# Investigation of the spatial orientation of architectural monuments of Khorezm

S.B. Bolelov[1], G.Yu. Kolganova[2], M.G. Nickiforov[3,4]
[1] The State Museum of Oriental Art, Nikitskiy Bulvar 12a, 119019, Moscow, Russia
[2] Institute of Oriental Studies, ul. Rozhdestvenka 12, 107031, Moscow, Russia
[3] Sternberg Astronomy Institute of the Moscow State University, Universitetskiy pr. 13, 119991, Moscow, Russia
[4] Moscow State Linguistic University, Ostozhenka st. 38, 119034, Moscow, Russia
michael.nickiforov@gmail.com

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**Abstract.** In this paper, the spatial orientation of the structures of ancient and medieval Khorezm is investigated. The analysis carried out reveals two statistically significant directions: orientation along the meridian line and the azimuth of the sunrise on the day of the winter solstice. It is shown that the importance of the second azimuth is reflected in the cultural tradition of the region.

**Key words:** Khorezm, spatial orientation, astronomy, culture.

## Introduction

The purpose of this paper is to investigate the astronomical orientation of the architectural monuments of Khorezm. By the term astronomical orientation, we mean the alignment of the elements of buildings (external defensive wall, citadel) on the astronomically significant directions: line of the meridian, the azimuths of the sunrise and/or sunset in the days of the solstice and azimuths of the rising/setting of the Moon in the extreme northern and southern states.

The question of spatial orientation study of architectural monuments has been considered in many publications. However, few of them have generalizing character. The greatest interest is M.S. Bulatov's (Bulatov 1978, p. 21-66.) sampling which consists of 36 objects belonging to different cultures: Mesopotamia, Egypt, Greece, Central Asia and India. Moreover, the objects in his sample also belong to different periods of time starting with Sumerian temples (IV millennium BC) and ending with Indian monuments of the early Middle Ages (VII century AD). The performed analysis shows the connection between architecture and astronomy in a variety of cultures, however, it does not allow us to draw more certain conclusions about a specific region. In addition, M.S. Bulatov used archaeological plans drawn up by third parties, so we do not have any information about the accuracy of azimuth determination (For example, a compass of Andrianov's system has an error up to 5 degrees.) and which of the meridians (magnetic or true) is indicated on the plan.

It should be noted that the detection of astronomically significant azimuth on one separate building may be a coincidence. In case we have several dozens of monuments with random orientation and the measurement error is about 2-3 degrees, the orientation of at least one of them with a large probability will correspond to one of the astronomically significant directions. It is clear that for each region one can select several monuments with a certain astronomical orientation, and then combine them into one set. Consequently, our task consists in the justification of the result and that can be done by statistic methods. If a certain azimuth is detected on a homogeneous group of monuments many times, this means that such an orientation is not accidental. Term "a homogeneous group of monuments" means a group in which all objects belong to a particular culture and specific chronological period.

Professor S.P. Tolstov was one of the first who paid attention to the description of the spatial orientation of the monuments of Khorezm (Tolstov 1948.). However, this question was a very secondary matter for him, so all his estimates are approximate.

## 1 Investigation of errors

The analysis of the accuracy of archaeological plans showed that it is impossible to use them for solving the problem of determining spatial orientation (Kolganova and ect. 2014, p. 26-30.). The comparison of different archaeological plans of the same building with each other and with satellite images revealed the difference in the orientation of the meridian line from several to a dozen degrees. The main reason for this situation is the lack of unification of the true north. The verification showed the presence of three different variants. The magnetic north is indicated in the first set of plans, the true north is present in another one and in the third group, we should assume an error in the sign of the magnetic inclination, which probably appeared when the magnetic meridian was recalculated to the true meridian. This can be argued with great confidence, because the value of the residual is equal to doubled magnitude of the magnetic declination. Finally, in some cases, the deviation from the true north is so great that it can only be explained by the presence of a gross error of an unknown character. The unsuitability of earlier archaeological plans for the analysis of spatial orientation badly limits the number of the objects that can be used for statistical analysis.

First, many monuments described in the works of the last century do not exist in our time. Yakke-Parsan's and Berkut-Kalas's oases were excavated and converted into collective farm fields, though the Yakke-Parsan and Berkut-kala themselves exist in present time. During the construction of hydroelectric units, some territories were flooded. The most famous monuments such as the fortress of Kaparas and the religious center Elharas are in the zone of flooding. Thus, at present, the number of monuments for analysis is smaller than in the 50's and 70's of the last century, when they were investigated.

Therefore, due to the impossibility of conducting new measurements, we have used satellite imagery of Google Earth (GE). This decision has obvious advantages, but at the same time it leads to some costs. The quality of the pictures depends on the terrain and it does not always allow us to find a monument in the image. For example, the monuments on the right bank of Khorezm, located near settlements and roads, are photographed at good resolution. The photographs of the left bank monuments, located in the desert (Kalaly Gyr-2, Gyur-kala-3, Deu-kala, etc.), were taken (for 2015) at a low resolution. In addition, there are badly damaged structures, poorly visible from cosmos even at a high resolution of the picture. Both of these factors reduce the number of monuments that can be used for analyzing.

As a result, for the study of spatial orientation we selected 37 objects, which have the shape of a convex polygon. The azimuth of the main axis of Koy-Krylgan-kala was added to this set on the assumption that it may be associated with the heliacal rising of the Pleiades (Kolganova and ect. 2014; Bolelov and ect., 2015.). We shall assume that the selective totality reflects the properties of the general one, which at present time is not available for research due to the above-mentioned reasons.

For each monument it is necessary to find a set of azimuths (or directions) that characterizes the spatial orientation of its walls. We use the term azimuth to mean the angle formed by the straight line passing through the walls of the monument and the meridian line. For rectangular shaped buildings we can determine four azimuths forming two pairs connected by an obvious relation: $A_{i+2} = A_i + 180°, i = 1, 2.$. Therefore, for further analysis, we shall leave only two of the four directions.

In the image analysis process, there are two main sources of errors, which lead to deviations of azimuth values. The error of the first kind is common for both the satellite image processing and the measurements conducted in the field terms. Another error is common only for the process of image processing.

The first type of error is related to the monument itself and depends on the degree of destruction of the building and/or its structural features. Obviously, that azimuths on a well-preserved objects such as Ayaz-kala 1, 3, Dzhanbas-kala can be defined much more accurately than that on partially destroyed buildings of Kulbascan-Kala 1, 2. It is also necessary to take into account that some of the well-preserved monuments that we have selected for the analysis have specific features of the layout. For example, the western and eastern walls of Kalala-Gyr-1 and the northern wall of Ayaz-kala 3 are deviating from the straight line, which can be either because of the adaptation of wall constructions to the features of the local relief or due to the errors, which were made during the construction. However, if the magnitude of the maximum deviation of the wall from the straight line is small relative to the length of the wall, then the azimuth can be found out, however, the accuracy of its determination will be lower than in the standard case.

The second error is caused by the quality of the image being dependent on the ratio of the geometric dimensions of the monument $L$ to the size of the smallest details that are visible in the image $l$. Deserted territories remote from large cities and infrastructure objects are often photographed at low resolution when $l_{desert} \gg l_{town}$. Therefore on such satellite images large details are significantly blurred and small ones are invisible at all. A larger ratio of $L/l$ corresponds to a higher accuracy of azimuth determination. The presence of this error leads to the fact that the spatial orientation of a compact monument with dimensions of 20-30 m (the citadel of Adamli-kala, Castle 2) can be determined with low accuracy even for relatively good image quality.

In addition, we have included into this category extra errors associated with lighting. In two identical images of the same monument, obtained at different times, the position of shadows falling from its walls will differ.

**Table 1.** Evaluation of image quality using code.

| Code | Legend |
|---|---|
| B1 | The wall is well preserved. It allows finding out the azimuth accurately. |
| B2 | The wall is partially destroyed, but its individual fragments make it possible to detect the azimuth confidently, for example, by using towers of walls. |
| B3 | The remains of the wall and image quality allow determining azimuth with satisfactory accuracy. |
| H1 | The remains of the wall allow determining the azimuth with low precision. |
| H2 | The wall or its fragment has small geometric dimensions; the azimuth can be defined with low precision. |
| H3 | The wall has significant curvature or the wall noticeably uneven, but the azimuth can be estimated from individual fragments or extreme towers with low precision. |
| H4 | A poor quality of the satellite image; the azimuth is defined with low precision. |
| X1 | The wall significantly destroyed; the azimuth cannot be defined. |
| X2 | The wall has an incorrect shape or other features; the azimuth cannot be defined. |
| X3 | The wall is not visible on the image; the azimuth cannot be defined. |

This may affect the definition of azimuth, especially when the monument is poorly preserved and the image quality is low.

Ground measurements can eliminate the error of the second type, or it can be reduced in the future when new satellite images of the corresponding territories of better quality are available. Evaluating azimuths it is necessary to take into account all characteristic types of errors as far as possible, so for each azimuth we have entered an alphanumeric code characterizing its quality (see Table 1).

The azimuths were determined from a pair of parallel walls, if these walls had the same alphanumeric cipher corresponding to equal quality of the azimuth estimate. In other case, we used the wall that has a better-quality cipher. The quality of ciphers is located in the following order: $B1 > B2 > B3 > H1 = H2 = H3 = H4$.

The ranking of ciphers is an attempt to give a qualitative estimate of the error, which we make in the process of azimuth evaluation. Such estimating "by eye" is somewhat subjective, however, it allows us confidently to order the azimuths by accuracy within one monument and choose from them the best one for the next analysis. Then, for further work, it is necessary to obtain a quantitative estimate of the error.

According to our previous calculations (Kolganova and ect. 2014, p. 26.), using standard tools of Adobe Photoshop program allows to determine the given azimuth on the GE image with an error $\varepsilon \approx 1.7°$ at confidence interval of $1\sigma$.

However, this evaluation was obtained on the azimuths of different quality. Now, after we have introduced the classification of azimuths depending on accuracy, it is possible to refine the result. Our aim is to determine the value of the error corresponding to each of the ciphers. Let us assume that the walls of all the monuments of the quadrangular form are strictly

**Table 2.** Evaluation of the deviation of walls from parallelism. Here we assume that the walls are strictly parallel, and the deviation from parallelism is our image processing error.

| | **Monument** | $\mathbf{A}_1$ | $\mathbf{A}_2$ | —$\Delta$**A**— |
|---|---|---|---|---|
| 1 | Aijr-kala | 30.0 | 30.0 | 0.0 |
| 2 | Angka-kala | 135.5 | 135.5 | 0.0 |
| 3 | Ayaz-kala-1 | 75.0 | 76.0 | 1.0 |
| 4 | Dzhanbas-kala | 155 | 156 | 1.0 |
| 5 | Dzhanbas-kala | 64.5 | 64.0 | 0.5 |
| 6 | Kalaly-Gyr-1 | 101 | 105 | 3.5 |
| 7 | Kazakli-Yatkan | 84.5 | 84.0 | 0.5 |
| 8 | Kosh-parsan | 84.5 | 84.0 | 0.5 |
| 9 | Kosh-parsan | -4.5 | -5.0 | 0.5 |
| 10 | Kyzyl-kala | 141.5 | 142.5 | 1.0 |
| 11 | Kyzyl-kala | 53.0 | 53.0 | 0.0 |
| 12 | Kyrkyz (Large) | 72.5 | 72.0 | 0.5 |
| 13 | Pil-kala | 90.5 | 90.0 | 0.5 |
| 14 | Vazir-rabat | 100.0 | 100.0 | 0.0 |
| 15 | Vazir-rabat | 2.5 | 2.0 | 0.5 |

parallel (We assume that the angle characterizing the "non-parallelism" of the walls is much smaller than the estimation error, which we make. Those cases, when this assumption is obviously not fulfilled, will be considered separately.), and the deviation from the parallelism of the azimuths of the opposite walls is explained only by the errors of the 1st and 2nd type which were described above. Then from the pairs of the corresponding azimuths one can compose four groups. The first group will include pairs of azimuths with codes "B1"-"B1", the second group - with codes "B2"-"B2", the third group - with codes "B3"-"B3", in the fourth - "H"-"H". For example, we presented the list of azimuths from the first group (see Table 2).

According to the given data, the average deviation of the walls from the parallel is $\delta_1(15) = 0.63 \pm 0.39$ for the first group. The largest discrepancy corresponds to the monument Kalaly-Gyr-1, however, the non parallelism of its northern and southern walls is clearly visible in the photo even with the naked eye. We assume that in this case the deviation of the walls from the parallel is determined by the structural feature of the monument but not by the error of our measurements. If we exclude Kalaly-Gyr-1 from consideration, we shall obtain the following value $\delta_1(14) = 0.42 \pm 0.87$. In the remaining groups, the mean deviation values are: $\delta_2(7) = 0.86 \pm 1.03$, $\delta_3(3) = 1.67 \pm 1.53$, $\delta_4(8) = 2.19 \pm 1.56$. At the same time, the small volume of the second and the fourth sets and a very small volume of the third group are striking evident. The discrepancy corresponding to Uj-kala can be excluded from the second group on the same basis as for Kalaly-Gyr-1. The satellite image shows that the western wall noticeably deflects from the meridian line, which passes with good accuracy along the eastern wall. The statistical hypothesis about the equality of the mean deviations in the first and the second groups does not allow us to identify significant differences between them at confidence level of $\alpha = 0.05$. This result remains stable in both cases whether we add Kalaly-Gyr-1 and Uj-kala to the set or not. This means that based on the available information, we can conclude that the accuracy of determining the azimuth in the first two groups is the same. Moreover, despite the difference in the pairs of values $(\delta_1, \delta_3)$, $(\delta_3, \delta_4)$, no significant statistical differences can be revealed between the mean values of the first and third groups, as well as the third and the fourth ones. This is due to the small number of the third group consisting of only three elements.

According to our calculations, there is a significant statistical difference between the first and the fourth groups at $\alpha = 0.05$, and between the second and the fourth ones at $\alpha = 0.10$. If we combine the observations of the first and the second groups into a single set, we will get: $\delta_{12}(22) = 0.70 \pm 0.90$ . Next, if we exclude from this set Kalaly-Gyr-1 and Uj-kala which have significantly large discrepancies from all the others, we'll get $\delta_{12}(20) = 0.45 \pm 0.39$.

Thus, the monuments with ciphers "B1" and "B2" correspond to the accuracy of azimuth determining a little less than 1 degree; however, we take this value for error evaluation. The azimuths with the letter "H" correspond to the accuracy of about 2 degrees. The code "B3" is somewhere between these two estimates, and, probably, we do not make a significant mistake if we take an average value.

It would be very useful to check precision of our evaluations. For this, it is necessary to compare the value of the azimuth determined during the ground-based measurements with the value obtained during the processing of the satellite image. Unfortunately, we have the only reliable ground measurement, which relates to the monument of Koy-Krylgan-kala (THAEE 1967, p. 23). We know the value of the magnetic azimuth and the value of the magnetic declination for this area. Although the accuracy of ground-based measurements is not indicated, the value of error does not exceed 10' even if the simplest theodolite of the T-30 model was used. Correct recalculation of the magnetic azimuth to the true azimuth gives the value of the direction of the main axis of the building $A = 80°$. This value exactly coincides with the azimuth that we obtained during processing of the satellite image (Bolelov and ect 2015, p. 182-183.). Such a coincidence may seem "absolutely accurate"; however, it is necessary to take into account evaluation error for each of the quantities. It can be assumed that the error does not exceed 10' for ground-based measurements, and we rounded the azimuth to 30' in our estimates. Therefore, it can be argued that both measurements of the azimuth correspond to each other within the limits of the error, and our evaluations of accuracy are quite reasonable. Note that the azimuth of Koy-Krylgan-Kala has the code "B2", which corresponds to high accuracy of the measurements.

## 2 Analysis of the spatial orientation of Khorezm monuments

Let us consider the spatial orientation of architectural buildings of the ancient (from the 4th century BC) and early medieval Khorezm (up to the

10th-11th centuries AD). Although the time interval of 1.5 millenniums may seem too long, we assume, as an initial hypothesis, that the building traditions (until a certain time) do not undergo significant changes. A priori, this hypothesis is confirmed statistically by analyzing an average distance between the towers located along the perimeter of the walls (Bolelov and ect 2016, p. 137-138.). If our assumption is erroneous, it can be established during the analysis.

The results of comparison of astronomically significant directions with the directions that we detected are given in Table 3.

In this calculation, we used the magnitude of the possible detuning (deviation) from the exact azimuth value equal to $|\Delta| \leq 3.4°$. It corresponds to the error in determining the angle from the satellite image taken at the level $2\sigma$. A total of 23 out of 38 (or 61%) monuments has a spatial orientation corresponding to 7 astronomically significant directions. While calculating the number of azimuths, we took into account that the orientation along the meridian line (the astronomical north) is closely correlated with the orientation toward the equinox east. This is because the geometric shape of the vast majority of monuments is close to rectangular, so if one wall targets the astronomical north, then the perpendicular wall will be directed along the east-west line.

According to Table 3, we have obtained that 10 monuments are oriented to the astronomical north and 6 along the east-west line. In the last list, we "have lack" of 4 azimuths corresponding to Ayaz-kala-3, Vazir-Rabat, Kulbaskan-kala 1 and 2. Their absence is explained by the following reasons. The Kulbaskan-kala-1 walls, located along the parallel, are not visible on satellite images, so we cannot establish their direction. The shapes of Vazir-Rabat and Kulbaskan-kala-2 differ slightly from rectangular, which is probably related to the error in the construction. As a result, the azimuths of the monuments $A_2(Vazir-Rabat) = 100°$ and $A_2(Kulbaskan-2) = 94°$ oriented towards the east, do not fall within the required range $90° \pm 3.4°$. Finally, Ayaz-kala-3 is a parallelogram that cannot simultaneously correspond to both azimuths. Thus, the first two lists give us 10 monuments.

To determine the statistical significance of azimuths from Table 3, we need to estimate the cutoff threshold, i.e. to find the frequency of the characteristic, the magnitude of which will be statistically significant. The simplest way of evaluation is as follows. Let us suppose that the azimuths are uniformly distributed around the circumference. Four azimuths correspond to a rectangular shape, so 90 degrees of circumference contain one azimuth. According to our evaluation the error of azimuth measurement is equal to 3.4°, it corresponds to the width of the interval 6.8°. Then the probability of a random hit of the azimuth in a given interval is $p = 6.8/90 \approx 0.076$ or 7.6%.

Using the binomial distribution law, we obtain that for $n = 38$ tests the probability of accidentally falling 10 times or more into a given range 6.8° is $p(k \geq 10) = 0.04\%$. This means that the azimuth of the true north with frequency $f_1 = 10$ is significant at $\alpha = 1\%$.(*According to Appendix 2, the number of azimuths that we associate with one of the significant directions from Appendix 3 is equal to 75. This number is obtained as follows. One azimuth corresponds to spatial orientation of the Koy-Krylgan-Kala and Kulbaskan-kala-1. Large Guldursun has a trapezoidal shape, so we have*

*3 directions for it, and for all other monuments we obtain -* $35 \cdot 2 = 70$ *azimuths. However, we have to note that the azimuths of buildings, which shape is close to the rectangle, are related by* $A_{i+1} = A_i + 90^\circ \pm \varepsilon$*, where* $\varepsilon$ *is a small error, probably made during construction. Therefore, if one of the azimuths hit a predetermined interval with a solution of* $6.8^\circ$*, the azimuth that forms a pair with it cannot in principle reach neither the same interval nor its vicinity. In fact, the situation when two astronomical directions are present on the same building is never realized. The only exception is orientation to the true north which correlated with orientation to the equinox East, however, we have considered this case. Consequently, we shall not make a great error if we assume that one azimuth for each monument will hit 90-degree sector. So, for average case we have* $n = 35 + (1 + 1 + 3)/2 \approx 38.$*)*

| **Orientation along meridian line (true north), A=0°.** | | | | |
|---|---|---|---|---|
| Number of buildings: n = 10. Rate: 25.6%. $\bar{A} = -0.05°$. $\sigma = 1.47°$ | | | | |
| **Name** | **A** | $\Delta$**A** | **Dating** | **Cipher** |
| Ayaz-kala-3 | -2.0 | -2.0 | IV-III cen. BC | B1 |
| Dzhanpik (citadel) | 0.5 | 0.5 | IX-X cen. | B1 |
| Hazarasp | -0.5 | -0.5 | IV-II cen. BC | B1 |
| Homestead-1[1] | -2.75 | -2.75 | IX-XII cen. | H1 |
| Kavat-kala | 0.0 | 0.0 | VI-VIII cen. | B3 |
| Kulbaskan-kala-1 | 0.25 | 0.25 | VI-VIII cen. | H1 |
| Kulbaskan-kala-2 | 0.25 | 0.25 | VI-VIII cen. | H1 |
| Pil-kala | 0.0 | 0.0 | IV cen. BC | B1 |
| Uj-kala | 1.5 | 1.5 | VI-VIII cen. | B2 |
| Vazir-rabat | 2.25 | 2.25 | IV-II cen. BC | B1 |
| **Orientation to the equinox east, A=90°.** | | | | |
| Number of buildings: n = 6. Rate: 15.4%. $\bar{A} = 90.50°$. $\sigma = 0.96°$ | | | | |
| Hazarasp | 89.0 | -1.0 | IV-II cen. BC | B1 |
| Homestead-1[1] | 91.75 | 1.75 | IX-XII cen. | H1 |
| Dzhanpik (citadel) | 91.0 | 1.0 | IX-X cen. | H2 |
| Kavat-kala | 90.0 | 0.0 | VI-VIII cen. | B3 |
| Pil-kala | 90.25 | 0.25 | IV cen. BC | B1 |
| Uj-kala | 91.0 | 0.0 | VI-VIII cen. | B1 |
| **Azimuth of sunrise at the day of WS, A=121.5°.** | | | | |
| Number of buildings: n = 7. Rate: 17.9%. $\bar{A} = 120.64°$. $\sigma = 1.97°$ | | | | |
| Aijr-kala | 122.0 | 0.5 | V-VII cen. | B1 |
| Castle-2[2] | 119.0 | -2.5 | IV cen. | H2 |
| Castle-60[3] | 120.0 | -1.5 | V-VIII cen. | B2 |
| Guldursun (Large)[4] | 119.0 | -2.5 | IV-II cen. BC | B1 |
| Kandym-kala | 120.0 | -1.5 | IV-II cen. BC | B2 |
| Kazakli-Yatkan | 120.0 | -1.5 | IV-III cen. BC | B1 |
| Teshik-kala | 124.5 | 3.0 | VI-VIII cen. | H2 |

| Name | A | ΔA | Dating | Cipher |
|---|---|---|---|---|
| **Azimuth of sunrise at the day of SS, A=57°.** Number buildings: n = 1. Rate: 2.6%. | | | | |
| Kurgashin-kala | 55.0 | -2.0 | IV-II cen. BC | H4 |
| **High Moon nothern azimuth, A=49°.** Number of buildings: n = 1. Rate: 2.6%. | | | | |
| Kumbaskan-kala | 49.0 | 0.0 | VII-VIII cen. | B2 |
| **High Moon southern azimuth, A=129°.** Number of buildings: n = 0. Rate: 0.0%. | | | | |
| **Low Moon nothern azimuth, A=64°.** Number of buildings: n = 1. Rate: 2.6%. | | | | |
| Dzhanbas-kala | 64.25 | 0.25 | IV cen. BC | B1 |
| **Low Moon southern azimuth, A=114.5°.** Number of buildings: n = 2. Rate: 5.3%. | | | | |
| Atsyz-kala | 112.0 | -2.5 | VI-VIII cen. | B3 |
| Bazar-kala | 114.5 | 0.0 | IV-II cen. BC | B1 |
| **Azimuth of heliacal rising of the Pleiades, A=78.5°.** Number of buildings: n = 1. Rate: 2.6%. | | | | |
| Koy-Krylgan-kala | 80.0 | 1.5 | IV-III cen. BC | B2 |

Table 3. Grouping of monuments along astronomically significant directions. Some examples are shown in the Appendix 1.

Legend: abbreviations SS and WS respectively mean the summer and winter solstice.

1. The monument was identified by triangulation method using the GE program and a map-scheme of B.V. Andrianov (Andrianov 1969, interstitial insertion, pp. 141-145).
2. This identification was made by S.B. Bolelov, who worked on this monument.
3. Castle №60. The monument was identified by triangulation method using the GE program and the map-scheme of the Berkut-kalas's oasis from the work of E.E. Nerazik (Nerazik 1966).
4. The walls of the fortress were rebuilt in the Middle Ages, however, they are located on the foundation of the ancient period.

We shall deal with the azimuth of the sunrise in the winter solstice with the frequency $f_2 = 7$. Similarly, we shall find out that the probability of falling within a given interval 7 times or more is equal to $p(k \geq 7) = 2.28\%$, which is significant at $\alpha = 5\%$. Note that in this calculation we do not take into account the conditional probability, i.e., the fact that 10 of the 38 azimuths already correspond to the orientation to the true north.

Another way of getting estimation is to assume that success is when the azimuth corresponds to one of the two given directions. Then, for the probability of a successful outcome in one experiment, $p = 2{\cdot}6.8/90 \approx 0.151$ and $n = 38$ trials, $p(k \geq 17) = 0.0013\%$. Note that this way of estimation can be applied only if the frequencies $f_1$ and $f_2$ are sufficiently large and close to each other in magnitude. Thus, the azimuth of the sunrise in the winter solstice with frequency $f_2 = 7$ is also statistically significant.

So, astronomically significant azimuths are found both in ancient and early medieval monuments. This confirms our assumption that the building tradition was stable and did not change with the passage of time.

Using the cipher makes it possible to estimate the accuracy with which we determine the azimuth. If the image quality evaluation is not worse than "B2", then the error of finding the azimuth is about $\varepsilon \approx 1.7^\circ$. For other ciphers, the error may be higher, in this set there are four such ciphers.

We have done a verification of only astronomical azimuths so far, now it is necessary to check the presence of other directions that can be statistically significant. To do this, let us take a sliding window with a width of 6.8 degrees (The width of the window is equal to the doubled value $\varDelta A = 3.4^\circ$.) and calculate for each direction the integral frequency, sequentially scanning all the azimuths in the range $[-15^\circ; 195^\circ]$. Our aim is to find the number of azimuths that fall in this window. The result of this calculation is shown in Fig. 1.

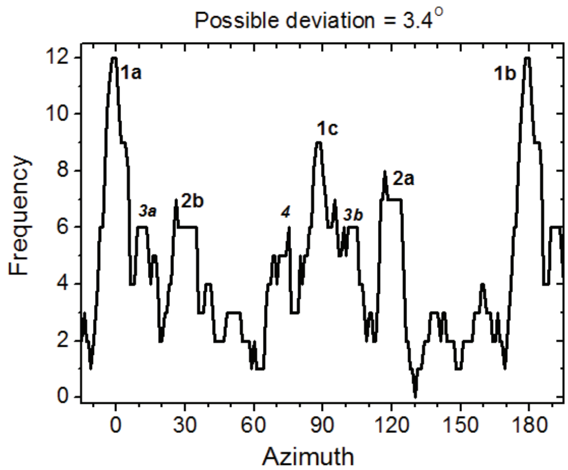



**Fig. 1.** Result of azimuths searching.

In Fig. 1 each of the local maximum corresponds to a certain direction. Let's consider the maxima in descending order. The first group of azimuths is formed by the peaks 1a, 1b and 1c with intensities $f_{1a} = f_{1b} = 11$ and $f_{1c} = 7$. The first two directions correspond to the orientation of the walls of the structure along the meridian line. The maximum matches to orientation to the East. It is obtained automatically if one pair of walls of rectangular building is oriented to the true north. The last requirement is fulfilled for the vast majority of constructions, and there are relatively few exceptions such as Ayaz-3 or Large Guldursun. In addition, we draw attention to the fact that the maxima of 1a and 1b are somewhat narrower than 1c. This can be explained by the fact that during the construction of the monument, the meridian line was chosen as the baseline for the marking, and the perpendicular to the west-east line was obtained from this line through geometric transformations. Since some errors appear during the markup process, they add up with the errors of determining the position of the north. We make one error during the procedure of calculation the west-east line and the second one appeared because of the fact that the meridian line was found inaccurately. As a result, the west-east line is defined with less precision than the north-south line, which we see from the figure.

The maximum 2a ($f_{2a}$=7) corresponds to the azimuth of the sunrise in the day of winter solstice, and the extreme 2b ($f_{2b}$=6), separated from it by 90 degrees, is a secondary. It is due to the fact that the shape of most monuments is close to a rectangle. As in the previous case, the intensity of the maximum of 2b is less than the intensity of the maximum of 2a, which has a larger width. (We compare the intensities of the signals at half-height.). This means that the builders oriented the monument along the azimuth corresponding to the maximum of 2a, which is astronomical.

The pair of maxima 3a and 3b connected with each other through the angle of 90 degrees, $A_{3a} = 11°$, $A_{3b} = 103°$ have the same intensity $f_{2a} = f_{2b} = 6$, however, neither of them can be identified with the motion of the Sun or the Moon. Knowing all input data, we can determine what monuments form this pair of peaks. They are Bederkent-kala ($A = 13.0°$), Berkut-kala ($A = 14.25°$), Duman-kala ($A = 14.0°$), Kalaly-gyr-1 ($A = 8.0°$), Mizdakhan ($A = 12.0°$) and Yakke-Parsan ($A = 9.0°$). In this group, the averaged azimuth value is $\bar{A} = 11.7°$ and the mean square deviation of $\sigma = 2.7°$. Relatively high standard deviation allows us to assume that these monuments are collected together accidentally. The values of azimuths suggest that this group can be divided into two subgroups, in each of which $\sigma$ does not exceed 1.0. However, in this case, both subgroups become so small that they are not suitable for statistical analysis.

The maximum 4 corresponds to the averaged azimuth $\bar{A} = 71.7$ (at $\sigma = 2.3°$) and its intensity is equal to $f_{2b} = 5$. The peculiarity of this case is that 3 out of 5 azimuths refer to the closely located monuments belonging to the same complex. Ayaz-1 ($A = 75.5°$), Ayaz-2 ($A = 70°$), Ayaz-3 ($A = 70.5°$), Large Kyrkyz ($A = 72.25°$), Toprak-kala ($A = 70°$). Although the monuments of Ayaz's complex belong to different time, according to archaeological data, new structures were built taking into account the spatial orientation of the old ones. For example, the walls of the Ayaz-3 monument were specially built around more ancient rectangular monument for the purpose of its protection. In this case, most likely, we cannot consider the azimuths of Ayaz's complex as three independent azimuths. If we combine these azimuths, then in this group there will be only three azimuths (Ayaz, L. Kyrkyz and Toprak), which are meaningless to analyze from the statistical point of view.

We have considered all the most intense maxima in Fig. 1 and could not allocate other azimuths in relation to those already found. So, other azimuths are absent.

## 3 Astronomical azimuths and the accuracy of the orientation of monuments

Calculations show that 15 out of 23 found azimuths fall into the interval $1\sigma$, when $\Delta < \varepsilon$, and the remaining 8 are within the doubled interval $\Delta < 2\varepsilon$. This means that the distribution of the discrepancies obtained by us is close to normal. The analysis of the signs of residuals shows that in 8 cases the sign is positive, 12 is negative, and in 3 cases it is zero, i.e. we have a slightly shifted subnormal distribution with the position of the center $x_c = -0.521.59$.

A similar picture will be obtained if we consider only the statistically significant azimuths: 11 out of 17 fall into the interval $1\sigma$, and the remaining 6 ones in $2\sigma$. The residual sign $\Delta$ will be positive in 7 cases, negative in 8, and in two cases it will be zero. In this case, $x_c = -0.381.69$.

It should be noted that the azimuths we are analyzing are heterogeneous. Unlike solar, lunar and stellar azimuths, the direction of the meridian line cannot be obtained by the direct observation of the event of rising or setting. To determinate the position of the north, builders are required with special knowledge and skills in the field of geometry and the error in determining the direction will depend on the applied method. The question of finding the meridian line can be the subject of a separate study.

The situation with other azimuths is also not so plain. Our model calculations showed (Nickiforov 2015, pp. 100-120.) that, due to the perturbation of the inclination of the lunar orbit according to the harmonic law with a period of $P = 173^d$ and an amplitude of $a = 10'$, its azimuths of rising and setting can shift by a value of up to $|\Delta A| \approx 0.4°$. Moreover, the most likely event is the observation of the moon's rising/setting with shifted positions. Therefore, the azimuths of the moon's rising and setting are determined on the average with a worse accuracy than the solar one, which we demonstrated in the example of Stonehenge (Ibid and Table 1.).

## Astronomical azimuths in the context of culture

The above analysis allows us to establish the presence of two statistically significant astronomical directions. It should be noted that some researchers have already paid attention to the practice of orienting monuments along the meridian line (Tolstov 1948, Bulatov 1978, etc.). So the detection of these azimuths wasn't unexpected. The novelty of the result obtained in the proposed study is that, unlike previous researchers, we used accurate estimates of azimuths, which are suitable for statistical analysis (Ibid.). In addition, unlike M.S. Bulatov, we examined the monuments belonging to a single cultural tradition and localized within the territory of one state.

From the point of view of the study of cultural traditions, in all likelihood, it is necessary to exclude the principal possibility to get an accurate answer to the question why in an ancient society some astronomically significant directions were relevant, while others were not used at all. We can only speculate based on the system of modern notions of expediency about practical application of these azimuths in the daily life of an ancient person.

On the other hand, it is possible to make an attempt to explain their presence by religious traditions and to search the reflection of these azimuths in written sources.

Let's pay attention to one empirical fact. The internal layout of fortresses and cities of Khorezm, as a rule, is aligned to the walls that serve for a fortification purpose. In particular, we can demonstrate the satellite image of the Toprak-kala complex, where the orientation of the buildings of the "urban", palace and temple parts is strictly parallel. Thus, if the walls of the monument are oriented along a certain azimuth, then all internal buildings, including religious ones, will have a similar orientation. The orientation of the monument along the meridian line may be relevant for two reasons. Alignment to the meridian makes it very easy to determine the moment of approach of the local midday, and sighting of the perpendicular walls of building allows to estimate the moment of the equinox approach. Consequently, the orientation of the construction along the north-south line can have concrete benefits in everyday life.

The cultural significance of the winter solstice in Khorezm and neighboring states can be confirmed according to the data of Biruni. The author writes that six months after Nauruz (which, at least in the Sasanian times corresponded to the summer solstice), the Persians, Sogdians and Khorezmians celebrated the festival associated with the winter solstice.

*"[Month] Faghakân. The 1st [day] is called Nim-sarda, i.e. the half of the year. The 2nd is a feast called Khwâra, when they assemble in their fire-temples and eat a certain dish which they prepare with the flour of millet, with butter and sugar. Some people out Nim-sarda five days earlier, i.e. on the 1st of Mihr-Mah, to make it agree with the Persian calendar, whilst, in fact, the middle of the year ought to be celebrated when after its beginning 6 months and 2.5 days have passed."* (Biruni 1879, p. 221.)

*"[Month Ûmrî]. The 1st day is the feast Azâd Kand Khwâr, i.e. the day of eating the bread prepared with fat. On that day they sought protection from the cold, and assembled for the purpose of eating the bread prepared with fat, around the burning fire-grates."* (Biruni 1879, p. 224.)

Thus, six months after Nauruz and the summer solstice, on the first day of the month of Faghakan, the inhabitants of Sogd celebrated the festival of Nim-Sarda, which means "half a year". The name of the holiday indicates the connection with Nauruz and the summer solstice, from which the calendar year is counted. The next day the Sogdians gathered in the temples of the fire (or houses of the fire) and ate a kind of food made from millet flour, butter and sugar. In the calendar of the Khorezmians, the sixth month after the summer solstice corresponds to the month of Umri, on the first day of which people ate bread baked with fat.

From the descriptions of Biruni it follows that the inhabitants of Sogd and Khorezm celebrated the same holiday associated with the onset of the winter solstice. This is clearly indicated by its Sogdian name, and also by a six-month interval that has elapsed since the beginning of the year (summer solstice). The significance of the winter solstice is characteristic not only for the Central Asian tradition.

*"17 [day of Kanun I]. Nothing mentioned This day people call the "Great Birth", meaning the winter solstice. People say that on this day the light*

*leaves those limits within which it decreases, and enters those limits within which it increases, that human beings begin growing and increasing, whilst the demons begin withering and perishing.*

*Ka'b the Rabbi relates that on this day the sun was kept back for Yosua the son of Nûn during three hours on clouded day. The same story is told by the simpletons among the Shî'a regarding the prince of the believers, 'Alîb 'Abî Tâlib.*

*Yahyâ b. Alî, the Christian writer of 'Anbâr, says that the rising-place of the sun at the time on winter-solstice is the true east, that he rises from the very midst of paradise; that on this day the sages lay the foundations of the altars. "* (Biruni 1879, p. 238.)

The last quotes focuse on Christians. Taking into account all available evidence, one can assume the existence of common cultural tradition to note the phenomenon of the turn of the sun and an increase in the duration of the day. Thus, the significance of the azimuth, which corresponds to the winter solstice, is reflected in cultural representations.

The remaining detected astronomical azimuths associated with the motion of the Moon are not statistically significant. We can make an attempt to increase the sample size by using poorly preserved monuments that are not visible on satellite images. However, it should be born in mind that there are not many such objects, and they will not be able to improve the statistics in a significant way. On the other hand, it is reasonable to assume that a sample of 38 structures can be considered sufficiently numerous to reflect the general trend of the total set.

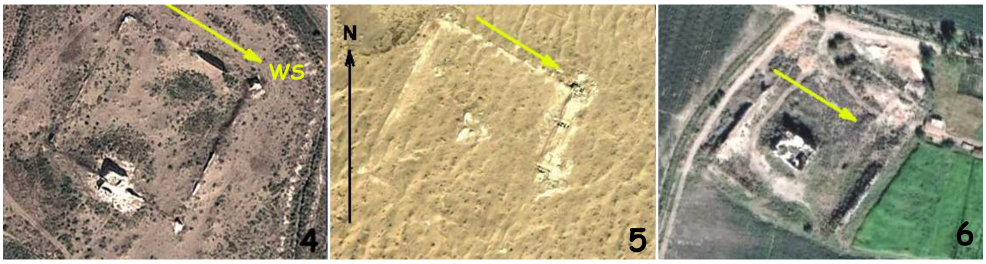
In practice, this means that we are not able to confirm the presence of other astronomical azimuths by statistical methods, even if they are actually present. However, the presence of a given azimuth can be justified by some cultural evidence, best of all - information from written sources.

## References

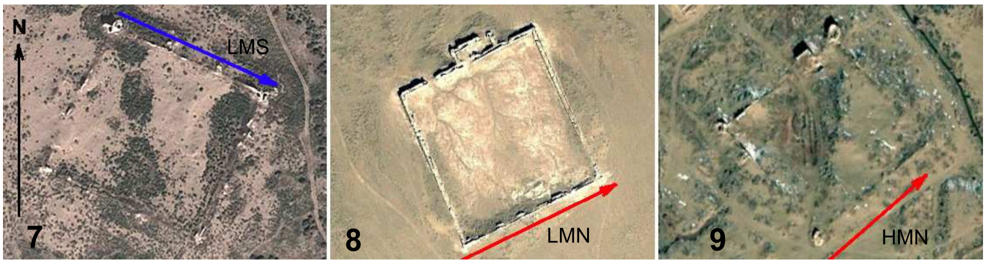
Andrianov B.V., 1969, Drevnie orositel'nye sistemy Priaral'ya, Moskva, Nauka.
Biruni A.R., 1879, The chronology of ancient nations, London.
Bolelov S.B., et al., 2015, *Kratkie Soobshcheniya Instituta Arheologii*, 237, 177.
Bolelov S.B., et al., 2016, *Kratkie Soobshcheniya Instituta Arheologii*, 242, 129.
Bulatov M.S., 1978, Geometricheskaya garmonizaciya v arhitekture Sredney Azii, Moskva.
Kolganova G.Yu., et al., 2014, *Bulgarian Astronomical Journal*, 23, 14.
Nerazik E.E., 1966, Sel'skie poseleniya afrigidskogo Horezma, Moskva, Nauka.
Nickiforov M.G., 2015, *Istoriko-astronomicheskie issledovaniya*, 38, 100.
THAEE, 1967, Trudi horezmskoi arheologo-etnograficheskoi ekspedicii, Vol. 5, Moskva, Nauka.
Tolstov S.P., 1948, Po sledam drevnehorezmskoi civilizacii, Moskva-Leningrad, Izdatelstvo AN SSSR.

## Appendix 1. Examples of monuments with an astronomically significant orientation.


**1. Orientation along the meridian line.**
Legend: 1 - Ayaz-kala-3; 2 - Kavat-kala; 3 - Pil-kala.




**2. Orientation toward sunrise in the winter solstice.**
Legend: 4 - Castle-60; 5 - Kazakli-Yatkan; 6 - Teshik-kala. *WS* - azimuth of sunrise in Winter Solstice.




**3. Orientation toward other astronomical significant directions.**
Legend: 7 - Bazar-kala; 8 - Dzanbas-kala; 9 - Kumbaskan-kala. *LMS* - azimuth of the rising of the Low Moon in the extreme South position; *LMN* - azimuth of the rising of the Low Moon in the extreme North position; *HMN* - azimuth of the rising of the High Moon in the extreme North position.

## Appendix 2. List of used monuments.

| Monument | Latt | Long | $\mathbf{A}_l$ | $\mathbf{A}_2$ | Evaluation |
|---|---|---|---|---|---|
| 1 Adamli-kala (cit.) | 41°44.450’ | 61°07.338’ | -3.5 | 85.0 | H1-X1-X1-H1<br>-3.5; –; –; -85 |
| 2 Aijr-kala | 41°51.268’ | 60°59.528’ | -30.0 | 122.0 | B1-B1-H2-B1<br>122; 30; 119; 30 |
| 3 Angka-kala | 41°45.500’ | 61°09.095’ | 44.0 | 135.5 | B1-H2-B1-B1<br>135.5; 43; 135.5; 44 |
| 4 Atsyz-kala | 41°56.300’ | 61°07.560’ | 21.0 | 112.0 | B3-X1-X1-B3<br>112; –; –; 30 |
| 5 Ashirtam-kala | 41°47.750’ | 60°57.173’ | -5.0 | 86.25 | H2-H2-H2-H2<br>84.5; -5.5; 88; -4.5 |
| 6 Ayaz-kala-1 | 42°00.854’ | 61°01.746’ | 75.5 | 170.5 | B1-B2-B1-B1<br>75; 170; 76; 170.5 |
| 7 Ayaz-kala-2[26] | 42°00.655’ | 61°01.630’ | 70.0 | 161.0 | B1-B1<br>70; 161 |
| 8 Ayaz-kala-3 | 42°00.320’ | 61°01.830’ | -2.0 | 70.5 | H3-B2-B1-B1<br>75; 170.5; 76; 170.5 |
| 9 Bazar-kala | 41°49.520’ | 61°11.383’ | 26.0 | 114.5 | B1-B2-B2-B2<br>114.5; 26; 114.5; 26 |
| 10 Bederkent-kala | 41°32.536’ | 59°59.964’ | 13.0 | 83.0 | B3-B3-X2-H1<br>83; 13 ; –; 12.5 |
| 11 Berkut-kala | 41°47.280’ | 61°03.312’ | 14.25 | 100.0 | B1-B2-X1-B1<br>100; 15; –; 13.5 |
| 12 Castle-2[27] | 41°56.987’ | 61°01.382’ | 37.0 | 119.0 | H2-H2<br>37; 119 |
| 13 Castle-60 | 41°45.317’ | 61°05.910’ | 30.0 | 120.0 | B2-B2-B2-B2<br>120; 29.5; 120; 30.5 |
| 14 Dzhanbas-kala | 41°51.480’ | 61°18.245’ | 64.25 | 155.5 | B1-B1-B1-B1<br>155; 64.5; 156; 64 |
| 15 Dzhanpik (cit.) | 42°01.605’ | 60°19.590’ | 0.5 | 91.0 | X3-B2-H2-B1<br>–; 0.0; 91; 0.5 |
| 16 Duman-kala | 41°44.310’ | 60°52.495’ | 14.0 | 106 | H2-H2-X1-X1<br>106; 14; –; – |
| 17 Hazarasp | 41°18.863’ | 61°05.534’ | -0.5 | 89.0 | B1-B1-H2-B2<br>89; -0.5; 88.5; -0.5 |
| 18 Homestead-1 | 41°51.892’ | 60°56.796’ | -2.75 | 91.75 | H1-H1-H1-H1<br>90; -1.0; 93.5; -4.5 |
| 19 Guldursun[28] L. | 41°45.590’ | 60°58.890’ | 25.0<br>36.0 | 119.0 | B1-B1-H2-B2<br>89; -0.5; 88.5; -0.5 |
| 20 Kalaly-Gyr-1 | 41°48.036’ | 59°11.000’ | 8.0 | 103.25 | B1-H3-B1-H3<br>101.5; 6.0; 105; 10.5 |

## Appendix 2. List of used monuments. (Continue.)

| Monument | Latt | Long | $A_l$ | $A_2$ | Evaluation |
|---|---|---|---|---|---|
| 21 Kandym-kala | 42°04.317' | 59°11.058' | 30.25 | 120.5 | B3-B2-B2-B2<br>121; 30; 120; 30.5 |
| 22 Kavat-kala | 41°51.248' | 60°54.688' | 0.0 | 90.0 | B3-H3-H3-B3<br>90.0; 3.7; 90.5; 0.0 |
| 23 Kazakli-Yatkan | 41°49.720' | 60°43.050' | 28.0 | 120.0 | B1-B3-B1-B3<br>120; 28; 120; 28 |
| 24 Kosh-parsan | 41°54.180' | 60°53.961' | -4.75 | 84.25 | B1-B1-B1-B1<br>84.5; -4.5; 84.0; -5.0 |
| 25 Kulbaskan-kala-1 | 41°48.375' | 60°59.347' | 0.25 | – | X3-H1-X3-H1<br>–; 0.5; –; 0.0 |
| 26 Kulbaskan-kala-2 | 41°48.440' | 60°59.342' | 0.25 | 94.0 | X3-H1-H1-H1<br>–; 0.5; 94; 0.0 |
| 27 Kumbaskan-kala | 41°43.707' | 61°01.666' | 49.0 | 138.25 | B2-B2-B2-B2<br>138; 49.5; 138.5; 48.5 |
| 28 Kurgashin-kala | 42°02.040' | 61°19.340' | 55.0 | 147.0 | X3-X3-H4-H4<br>–; –; 147; 55 |
| 29 Kyzyl-kala | 41°55.811' | 60°47.050' | 53.0 | 142.0 | B1-B1-B1-B1<br>141.5; 53; 142.5; 53 |
| 30 Kyrkyz-kala L. | 42°00.467' | 61°09.470' | 72.25 | 163.75 | B1-B2-B1-B1<br>72.5; 164 72; 163.5 |
| 31 Mizdakan[29] | 42°24.070' | 59°23.360' | 12.0 | 102.0 | B3-B3<br>102; 12 |
| 32 Pil-kala (Cit.) | 41°42.300' | 60°44.250' | 0.0 | 90.25 | B1-B1-B1-B3<br>90.5; 0.0 90.0; 0.0 |
| 33 Teshik-kala[30] | 41°45.086' | 61°02.644' | 38.5 | 124.5 | H2-B3-H2-B3<br>125; 37 124; 40 |
| 34 Toprak-kala[31] | 41°55.635' | 60°49.375' | 70.0 | 158.0 | B2-B2<br>158; 70 |
| 35 Uj-kala[32] | 41°52.083' | 61°04.733' | 1.5 | 91.0 | B1-B1-B1-B1<br>100; 2.5; 100; 2.0 |
| 36 Vazir-rabat | 42°17.482' | 58°24.035' | 2.25 | 100.0 | B1-B1-B1-B1<br>100; 2.5; 100; 2.0 |
| 37 Yakke-parsan | 41°55.270' | 61°01.105' | 9.0 | 101.5 | B3-B3-X1-B3<br>101.5; 8; –; 10 |
| 38 Koy-Krylgan-k. | 41°45.317' | 61°07.020' | 80.0 | – | B2<br>80 |

## Appendix 3. List of astronomically significant azimuths, used for the analysis of spatial orientation of monuments.

| | Astronomical direction | Azimuth |
|---|---|---|
| 1 | True North (the meridian line). | 0 |
| 2 | Orientation to the equinox east. | 90 |
| 3 | Azimuth of sunrise at the day of winter solstice. | 121.5 |
| 4 | Azimuth of sunrise at the day of summer solstice. | 57 |
| 5 | High Moon, northern azimuth. | 49 |
| 6 | High Moon, southern azimuth. | 129 |
| 7 | Low Moon, northern azimuth. | 64 |
| 8 | Low Moon, southern azimuth. | 114.5 |
| 9 | Azimuth of heliacal rising of the Pleiades. | 78.5 |