Rectangular torus dynamo model and magnetic fields in the outer rings of galaxies

E.A.Mikhailov, A.D.Khokhryakova
Faculty of Physics, M.V.Lomonosov State University, Moscow, Russia

---

XI Bulgarian-Serbian Astronomical Conference
Belogradchik, Bulgaria

Introduction

- Today it is no doubt that some galaxies have magnetic fields of several microgauss (Beck et al. 1996)
- Their existence has been proved using both observations and theoretical models.
- Some of the galaxies have outer rings. It is quite interesting to study the possibility of the magnetic field generation there.

Dynamo mechanism

- The generation of the magnetic field is described by dynamo mechanism.
- The dynamo is connected with transformation of energy from turbulent motions to magnetic field.
- This process is based on joint action of differential rotation and alpha-effect.

Basic equation

- The large-scale magnetic field evolution is described by Steenbeck – Krause – Rädler equation:

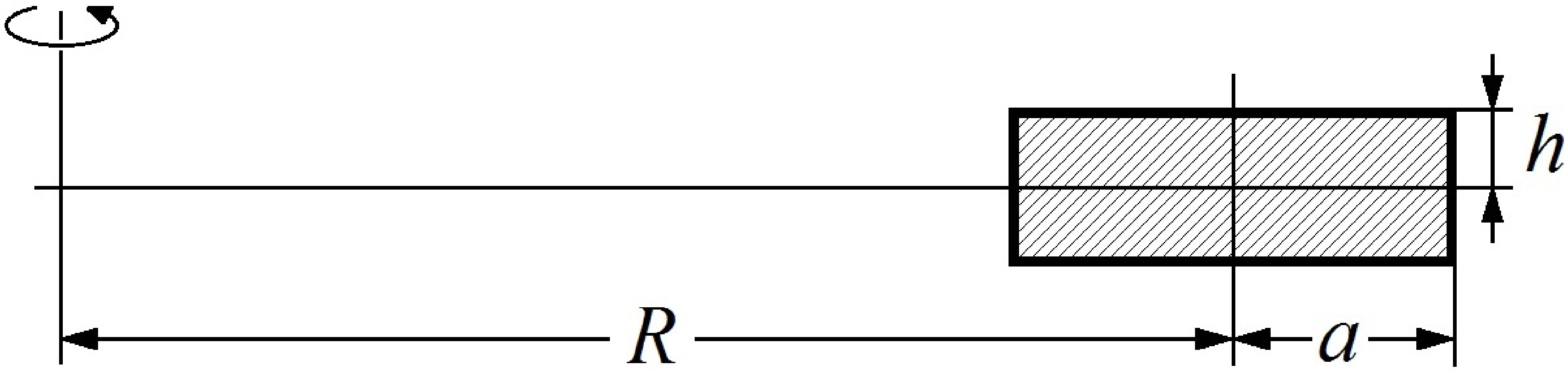
$$\frac{\partial \mathbf{B}}{\partial t} = \mathrm{curl}\left[\mathbf{V}, \mathbf{B}\right] + \mathrm{curl}\left(\alpha \mathbf{B}\right) + \eta \Delta \mathbf{B},$$

where $\mathbf{B}$ is the magnetic field, α characterizes the alpha-effect, η is the turbulent diffusivity coefficient, V is the large-scale velocity of the medium.

Models for the magnetic field

- Direct solution of the Steenbeck – Krause – Rädler equations is quite complicated.
- Usually some two-dimensional models for the magnetic field are used.
- As for galaxies, the equations are usually solved using so-called no-z approximation (Moss 1995).
- We can use this approximation for the outer ring (Moss et al. 2016).

The outer ring



No-z approximation

- The galaxy is quite thin, so we can assume that the magnetic field lie in the equatorial plane, so we can omit the equation for its z-component.
- Some of the derivatives of the magnetic field can be changed by algebraic expressions.

Equations of no-*z* approximation for outer ring

- If we measure the field in units of h^2/η, distances in galaxy radius R, the equations of the magnetic field will be:

$$\frac{\partial B_r}{\partial t} = -S_\alpha B_\varphi - \frac{\pi^2 k^2 B_r}{4} + \lambda^2 \frac{\partial}{\partial r}\left(\frac{\partial}{r\partial r}(rB_r)\right);$$

$$\frac{\partial B_\varphi}{\partial t} = -S_\omega B_r - \frac{\pi^2 k^2 B_\varphi}{4} + \lambda^2 \frac{\partial}{\partial r}\left(\frac{\partial}{r\partial r}(rB_\varphi)\right);$$

- where S_α characterizes alpha-effect, S_ω – differential rotation, $\lambda=a/R$ – thickness of the galaxy disc, $k=a/h,$ where a is the half-width in the galaxy, h is the half-thickness of the galaxy (Mikhailov 2018).

Nonlinear saturation

- The magnetic field growth should stop if its induction becomes close to the equipartition value B_{max}.
- It can be described by nonlinear modification of alpha-effect:

$$S_\alpha \to S_\alpha \left(1 - \frac{B_r^2 + B_\varphi^2}{B_{\max}^2} \right).$$

- It is quite convenient to measure the field in units of equipartition field.

Nonlinear equations

- The nonlinear equations are:

$$\frac{\partial B_r}{\partial t} = -S_\alpha B_\varphi \left(1 - B_r^2 - B_\varphi^2\right) - \frac{\pi^2 B_r}{4} + \lambda^2 \frac{\partial}{\partial r}\left(\frac{\partial}{r\partial r}\left(rB_r\right)\right);$$

$$\frac{\partial B_\varphi}{\partial t} = -S_\omega B_r - \frac{\pi^2 B_\varphi}{4} + \lambda^2 \frac{\partial}{\partial r}\left(\frac{\partial}{r\partial r}\left(rB_\varphi\right)\right).$$

- Usually the typical values of the parameters are $S_\alpha=1$, $S_\omega=10$, $\lambda=0.1$, $k=2$.
- We will solve the equations for values:

$$1-\lambda < r < 1+\lambda.$$

- At the boundaries we assume that $B=0$.

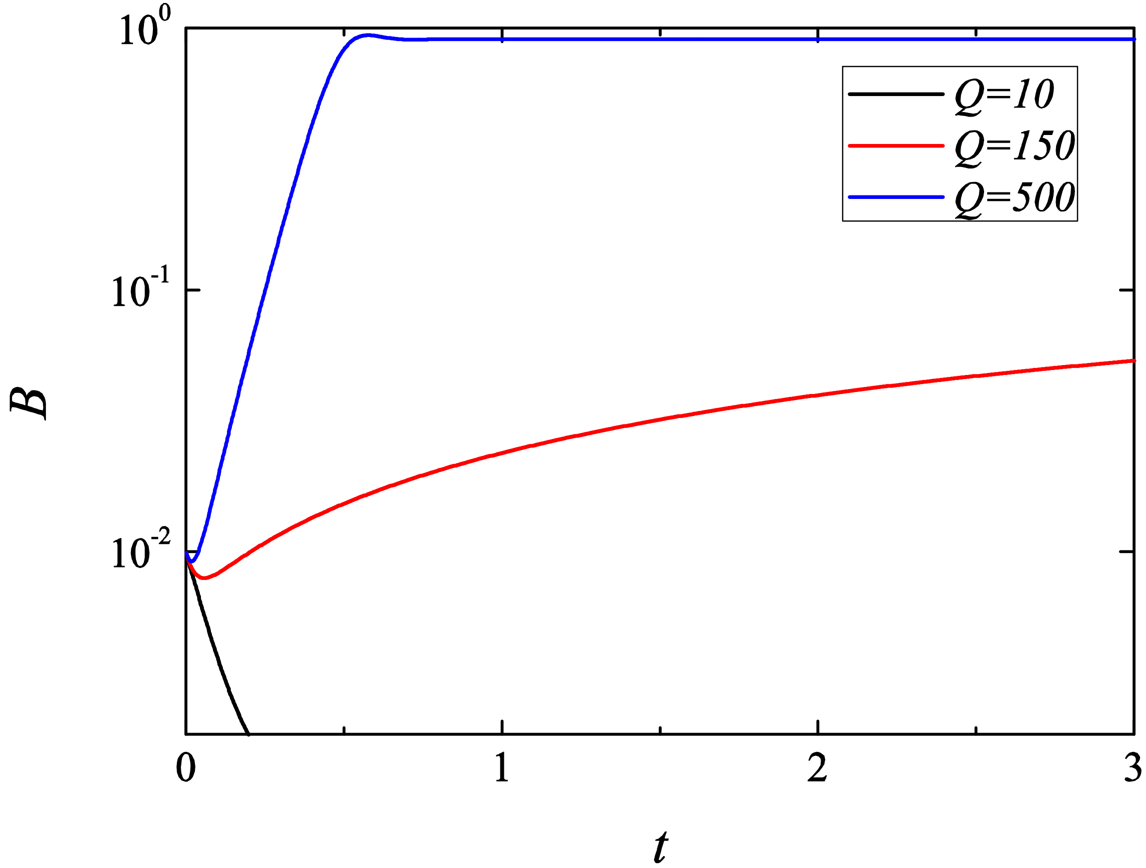
Magnetic field generation

- The possibility of the magnetic field generation is described by the dimensionless number:

$$Q=S_\alpha S_\omega.$$

- For higher values of Q the magnetic field grows faster, and for lower ones it grows slower or decays.

Magnetic field generation



Problems of the no-*z* model

- The no-z approximation was constructed for the main part of the galaxy, where the thickness of the galaxy disc is much smaller than its radial lengthscale.
- As for the outer ring, we should use the model where we take into account the z-component.

Torus dynamo models

- Previous works described the magnetic field using round torus dynamo model (Mikhailov 2017).
- However, it is better to take into account that the ratio between thickness and width of the ring can be different.

Rectangular torus dynamo model

- It is much more convenient to use the rectangular torus model, which takes into account these details.
- We will describe the axisymmetric model for the magnetic field which is described as a combination of toroidal magnetic field and vector potential of the poloidal field (Deinzer et al. 1993):

$$\mathbf{B} = B\mathbf{e}_{\varphi} + \mathrm{rot}\left(A\mathbf{e}_{\varphi}\right).$$

Equations for the magnetic field using torus approximation

- Using the same units, the equations for the magnetic field will be (Mikhailov 2017)

$$\frac{\partial A}{\partial t} = S_\alpha z B\left(1 - B^2\right) + \lambda^2 \Delta A;$$

$$\frac{\partial B}{\partial t} = S_\omega \frac{\partial A}{\partial z} + \lambda^2 \Delta B.$$

Boundary conditions

- We will solve the equations for values:

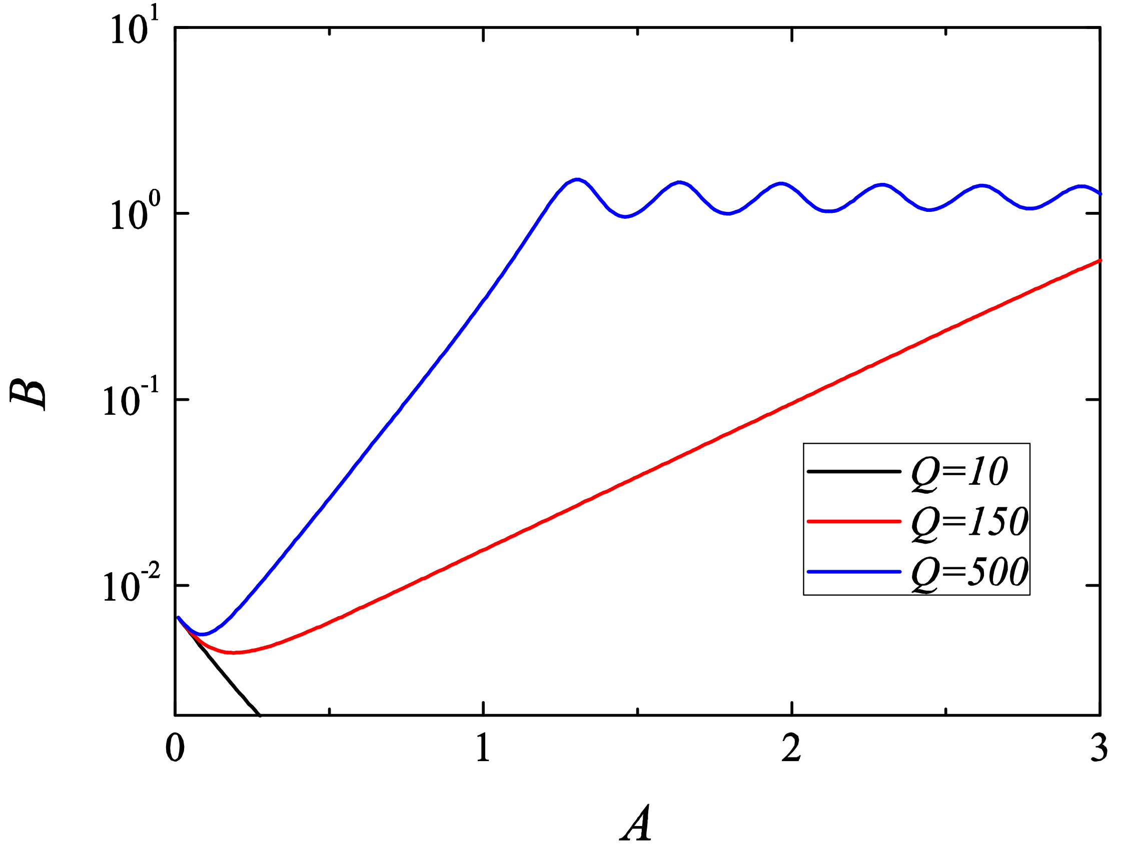
$$1-\lambda < r < 1+\lambda;$$

$$-\frac{\lambda}{k} < z < \frac{\lambda}{k}.$$

- At the boundaries we will use the conditions:

$$B = 0, \frac{\partial A}{\partial n} = 0.$$

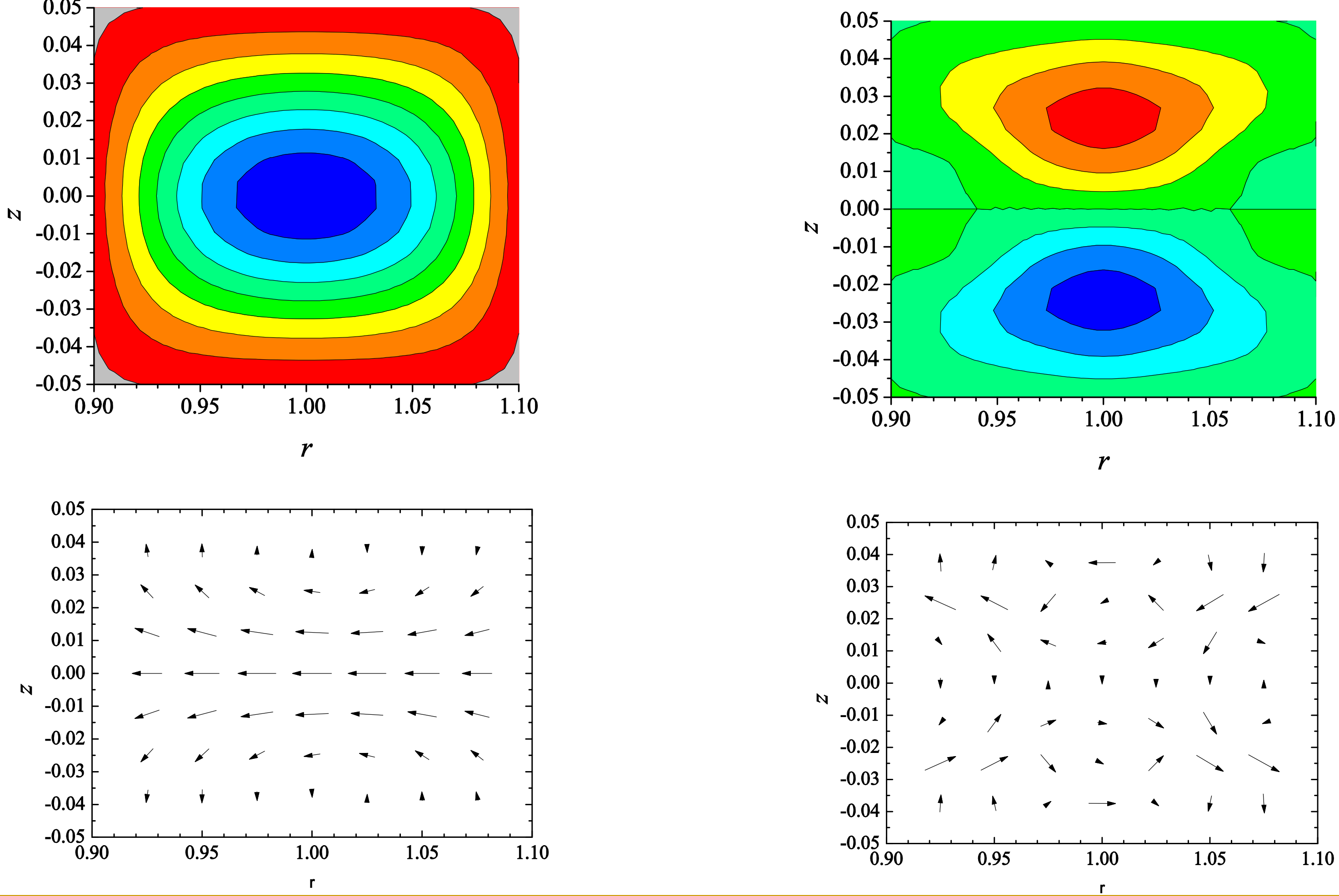
Magnetic field in rectangular torus dynamo



Dypolar and quadrupolar fields

- The magnetic field in the rectangular torus dynamo model grows slower than in the no-z approximation
- For high Q numbers the magnetic field in the outer ring can have not only the quadrupolar symmetry, it can also have the dypolar symmetry.

Quadrupolar and dypolar magnetic field (poloidal component)



Conclusions

- We have studied the magnetic field generation using rectangular torus dynamo.
- The magnetic field grows slower than in the no-z model.
- This model can describe the field with dypolar symmetry.

References

- R. Beck, A. Brandenburg, D.Moss et al., Ann. Rev. Astron. Astrophys., **34,** 155, 1996.
- D.Moss, Mon. Not. R. Astr. Soc., **275**, 191, 1995.
- D. Moss, E. Mikhailov, O. Sil’chenko et al., Astron. Astrophys., **592**, A44, 2016.
- E.Mikhailov, Astrophysics, **61,** 2, 2018.
- E.Mikhailov, Ast. Rep., **61**, 9, 2017.
- W. Deinzer, H. Grosser, D. Schmitt, Astron. Astrophys., **273**, 405, 1993.

THANKS FOR ATTENTION!