Manev, Bulgarian Author of a Classical Alternative to Relativity

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Abstract

Georgi Manev, a Bulgarian scientist, is the author of a theory that offers a classical alternative to special relativity. In the so-called planetary approximation, it is the natural analog of the Schwarzschild model. Unfortunately, Manev's model fell into oblivion for more than half a century. Romanian scientists rediscovered it in the last years and returned it to the celestial mechanics.

The dynamics of systems in Manev-type fields models a large class of concrete situations belonging mainly to astronomy. The qualitative study of the motion in such fields reveals new, surprising properties, which cannot be met within the Newtonian framework. We put in evidence such characteristic features.

1 Rediscovery of Manev's model

In the early nineties, Florin Diacu, a Romanian scientist, today professor at the University of Victoria, Canada was searching for a non-Newtonian model in order to study the corresponding dynamics via McGehee transformations. He learned from Hagihara's *Celestial Mechanics* (Vol. 2, Part I, 1975) about Manev's field, which, at the solar system level at least, provided the same good theoretical approximations as the general relativity, however without leaving the classical mechanics. This led to the discovery of four forgotten papers, published by the Bulgarian physicist Georgi Manev between 1924 and 1930.

This constituted the start of a systematic research performed by a group of Romanian astronomers, aimed to describe the dynamics of systems in Manev's field, or in Manev-type (featured by the $A/r + B/r^2$ potential, with A and B real parameters) fields. This new and promising framework provided rich and really surprising results, which match the astronomical reality better than the Newtonian model (without however leaving the classical mechanics).

Such results attracted and continue to attract more and more astronomers, mathematicians and physicists towards the study of this classical model. Scientists from many countries brought consistent contributions to this domain (see the last section of this paper).

But this was not all. As a new evidence of the spiritual links between the two peoples separated by the Danube, the Romanian endeavours found a large echo in the Bulgarian scientific community. The common efforts of the Bulgarian and

Romanian astronomers shed a new light on the personality of Georgi Manev. Political reasons had doomed him to oblivion in his own country; this made him unrecognized and eventually forgotten for decades.

This rediscovery led to the apparition of the book *The Bulgarian Physicist-Theorist Georgi I. Manev (1884-1965). Rediscovery after Oblivion* (Physics on Stage, Sofia, 2000), intended to vindicate *post factum* the name and the achievements of the great Bulgarian physicist.

2 Manev's model in celestial mechanics

One of the most important problems of celestial mechanics is to find a model able to maintain the dynamical astronomy within the framework of classical mechanics (keeping the simplicity and the advantages of the Newtonian model), offering at the same time equally good justifications of the observed phenomena as the relativity. Such a model is that based on the $A/r + B/r^2$ potential, which we shall hereafter call Manev-type potential. The study of the dynamical systems associated to this field is fully justified from the physical point of view, since, as we already said, Manev's model offers the same good approximations as relativity. Today's methods of experimental physics show that, if considering the corresponding field equations, theory and practice match well together. Our point of view is to offer a projection of the field equations in a classical mechanics framework in order to satisfy the needs of celestial mechanics.

One knows that the perihelion advance of the inner planets (especially that of Mercury) cannot be fully explained within the framework of the classical Newtonian law, even resorting to perturbation theory. The many pre- and post-relativistic laws (as those proposed by Hall and Newcomb, for instance) usually answered this question, but failed to explain other issues, as the secular motion of the Moon's perigee. As regards the general relativity, it succeeded in explaining well such phenomena, both quantitatively and qualitatively. Unfortunately, this powerful theory, which answered many momentous questions in physics and astronomy, is not of much help for celestial mechanics. All attempts to formulate a meaningful relativistic n-body problem have failed to provide valuable results. Although works as those of Einstein, Infeld and Hoffmann or Levi-Civita have had important consequences in physics, and even a notable impact in celestial mechanics, they had never been developed towards understanding the dynamics of gravitating particle systems.

Therefore the problem is to find a model able to respond to the theoretical needs of celestial mechanics and also to describe correctly the orbits coming close to collisions, in order to match theory with observations. Such a model is that given by Manev's potential.

3 Brief history

The history of this force field is more than three centuries old. Isaac Newton himself considered it. In his *Principia* (Book I, Article IX, Proposition XLIV, Theorem XIV, Corollary 2), he showed that a force generated by a potential of the form $A/r + B/r^2$ (with positive A and B) entails a precessionally elliptic relative orbit. In other words, the trajectory of a point mass with respect to a fixed frame, originated in another point mass, is an ellipse whose focal axis rotates in the plane of the motion. Except for this result, Newton's research on such a model remained unpublished during his lifetime. However, the 1888 catalogue of the Portsmouth Collection of manuscripts - today famous among historians of science - revealed his interest in this subject. The reason was the impossibility of explaining the Moon's apsidal motion within the framework of his inverse-square force model.

Clairaut later reconsidered Newton's Manev-type model for the same reason: the impossibility to explain the Moon's perigee motion. Later, however, he found an argument within the framework of the classical Newtonian model and gave up his endeavours on the Manev-type law.

Poincaré himself questioned the validity of the Newtonian model, when, in a popular account on the stability of the solar system, published in 1898, he wrote: "One of the questions with which researchers have been most preoccupied is that of the stability of the solar system. This is, if truth be told, more a mathematical question than a physical one. Even if one were to discover a general and rigorous proof, one could not conclude that the solar system is eternal. It may, in fact, be subject to forces other than those of Newton." This came at a time when the world of physics was already seeking a new path, a search that would eventually lead towards the discovery of quantum mechanics and relativity.

In the twenties, Manev (Maneff in French or German spelling) proposed a gravitational model based on the $A/r + B/r^2$ potential, with $A = \mu$, $B = 3\mu^2/(2c^2)$ (μ = gravitational parameter of the field-generating body, c = speed of light). His arguments were based on the following facts:

In *Électricité et optique*, published in 1901, Poincaré had noticed that Lorentz's theory concerning the electrodynamics of moving bodies, on which the special relativity would be founded, failed to satisfy the action-reaction principle. In 1903, M. Abraham defined the quantity of electromagnetic movement, which helped F. Hasenöhrl prove the accuracy of Lorentz's contraction principle. In 1908, using this new quantity of electromagnetic movement, Max Planck stated a more general action-reaction principle, verified by the special relativity, and from which Newton's third law followed as a theorem. On the basis of these results, Manev showed that, applying the more general action-reaction principle to classical mechanics, one is naturally led to a law given by a potential of the type $A/r + B/r^2$

Manev considered that this model could be used as a substitute to general relativity. In fact, Diacu, Mioc and Stoica (2000) showed that in the so-called planetary approximation Manev's model is the natural classical analog of the Schwarzschild model.

Manev's model does not try to *replace* the relativity. It simply offers a *classical alternative* to it, more suited to the needs of celestial mechanics.

4 Modellable situations

Consider a more general Manev-type model, in which the field parameters A and B can take any real value. The dynamics of systems in such fields models a large class of concrete situations belonging mainly to astronomy, but also to (astro)physics, space dynamics, mechanics, even atomic physics. Let us give some examples.

The motion in certain relativistic fields, truncating the negligible terms, is such a situation. For instance, Fock's field has $A > 0$, $B > 0$, while the field described by Reissner-Nordström metric corresponds to $A > 0$, $B < 0$.

A photogravitational field generated by a radiative source is another example. In this case B can be zero or nonzero, as the gravitational component of the field is Newtonian or not. In turn, A can be positive, zero, or negative, as the Newtonian part of the gravitational force is stronger than, equal to, or weaker than the repelling radiative force. Moreover, considering that the luminosity of the source is not constant (the case of a variable star, for instance), we are in front of a perturbed Manev-type potential.

The Manev-type problems include, as particular cases, different well-known classical models (Newton's law, radiative force, force-free field).

For atomic physics, the potential energy of the outward electron in the field of the nucleus is given in a second approximation by a Manev-type model.

For mechanics, the Manev-type model with $A = 0$, $B > 0$ has many interesting properties. It is a bifurcation between two very different types of orbit behaviour. Also, in such a field, the rectilinear n-body problem is completely integrable. The time of simultaneous total collapse for the associated general n-body problem can be computed.

To end this series of examples, we resort to some supplementary astronomical arguments. In Manev-type fields, the natural, unperturbed motion of the celestial bodies is precessional, as relativity had foreseen. In such fields, the set of initial data leading to collisions has positive Lebesgue measure, and in the solar system the collisions are not so rare as the Newtonian model asserts. Finally, we mention a scenario concerning the formation of a pulsar from two white dwarfs that spiral in towards each other, eventually merging into a single object. Such a black hole effect (spiral collision) is possible in a Manev-type field, but not in Newton's field.

5 Some results

Diacu's (1993) study on the isosceles three-body problem in Manev's field constituted the departure point for the series of studies focusing Manev's model. For the two-body problem with this law, Mioc and Stoica (1995) used Sundman-type transformations to regularize the motion equations and found a general solution of the regularized equations. Diacu et al. (1995) obtained an analytic solution and the local flow near collision. Ureche (1995) used this field to an astrophysical problem: the free-fall collapse of a homogeneous sphere.

The Manev potential for arbitrary $A > 0$, $B > 0$ was also tackled. Lacomba, Llibre and Nunes (1991) studied it in the Hamiltonian formalism for negative total energy. They also applied the KAM theory to a perturbed potential of this kind to prove a crucial result: if the motion equations undergo a slight perturbation, not necessarily Hamiltonian, then most invariant cylinders and tori are topologically preserved. From the astronomical point of view, this means that the natural, unperturbed orbit of a celestial object around the Sun (for instance) is not an ellipse (parabola, hyperbola), but a precessional ellipse (parabola, hyperbola), the precessional effect becoming more evident the close the orbit is to the Sun. Under the perturbing action of the other celestial bodies, the orbit continues to be, in general, a precessional conic section. There are exceptions, but they are unlikely in the sense that the set of initial data for which they occur has measure zero.

The Melnikov integral associated to nonhyperbolic equilibria was computed by Casasayas, Fontich and Nunes (1993). Aparicio and Floria (1996) showed that only the class of Hamiltonians to which Manev's model belongs admits linearization (reduction to harmonic oscillators). Diacu (1996) proved that the Manev case represents the only bifurcation of the flow among all quasihomogeneous potentials. Stoica and Mioc (1996) depicted the problem geometrically in the usual phase planes, while Delgado et al. (1996) provided the complete analytic, geometric, and physical description of the global flow in the space of McGehee's coordinates.

The anisotropic Manev problem (important for understanding the connections between classical and quantum mechanics), suggested by Diacu (1993), was investigated by Craig et al. (1999). They obtained the local flow near collision, some elements of the global flow, and the complete picture of the zero energy case.

Mioc and Stavinschi tackled Maxwell's $(n+1)$-body problem (model for planetary rings) in a Manev-type field. They analyzed the stability of the configuration, and proved the existence of the Jacobi integral in the restricted problem associated to equilibria. They also studied the nonlinear stability of equilibria in the two-body problem, using the algorithm of Zombro and Holmes (Mioc and Stavinschi 1999). Mioc and Blaga (1999) tackled the rhomboidal five-body problem in Manev's field, and showed that a continuum of equilibria does not exist in this problem.

6 Specific dynamical features

The global flow of the Manev-type problem was fully described by Diacu, Mioc and Stoica (2000). They resorted to McGehee transformations of the second kind to regularize the motion equations and the first integrals of energy and angular momentum. Then, exploiting the rotational symmetry of the problem, the dimension of the phase space was reduced from 4 to 3, and clear pictures of the flow were obtained.

Performing a bifurcation analysis for the whole interplay among the field parameters, the angular momentum and the level of energy, the global structure of the phase space was understood and described. We shall emphasize only those dynamical features that cannot be met within the framework of classical models.

A very important characteristic is the motion on precessional conic sections (for nonzero angular momentum). We must point out here the fact that almost all motions on noncollisional precessional ellipses are quasiperiodic (except a set of measure zero of periodic orbits).

Another essential feature is the occurence of the black hole effect. The collisions are much more probable than in the Newtonian model, because the set of initial data leading to them has positive Lebesgue measure.

The existence of bounded orbits for nonnegative energy levels is unusual, too. There are stable circles for zero energy, or precessional parabolas and hyperbolas of the type ejection-collision.

We also have to mention the coexistence, for the same angular momentum and the same energy level, of the trajectories of the types ejection-collision and capture-escape.

The existence of unstable circular motion (for nonzero angular momentum) and unstable rest (for zero angular momentum) deserves to be mentioned, too. This generates radial or spiral motions of the types ejection-equilibrium, equilibrium-collision, equilibrium-escape, or capture-equilibrium, which cannot be recovered in classical models.

Another unusual motion is that performed on precessional hyperbolas that turn their convexity to the central body.

To end the list of these specific features, we have to emphasize the existence of the stable rest and that of radial librations.

The variety of motions and the large class of concrete astronomical and physical situations modellable in this way make the Manev-type problems deserve to be further studied.

Without trying to replace it, *Manev's theory seems more suited to the needs of celestial mechanics than the relativity.* Moreover, on the one hand, *the Manev-type models seem to build a bridge between classical and relativistic celestial mechanics*; on the other hand, *they bring a unifying standpoint for many classical astronomical, physical, and mechanical problems.*

7 Selective list of papers dealing with Manev's model

I. APARICIO, L. FLORIA, *On perturbed two-body problems and harmonic oscillators*, C. R. Acad. Sci. Paris, sér.IIb, **323** (1996), 71-76.

I. APARICIO, L. FLORIA, *A BF-regularization of a nonstationary two-body problem under the Maneff perturbing potential*, Extracta Math. **12** (1997), No. 3.

I. APARICIO, L. FLORIA, *Maneff-type problems in DEF-variables*, Rev. Acad. Ciencias Exactas, Fis.-Quim. Nat. Zaragoza **53** (1998), 233-248.

A. V. BOBYLEV, P. DUKES, R. ILLNER, H. D. VICTORY, *On Vlasov-Manev equations. I. Foundations, properties, and nonglobal existence*, J. Statist. Phys. **88** (1997), 885-911.

J. CASASAYAS, E. FONTICH, A. NUNES, *Transversal homoclinic orbits for a class of Hamiltonian systems*, in "Hamiltonian Systems and Celestial Mechanics", Advanced Series in Nonlinear Dynamics, Vol. 4, World Scientific, Singapore, 1993, pp. 35-44.

S. CRAIG, F.N. DIACU, E. LACOMBA, E. PEREZ, *The anisotropic Manev problem*, J. Math. Phys. **40** (1999), 1359-1375.

J. DELGADO, F.N. DIACU, E.A. LACOMBA, A. MINGARELLI, V. MIOC, E. PEREZ, C. STOICA, *The global flow of the Manev problem*, J. Math. Phys. **37** (1996), 2748-2761.

F.N. DIACU, *The planar isosceles problem for Maneff's gravitational law*, J. Math. Phys. **34** (1993), 5671-5690.

F.N. DIACU, *Near-collision dynamics for particle systems with quasihomogeneous potentials*, J. Diff. Eq. **128** (1996), 58-77.

F.N. DIACU, A. MINGARELLI, V. MIOC, C. STOICA, *The Manev two-body problem: quantitative and qualitative theory* , in "Dynamical Systems and Applications" (R.P. Agarwal, ed.), World Scientific Series in Applicable Analysis, Vol. 4, World Scientific, Singapore, 1995, pp. 213-227.

F.N. DIACU, V. MIOC, C. STOICA, *Phase-space structure and regularization of Manev-type problems*, Nonlinear Analysis **41** (2000), 1029-1055.

D. K. HRISTOV, M. K. TSVETKOV, V. E. TODOROV, K. P. TSVETKOVA, A. G. MANEV, M. I. DONEV, L. A. DASHOVSKA, S. D. STOIANOVA, F. DIACU, M. STAVINSCHI, V. MIOC, D. P. PARVANOVA, G. M. DESIMIROV, R. S. RADKOV, T. I. FIZYEVA, T. I. DAVIDOVA, I. L. DAVIDOV, N. H. MOLLOVA-GANOVA, E. I. DOBREVA, E. S. BRANKOVA, V. S. TENEVA, S. B. ZLATEVA, T. M. BUZOVA, M. N. APOSTOLOVA, H. B. DAREVA, *The Bulgarian Physicist-Theorist Georgi I. Manev (1884-1965). Rediscovery after Oblivion*, Physics on Stage, Sofia, 2000.

R. ILLNER, H. D. VICTORY, P. DUKES, A. V. BOBYLEV, *On Vlasov-Manev equations. II. Local existence and uniqueness*, J. Statist. Phys. **91** (1998), 625-654.

E.A. LACOMBA, J. LLIBRE, A. NUNES, *Invariant tori and cylinders for a class of perturbed Hamiltonian systems*, in "The Geometry of Hamiltonian Systems" (T. Ratiu, ed.), Proceedings of a Workshop held June 5-16, 1989, Springer-Verlag, New York, 1991, pp. 373-385.

V. MIOC, *Specific dynamical features of Manev-type problems*, Publ. Astron. Obs. Belgrade **60** (1998), 126-133.

V. MIOC, C. BLAGA, *A class of relative equilibria in the Manev five-body problem*, Hvar Obs. Bull. **23** (1999), 41-48.

V. MIOC, M. STAVINSCHI, *Nonlinear stability of equilibria of the Manev-type two-body problem*, Baltic Astron. **8** (1999), 411-424.

V. MIOC, M. STAVINSCHI, *On Maxwell's (n+1)-body problem in the Manev-type field and on the associated restricted problem*, Phy. Scripta **60** (1999), 483-490.

V. MIOC, C. STOICA, *Discussion et résolution complète du problème des deux corps dans le champ gravitationnel de Maneff (I)*, C. R. Acad. Sci. Paris, sér.I, **320** (1995), 645-648.

V. MIOC, C. STOICA, *Discussion et résolution complète du problème des deux corps dans le champ gravitationnel de Maneff (II)*, C. R. Acad. Sci. Paris, sér.I, **321** (1995), 961-964.

V. MIOC, C. STOICA, *Unperturbed trajectories in Maneff's gravitational field are ellipses in velocity plane*, Bull. Astron. Belgrade **152** (1995), 43-47.

V. MIOC, C. STOICA, *Solution of regularized equations for the two-body problem in Maneff-type fields*, Rom. Astron. J. **5** (1995), 165-170.

V. MIOC, C. STOICA, *Phase and physical orbits in Maneff's field*, Rendiconti Sem. Fac. Sci. Univ. Cagliari **66** (1996), 101-106.

V. MIOC, C. STOICA, *On the simultaneous total collision for particle systems with quasihomogeneous potentials*, Rom. Astron. J. **6** (1996), 165-169.

V. MIOC, C. STOICA, *Radial Motion with Zero Initial Velocity in Maneff-Type Fields*, Studia Univ. Babeş-Bolyai, ser. Mathematica, **42** (1997), No.3, 103-109.

V. MIOC. C. STOICA, *On the Manev-type two-body problem*, Baltic Astron. **6** (1997), 637-650.

V. MIOC, C. STOICA, *Lagrange-Jacobi and Sundman relations for a sum of homogeneous potentials*, Studia Univ. Babeş-Bolyai, ser. Mathematica, **42** (1997), No.4, 57-62.

R. MONTGOMERY, *The N-body problem, the Braid group, and action-minimizing periodic solutions*, Nonlinearity **11** (1998), 363-376.

E. PEREZ-CHAVELA, L. V. VELA-AREVALO, *Triple collision in the quasi-homogeneous collinear 3-body problem*, J. Diff. Eq. **148** (1998), 186-211

C. STOICA, *Bounded noncollisional orbits in generalized Maneff-type fields*, Rom. Astron. J. **5** (1995), 157-163.

C. STOICA, *Bounded noncollisional orbits in generalized Maneff-type fields (II)*, Rom. Astron. J. **7** (1997), 177-182.

C. STOICA, *The Generalized Maneff Two-Body Problem: Quantitative and Qualitative Theory*, Ph.D. Thesis, Astronomical Institute of the Romanian Academy, Bucharest, 1997.

C. STOICA, V. MIOC, *Radial Motion in Maneff's Field*, Studia Univ. Babeş-Bolyai, ser. Mathematica, **40** (1995), No.4, 99-105.

C. STOICA, V. MIOC, *Two-Body Problem for Maneff's Post-Newtonian Gravitational Field*, Rev. Roum. Sci. Techn., sér. Mécanique Appliquée, **41** (1996), 185-191.

C. STOICA, V. MIOC, *On the two-body problem in Maneff-type fields*, Bull. Astron. Belgrade **154** (1996), 1-8.

C. STOICA, V. MIOC, *The Maneff-type two-body problem in velocity plane*, Studia Univ. Babeş-Bolyai, ser. Mathematica, **42** (1997), No.4, 91-103.

C. STOICA, V. MIOC, *Near-escape orbits in Maneff-type problems*, Rom. Astron. J. **7** (1997), 183-188.

C. STOICA, V. MIOC, *Geometrical and physical description of the two-body problem in Maneff's gravitational field*, Bull. Astron. Soc. India **25** (1997), 217-225.

V. URECHE, *Free-fall collapse of a homogeneous sphere in Maneff's gravitational field*, Rom. Astron. J. **5** (1995), 145-148.

V. URECHE, *Gravitational redshift in Maneff's field*, Rom. Astron. J. **9** (1999), 119-124.