t) = \int_t^{t+\tau} I(t')dt' \tag{24}$$

Generally, the quantity U follows some unknown distribution $p(U)$. In this case a distribution of counts is described by the fundamental *Mandel equation*

$$p(n,\tau) = \int_0^\infty \frac{U^n}{n!} \cdot e^{-U} p(U)dU \tag{25}$$

where the left-hand p is that can be measured, the right-hand p is that has to be evaluated. It must be noted that $p(U)$ is the unknown function, so we have the Fredholm integral equation of the first kind.

The simplest way to solve it consists in applying the method known as the method of moments. The distribution function $p(U)$ can be defined through its moments

$$< U^k >, k = 1, 2, 3, ...\infty$$

where $< ... >$ denotes a time average. One can show that the moments of U have a form

$$< U^k >=< \frac{n!}{(n-k)!} >=< n(n-1)...(n-k+1) >= n_{[k]} \tag{26}$$

where $n_{[k]}$ are so-called the factorial moments. It is convenient to use the normalized factorial moments

$$h_{[k]} = \frac{n_{[k]}}{< n >^k}$$

In the case of the Poisson statistics it may be shown that all the moments $h_{[k]} \equiv 1$ identically for any k. Hence if any $h_{[k]}$ differs from one, we may affirm that variability is detected.

Next step consists in estimation of the variability characteristics. To do this we must concrete the signal model.

Let $U = \overline{U} + \Delta U$ and $n = \chi + \psi$ be two samples with $\overline{U} = const$ and ψ with the Poisson density. We take ΔU and ψ to be the unknown distributions. Under (26) ΔU has a distribution dependent of ψ. Using the $Vandermonde$ theorem for the factorial moments and the binomial theorem, we have

$$(\chi + \psi)_{[k]} = \sum_{i=0}^{k} C_i^k \chi_{[k-i]} \psi_{[i]}, \tag{27}$$

$$(\overline{U} + \Delta U)^k = \sum_{i=0}^{k} C_i^k \overline{U}^{k-i} (\Delta U)^i \tag{28}$$

where C_i^k are the binomial coefficients. From the expression (26) we have the following equalities

$$< (\Delta U)^k >= \psi_{[k]},$$

$$\begin{aligned} n_{[1]} &= < \chi > + \psi_{[1]}, \\ n_{[2]} &= < \chi >^2 + 2 < \chi > \psi_{[1]} + \psi_{[2]}, \\ n_{[3]} &= < \chi >^3 + 3 < \chi >^2 \psi_{[1]} + 3 < \chi > \psi_{[2]} + \psi_{[3]} ... \end{aligned}$$

Assuming $\psi_{[1]} = < \psi > = 0$ without loss of generality, we have

$$\begin{aligned} \psi_{[2]} &= < \chi >^2 (h_{[2]} - 1), \\ \psi_{[3]} &= < \chi >^3 [(h_{[3]} - 1) - 3(h_{[2]} - 1)] ... \end{aligned}$$

and all the moments of unknown integrated intensity $< (\Delta U)^k >$ can be expressed through normalized factorial moments which may be evaluated from the sets of the photometer readings. In particular, if the variable component ψ has zero mean,

$$\psi_{[2]} = < n >^2 \cdot (h_{[2]} - 1)$$

The second order moment $h_{[2]}$ has the greatest practical utility. It specifies the relative power of fluctuations in some frequency domain and can be expressed through measurable quantities σ^2 and $< n >$

$$h_{[2]} = \frac{\sigma^2 - < n >}{< n >^2} + 1 \tag{29}$$

The standard deviation for $h_{[2]}$ is

$$std(h_{[2]}) = \frac{1}{< n >} \surd \left(\frac{2}{N} \right) \tag{30}$$

where N is the length of data segment. For the coherent harmonic signal with an amplitude a we have

$$a_{min} = < n >^{\frac{1}{2}} \cdot \left(\frac{72}{N} \right)^{\frac{1}{4}}$$

where a_{min} denotes the minimum amplitude at the 3σ confidence level. Thus, to detect the sinwave signal we need about one hundred of measurements.

Such an elegant method is an example of high technology in stellar photometry.

Further capabilities of detecting of low-amplitude variability of stars come up with many-telescope synchronous monitoring.

2.1 Application to stochastic variability

To detect activity of the flare star EV Lac on the shortest time-scales the above mentioned Mandel relation was applied. The expression (27)

$$\varepsilon = \frac{\sigma^2 - < n >}{< n >^2} = h_{[2]} - 1$$

specifies the relative power of fluctuations ε in the frequency domain $\Delta f = (\frac{1}{2\Delta t} - \frac{1}{N\Delta t})$, where Δt is the sampling time, N the length of the data segment. Choosing an appropriate value of N, one can calculate the power spectrum of fluctuations by averaging of ε over time. The actual value of ε caused by atmospheric scintillation can be determined from the measurements of a reference star. The difference in ε between the observed relative power of the star and that of the atmospheric scintillation, which is derived from the observation of control star, is taken to be the intrinsic ε - spectrum of star.

Stochastic variability of EV Lac was detected in quiet state. Figure 3 shows that relative power of the brightness variations of EV Lac is about 2.5% in U band in the frequency interval $0.05 \div 3.5$ Hz. It is higher than those of the control star by a factor of 30.

So far the time-averaged flare power for EV Lac was evaluated on time scale extending to some hours only (Abranin et al. [1]). It may amount to about 2% of the total luminosity in the U band. Our new approach permit us to evaluate the micro-flare power in the frequency range up to several Hz.

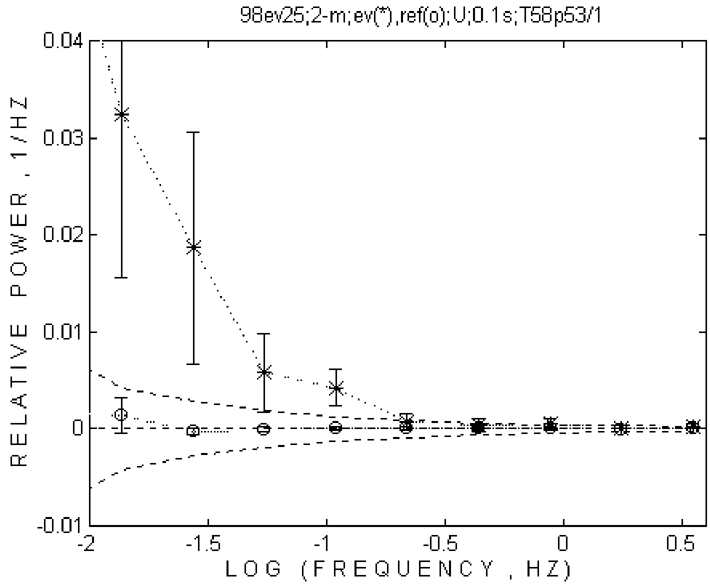

Figure 3: The relative power spectra of EV Lac (stars) in quiescence state, and of the control star (circles). Peak Terskol, the 2 m telescope, U band.

3 Integral transforms of the light curves

This section is concerned with detection of fine-scale variability with several synchronously operated telescopes. The key idea contributed by the synchronous network of optical telescopes ($SNOT$) is as follows: operations of distant telescopes are synchronized to UTC time within the sampling time; consequently, the different datasets can be considered as sets with the same valid signal and uncorrelated atmospheric and photon noises. In this case, using available statistical procedures for simultaneous photometer readings, it is possible to identify instrument faults, to increase the signal-to-noise ratio and to detect signals that may be comparable to that contributed by intrinsic noise sources.

We shall consider the problem for the Poisson density function (23) with the mean and current intensities n_0 and n. Let convert the light curve using the cumulative Poisson distribution

$$P(n, n_0) = \sum_{k=0}^{n} \frac{n_0^k}{k!} \cdot e^{-n_0} \tag{31}$$

which is equal to probability to get the reading value of n. For large intensity ($n_0 \geq 30$) it is desirable to have an asymptotic approximation

$$P(n, n_0) \approx 1 - \Phi[2(n+1)^{\frac{1}{2}} - 2n_0^{\frac{1}{2}}]$$

where $\Phi(n, n_0)$ is the standard normal cumulative. Then

$$SL = 1 - P(n, n_0) \simeq 1 - \Phi[n, n_0]$$

is the significance level to obtain the reading value of n in the light curve.

Large values of the statistic P may correspond to the true photometric events in the light curve. In particular, it can be applied to detection of photometric events at selected critical value of probability.

Since the SL statistic can take on values anywhere between 0 and 1 equiprobably the quantity $-2\ln(SL)$ follows the chi-square distribution with two degree of freedom.

If we consider the problem for several synchronously operated telescopes, a good test can be constructed using the statistic SL. To calculate the joint probability we may write

$$SL_m = -2\sum_{i=1}^{m} \ln(SL_i) \tag{32}$$

where m is the number of the synchronously operated telescopes. It is clear that the statistic SL_m follows the χ^2_{2m} distribution with $2m$ degree of freedom.

If some flare event is registered with a single telescope at the 2σ confidence level ($SL = 0.025$ and the confidence probability is 97.5%), such an event cannot be certified as very significant. However, if such an event has been registered with two or three instruments simultaneously its significance level increases up to $SL = 0.005$ and 0.001, respectively. Its confidence probability amounts to about 99.5 and 99.9% ! Thus, using the $SNOT$ and the *Cumulative Probability Curve* (CPC) algorithm good results may be obtained. The problem of detection of small-scale stellar variability can be reduced to a problem of calculating of the SL_m statistic advanced at the first time by *R.Fisher* in 1970 to evaluate the significance level in two or more statistical trials.

3.1 Application to extreme small-scale stellar flares

On short time scales variability is completely masked by noises. Figure 4 shows that the *SNOT + CPC approach* may lead to substantial possibilities in detecting of extreme small-scale stellar variability. The light curves (upper panel) with their associated CPC curves (lower panel) of EV Lac obtained in quiet state in the V and R bands simultaneously, are shown. The CPC algorithm has detected short-lived low-amplitude flare ($\Delta V = 0.030$ mag, $\Delta R = 0.013$ mag and $\Delta T = 3$ sec) at the 99.81% combined confidence level. The individual values are equal to 95.55 and 99.53%, respectively. It would be hopelessly to speak of reality of this flare event without the CPC approach.

Conclusion

Statistical photometry carries new approaches allowing one to detect unprecedented small-scale and high-speed astrophysical variability. These might well open up a new field in stellar astronomy providing information on the possible

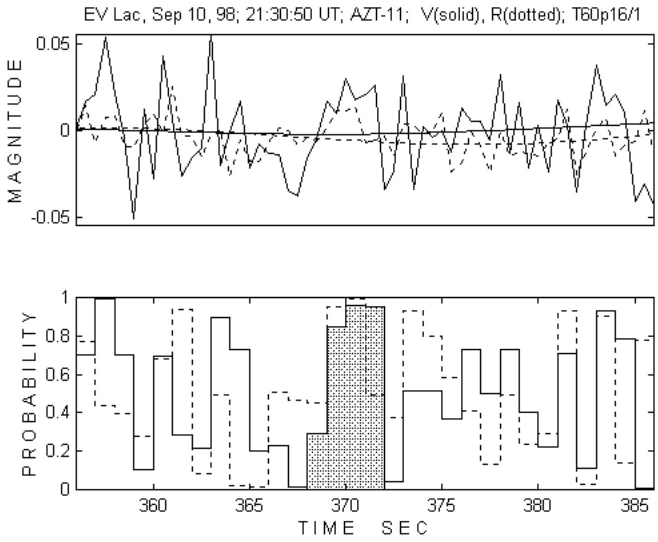

Figure 4: The V (solid) and R (dotted) light curves (upper panel) of EV Lac and their $CumulativeProbabilityCurves$ (lower panel). The filled area corresponds to the small flare event detected. The Crimean AZT-11 telescope, 11 September 1998, 21:30 UT

very rapid variability in astronomical objects on time scales down to milliseconds, to reach into the domain of millimagnitude variability.

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