Forward orbital evolution of the Vesta Family with and without the Yarkovsky effect

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Abstract.
Vesta family members (VFMs), totally 17164, were selected by means of hierarchical clustering method (HCM) from the data base containing 393347 synthetic proper elements of numbered asteroids from the ASTDyS Catalogue (2015) updated in May 5, 2015. Keplerian elements from the Lowell Catalogue (2015) were used for studying orbital evolution of all 17164 VFMs in the time interval 1 Gy forward. Two cases were considered: evolution pass without the Yarkovsky effect (YN) and evolution pass with it (YY). It has been found that swarm of asteroids disperses about 28 times more efficient for the case YY than in the case YN. Efficiency of dispersion was studied versus semiaxis of asteroids relative to Vesta (smaller or larger than semiaxis of Vesta) as well as versus the sizes of asteroids. Weak relationships between size and efficiency of dispersion on YE have been found for the both cases YN and YY. The loss of number of the asteroids from VF weakly depends on their sizes. The total lost by number as well by mass is about 10% per 1 Gy.
Key words: Hierarchical Clustering Method (HCM). Vesta Family (VF), Yarkovsky Effect (YE)

List of symbols and abbreviations
a, e, i - Orbital synthetic proper elements
R - Radius of an asteroid
v - Subscript denoting asteroid Vesta
HCM - Hierarchical clustering method
LG and RG - the Left Group and the Right Group of asteroids, respectively
MMR - Mean motion resonance
VF - Vesta family
VFM - Vesta family member
YE - Yarkovsky effect
YN - Without Yarkovsky effect (Yarkovsky Not)
YY - With Yarkovsky effect (Yarkovsky Yes)

Introduction

Vesta Family Members (VFMs) were identified by means of the Hierarchical Clustering Method (HCM). More precisely the HCM only allows to identify the VFMs-candidates. However, some of the candidates could be the interlopers that are not genetically related to the family. Farther on we treat all 17164 asteroids chosen by HCM as the VFMs. It is well known that the Yarkovsky Effect (YE) influences orbits of small bodies of the Solar System, particularly the orbits of asteroids. The goal of this work is comparing evolution of the Vesta Family (VF) without the Yarkovsky effect (YN) with their evolution when the YE is considered (YY). In fact it is not so important to consider all selected asteroids as the real VFMs or only as the potential VFMs that are as the real ones admixed by interlopers. Essential is a study of orbital evolution of a large set of asteroids following two different models. So, in this paper no effort is done to eliminate interlopers. We use an abbreviated form VFMs instead of the correct form VFMs-candidates. Orbital evolution, that is the changes Δa, Δe, and Δi of semiaxis a, of eccentricity e, and of orbital inclination i vs time is studied for all 17164 asteroids identified as VFMs. Orbital evolution of them is performed 1 Gy forward. Numerical algorithm is organized in such a manner that an access is granted to partial results, i.e. to the instantaneous values of orbital parameters of the all asteroids considered in each moment of orbital integration.

1. Identification of the VFMs by means of the HCM

The AstDys Catalogue (2015) is the primary source of data for this work. It contains analytic and synthetic proper elements of the numbered asteroids and the multi-opposition asteroids. In our previous work (Wlodarczyk and Leliwa-Kopystynski, 2014) we used analytic proper elements given by AstDys Catalogue updated in February 2012. By means of the HCM it was found that the data set of 292003 numbered asteroids contains 14728 VFMs (interlopers not eliminated) that is 5.04% of the total. We described application of HCM (Fig. 1 and Table 2 in Wlodarczyk and Leliwa-Kopystynski, 2014) for five asteroid families, particularly for the VF. However, in the present work to achieve more accurate results synthetic proper elements were used instead of analytic proper elements. This choice is supported by detailed analysis given in Knezevic and Milani (2000), in Knezevic and Milani (2003) and in Milani et al. (2014). The latter showed that the analytic proper elements are less accurate than the synthetic ones by a factor of about 3 for the low eccentricity e and for the orbital inclination i. For high e and i it is even difficult to compute analytically the proper elements. Hence, Milani et al. (2014) recommended using catalogs of synthetic proper elements. Moreover, until now (AstDys Catalogue, 2015), synthetic proper elements are not available for multi-opposition asteroids, therefore we used only the numbered asteroids. From the data base (AstDys Catalogue, 2015) that contains 393347 synthetic proper elements of numbered asteroids were identified by means of HCM the 17164 VFMs candidates, that is 4.36% of the total. Result of identification of VFMs is seen in Fig. 1, that is the population of the family depends primary on the size of the data base. The parameters are the cut-off velocity v_{cut} and the threshold of absolute magnitudo H.

2. Time evolution of the VF

2.1. Initial conditions.

All identified VFMs, 17164 totally, have their present-day keplerian orbital elements (Lowell Catalogue, 2015). They are presented in the planes (a, e), (a, i), and (e, i) in Fig. 2 - left panels. These figures illustrate initial data for calculations of orbital evolution. Figure 2 - top and middle of left panels

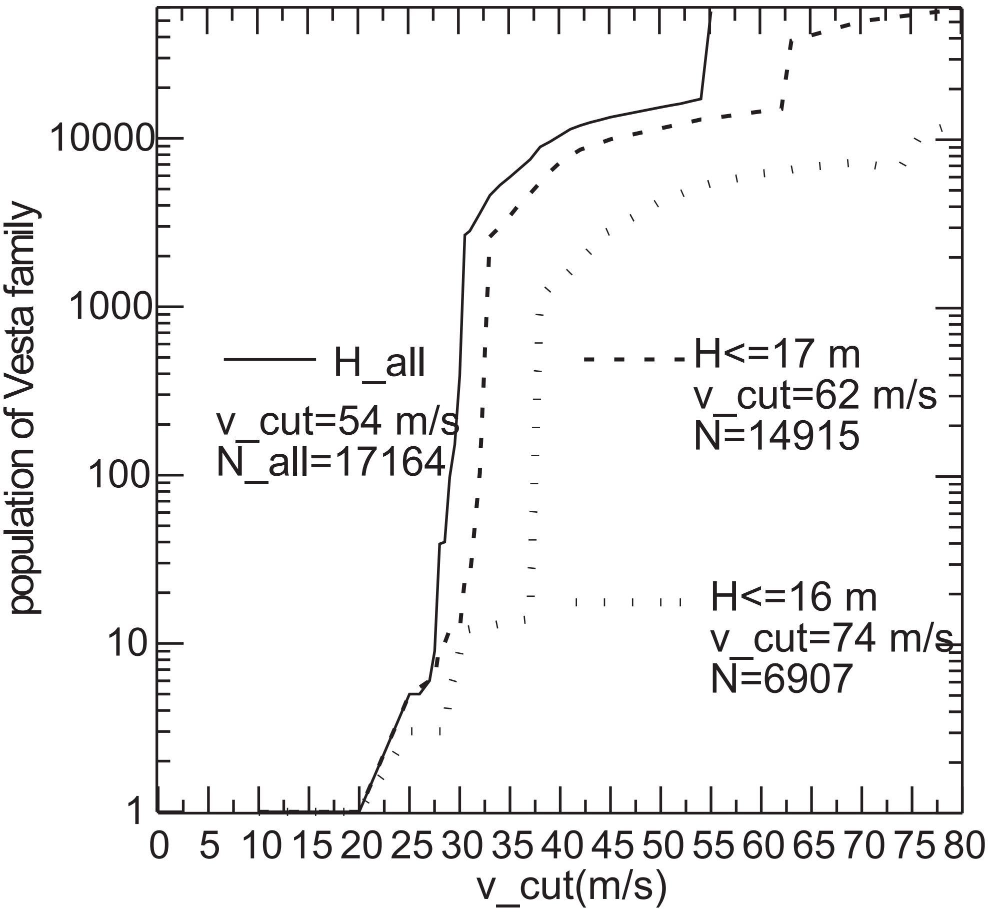

Fig. 1. Population of the VF versus cut-off velocity v_{cut}. Absolute magnitudo threshold H is a parameter and N is the number of the VFMs that corresponds to this H. To the chosen value $v_{cut} = 54$ m/s corresponds VFMs the population 17164, adopted in this paper as definitive (with interlopers). A more-than-one-step form of the plots can be an argument supporting hypothesis that the present-day VF is a product of more than one large impact. We suppose that the difference in value of the parameter v_{cut} between our previous work (Wlodarczyk and Leliwa-Kopystynski, 2014: $v_{cut} = 57$ m/s) and its presently selected value $v_{cut} = 54$ m/s results from using of different data base: analytic proper elements of 292003 numbered asteroids from the ASTDyS updated in February 2012, and synthetic proper elements of 393347 numbered asteroids from the ASTDyS updated in May 5, 2015, respectively.

show that the VF is placed within certain limits of the solar distance. The left-hand border of a solar distance is close to the orbital mean motion resonance (MMR) with Jupiter 7:2J that corresponds to the solar distance $a = 2.256$ au. The right-hand border is close to the MMR with Jupiter 3:1J that corresponds to the solar distance $a = 2.500$ au. However, surprisingly, the swarm of VFMs is shifted about 0.02 au toward the Sun. The left-hand border is fuzzy and it spreads over the distances from $\sim$2.2 au up to 2.256 au. The right-hand border $a = (2.480 \pm 0.005)$ au is much more sharp.

2.2. Evolution without Yarkovsky effect (YN).

The whole ensemble of 17164 asteroids forming the VF was divided into 85 groups containing 200 asteroids each, and one group with 164 asteroids. This division was only of technical character to perform calculations of orbits of the family members faster and to undergo easier inspection. From the physical point of view the grouping has no meaning since mutual interactions (gravitational or collisional) among the asteroids are not considered. The orbits of asteroids are influenced only by the Sun and the planets from Mercury to Neptune.

As initial orbital elements we took the osculating keplerian elements from the Lowell Catalogue (2015) for the epoch 2015-06-27. It contains all the objects that are listed in the AstDys Catalogue, (2015) of synthetic proper elements. Then the initial orbital elements of VFMs and planets were computed for the same epoch using the software *Mercury* (Chambers, 1999). Time evolutions of orbital elements of all 17164 asteroids are calculated by means of the software *swift_rmvs* developed by Broz (2006). This software takes into account gravitational influence of all planets (variant *swift_rmvs3_f*) and additionally it allowed to introdece diurnal and seasonal variations of the Yarkovsky effect (variant *swift_rmvs_f_y*). Our calculations were done for the case without Yarkovsky effect (YN) and with it (YY). The integration was performed 1 Gy forward. The orbital elements of the all 17164 asteroids were outputted every 1 My.

After 1 Gy of evolution without Yarkovsky effect (YN) population of VF diminished from 17164 to 17095. It means that the VF lost only 69 members that is only 0.4% of the initial population. Nevertheless, the plots in the middle column of Fig. 2 differ considerably from the plots in the leftmost column of Fig. 2 presenting initial positions: a halo, formed from some AFMs dispersed out of their initial positions, appears.

2.3. Evolution with Yarkovsky effect (YY).

Apart of the orbital parameters of the individual asteroids there is a set of physical data that is necessary for performing calculations by means of the HCM and to present them versus size of the asteroid family members. Some additional parameters are required to consider the YE as one of the causes of evolution of the asteroid orbits through time. These parameters are listed below. The data for Vesta, given in brackets, are according to Russel et al. (2012):

→ H (3.20) - the absolute magnitudo, according to IRAS catalogue (Tedesco

et al., 2004). For each asteroid is taken its individual value of H from the Lowell Catalogue (2015).
→ p (0.4228) - the visual geometric albedo according to the IRAS catalogue (Tedesco et al., 2004). In our previous work (Wlodarczyk and Leliwa-Kopystynski, 2014) we assumed that all VFMs have albedo equal to that of Vesta but VFMs which albedo are outside assumed range (0.24-0.60) are eliminated as interlopers. Presently, we adopted for all selected VFMs mean value of geometric albedos of VFMs which are published in WISE catalogue.
→ R (2.65×10^5 m) - the mean radius of an asteroid. It is calculated by means of the formula given by Harris (1998)

$$2R = 1329 \times 10^{-\frac{H}{5}} p_v^{-1/2} \tag{1}$$

→ $\rho = 2000$ kg m^{-3} (3456 kg m^{-3}) - the mean density. Arbitrary adopted.
→ $\rho_s = 1500$ kg m^{-3} - the mean near sub-surface density of all VFMs. Arbitrary adopted.
→ $K = 0.01$ W m^{-1} K^{-1} - the heat conductibility of the near-surface layer of all VFMs. We adopted this value after Table 5 from Broz (2006).
→ $c = 680$ J kg^{-1} K^{-1} - the specific heat of the near-surface layer of all VFMs. We adopted this value after Carruba et al. (2005).
→ $C_s = (1360 \text{ J m}^{-2} \text{ s}^{-1})/(a \text{ in au})^2$ - the mean solar constant for an asteroid with the major semiaxis a.

The spin parameters are used in calculations as well. They are the rotation period P and the orientation of the rotation axis (its ecliptic longitude and latitude) in the ecliptic reference frame. Only these for Vesta are known. The ecliptic longitude and latitude for the others VFMs are randomly selected. Their rotation periods were chosen by means of the assumption of Broz (2006) that any asteroid with the mean radius R = 500 m has the period of rotation equal to 5 hours. The formula for the angular velocity of rotation w of an asteroid with any radius R fits to this assumption:

$$\omega = (\frac{2\pi}{5h} / (\frac{R}{500}) = \frac{0.1745}{R} [rd/s], \tag{2}$$

where R is expressed in meters. In the software $swift\_rmvs3\_f\_y$ in order to eliminate initial values of spins on evolution of VF a random variability of spins versus time is assumed. We adopted that spins varied every 10000 years and this value is given as reorientation time step in the input parameter yarko.in in the software $swift\_rmvs3\_f\_y$.

After 1 Gy of evolution with Yarkovsky effect (YY) the population of VF diminished from 17164 to 15227. It means that as many as 1937 (11.3%) asteroids escaped the family. They fall on the Sun, impacted the planets, or they escaped from the Planetary System. Positions of 15227 asteroids that remained in the region, are illustrated on the right column of Fig. 2. These diagrams compared with that on the left and on the middle columns indicated that the family evolved with YE is less dense, especially in the peripheral area where the VFMs are more dispersed forming the halo.

Diagrams presented in Fig. 2 contain so many asteroids each that neither discussion of evolution nor discussion of influence of YE on the evolution

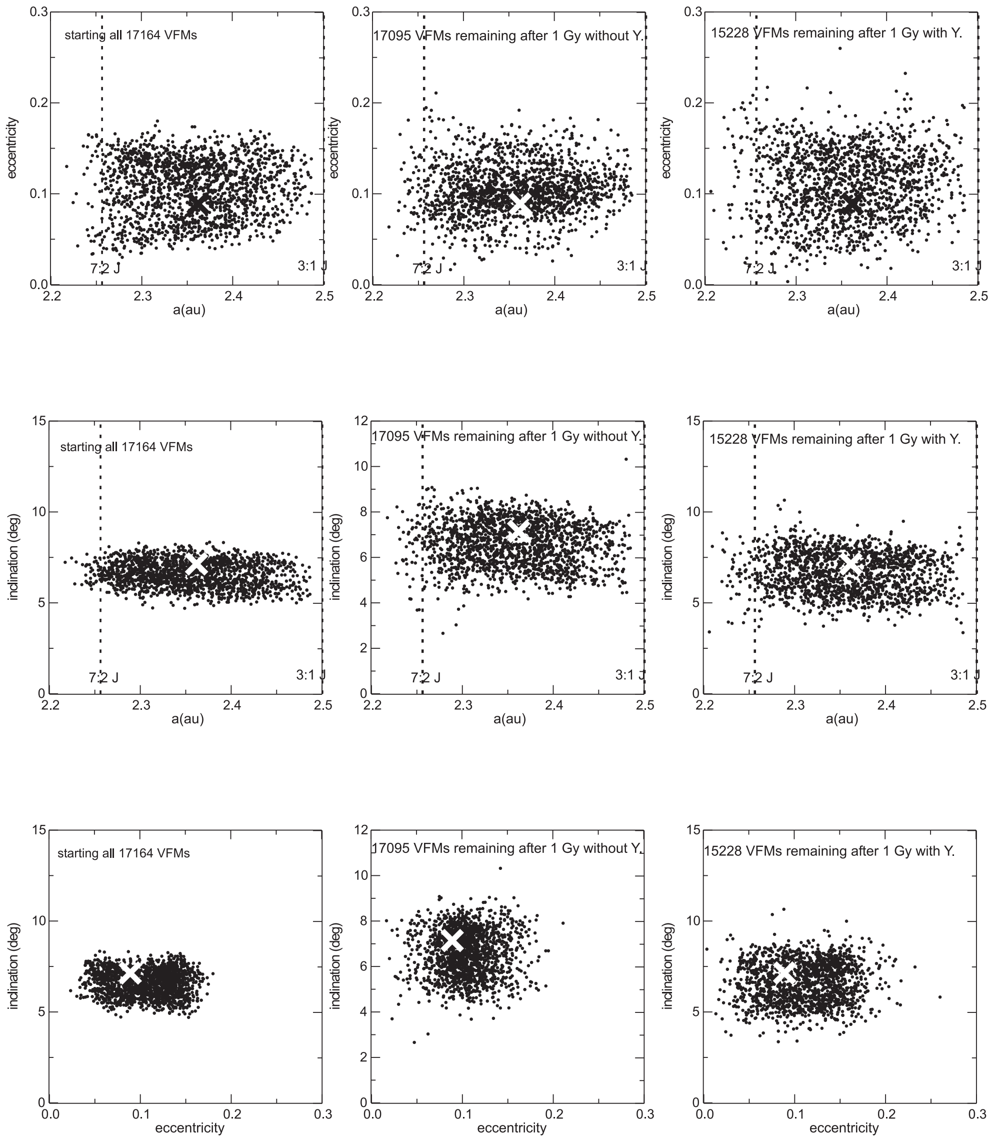

Fig. 2. Presentation of evolution of the cloud VFMs on the (a, e), (a, i), and (e, i) planes. Every tenth VFM is drawn. Ranges: $a = (2.2149$ au, 2.4904 au$)$, $e = (0.02328, 0.18449)$, and $i = (4.2987, 8.5654)$. Cross denotes Vesta position $a_v = 2.3619$ au, $e_v = 0.08883$, $i_v = 7.1401$. Orbital resonances Vesta/Jupiter 7:2J and 3:1J are marked in the two upper rows. They correspond to solar distance $a_{7:2} = 2.256$ au and $a_{3:1} = 2.500$ au, respectively. The left column shows initial positions of all 17164 VFMs chosen by the HCM. The middle column containing 17095 asteroids shows evolved position of the cloud after 1 Gy, for the case YN. Nevertheless the contents of this column is only 69 asteroids less numerous than the content of the left column. Blurry of the cloud is clearly visible. The right column illustrates positions of 15227 VFMs remaining after 1 Gy in the same boxes for the case YY; 1937 asteroids were lost from the boxes. Cloud is even more dispersed than in the middle column.

is possible. Therefore, we decided to focus not on the asteroids that still belong to the Vesta family but on these VFMs that escaped from the considered region (a, e). To discuss time evolution of the orbits of the VFMs it is convenient to divide them onto two groups:
→ Left Group (LG), containing the VFMs whose semimajor axes are smaller than the semimajor axes of Vesta $a_v = 2.3619$ au.
→ Right Group (RG), containing the VFMs whose semimajor axes are larger than a_v.
Table 1 summarizes results illustrated in Fig. 1 and Fig. 2. The figures listed in the bottom row of Table 2 indicate for asymmetry of escaping of the VFMs from the proper element boxes. In Fig. 3 (case YN) and in Fig. 4 (case YY) there are presented only these asteroids that escaped from the Vesta family during the time interval equal to 1 Gy. It turns out that this escaping is highly asymmetric: the LG and the RG during their orbital evolution lost very different number of asteroids. In other words the VF loses different amounts of its members into the inbound direction relative to the Vesta orbit (LG) and into the outbound direction (RG). Moreover, the losses differ depending on whether the YE is not considered (YN) or it is considered (YY):

→ LG, YN: (9537 - 9468) = 69 that is 0.72 % of initial population of the LG

→ LG, YY: (9537 - 8356) = 1181 that is 12.4 % of initial population of the LG

→ RG, YN: (7627 - 7627) = 0

→ RG, YY: (7627 - 6871) = 756 that is 9.9 % of initial population of the RG

→ LG and RG together YN: (17164 - 17095) = 69 that is 0.40 % of the total population of the VF

→ LG and RG together YY: (17164 - 15227) = 1837 that is 10.7 % of the total population of the VF

The percent of asteroids escaping from VF is almost negligible for the case YN. The loss of asteroids from the LG and from the RG that is 0.72% and 0% respectively are below the values of the error bars. However, the loss is significant for the case YY being of the order of 10% for both left- and right-group of the VFMs. Unique explanation for this is such that YE shifts asteroids inbound or outbound the Sun.

Figure 2 illustrates the fact that the VFMs spread out from the Vesta position up to the MMRs with Jupiter 7:2J and 3:1J on the left-hand and on the right-hand sides, respectively. If during orbital evolution a major semiaxis of an asteroid approaches to any of these resonances it can be ejected from the family more efficiently than from the other regions. This is illustrated on Fig. 3 where initial positions are shown of the asteroids which then are ejected. They are mostly in the vicinity of the MMRs points. From the total loss of 69 VFMs the 58 are ejected from the point 7:2J and the 11 from the point 3:1J. However, if the process of ejection of asteroids from the family is supported by the YE removal mechanism is active in the whole (a, e) region and it is about $1937/69 = 28$ times more efficient than without YE. Figure 4 presents initial positions of 1937 asteroids next ejected from the considered ranges of orbital elements.

Table 1. Fate of these VFMs that did not survived in the VF after 1 Gy of orbital evolution. From the total initial population 17164 of the VF only as few as 69 asteroids are escaping in the case YN, and as many as 1937 asteroids are escaping in the case YY.

Event	YN; Fig. 3			YY; Fig. 4		
	Left	Right	Total	Left	Right	Total
Collisions with the Sun	39	10	49	86	86	172
Collisions with the planets	7	0	7	1066	644	1710
Ejected from the Planetary System for the orbits with a >1000 au	12	1	13	29	26	55
Total not surviving as VFMs after 1 Gy integration	58	11	69	1181	756	1937

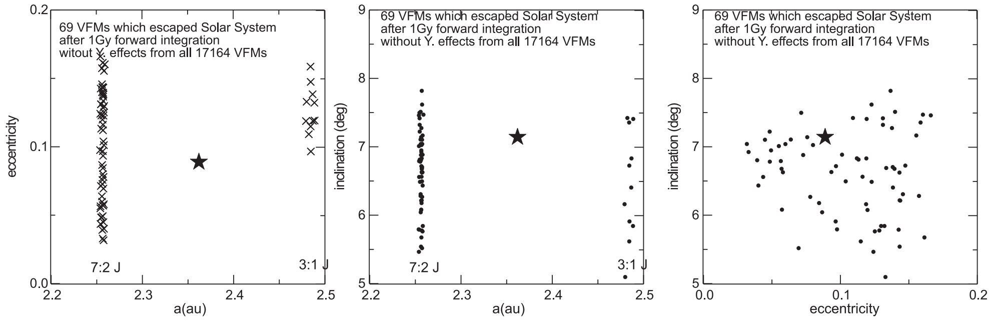

Fig. 3. Initial positions of 69 VFMs in the planes (a, e), (a, i), (e, i) that will escape the family during the next 1 Gy for the case YN. Note the high concentration of the asteroids to be ejected from very near vicinity of the MMR points.

3. Distributions of VFMs vs their sizes: the initial state and the cases YN and YY after 1 Gy

Size distributions of VFMs are presented in Table 2 illustrated by the appropriate figures. The axis of radii R of the VFMs spread over an interval from 0.2 km to 2.5 km. The lower limit corresponds to detection limit of the VFMs. Above the upper limit there are only a few, compared with total, the largest VFMs. The R axis is divided into bines with width 100 m each. The center of the first not-empty bin is at 0.25 km. Table 2 shows that the present-day number 17164 of VFMs is divided onto 9537 (LG) plus 7627 (RG). The small differences between the numbers in the same rows of columns 2 and 3 (LG) and columns 6 and 7 (RG) on Table 2 are related to migrations of VFMs between the bins. It is intriguing that the RG is significantly less numerous than the LG. This can be explained e.g. by the geometry of that particular impact which formed VF: the post-impact distribution of speed-vectors of the fragments may be such that favors throwing

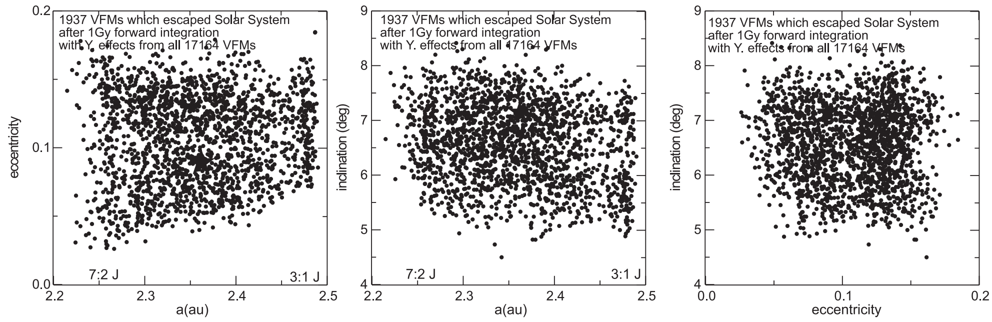

Fig. 4. Initial positions of 1937 VFMs in the planes (a, e), (a, i), (e, i) that will escape the family in the next 1 Gy for the case YY. Asteroids, which will escape their (a, e) region were initially distributed in this region quasi uniformly. The role of positions of asteroids in the vicinity of MMR points seems to be only marginal. However in the left and in the middle plots a slightly higher concentration of the asteroids to be ejected is visible in the vicinity of the MMR 3:1J.

fragments onto the orbits with semimajor axis smaller than that of Vesta. Losses, that is, the throwing out of asteroids from the initially occupied box of the orbital elements space (a, e, i) presents only a weak dependencies on asteroids' radii R (Fig. 5 and Table 2). However, in general, the smaller VFMs tend to escape more efficiently from the box than the larger ones. It is visible most clearly in Fig. 5 - right panel. For the case YY the bins containing smaller-sized VFMs lost relatively larger amount of the VFMs than the bins containing larger-sized VFMs. This observation is important since it agrees with the theory of mechanism of the YE.

4. Evolution of orbital elements versus sizes of the VFMs

In this section we follow changes of the mean changes of orbital elements Λa_k, Δe_k, and $\Delta \sin(i_k - i_v)$ versus sizes of VFMs in the k-bin. The LG and the RG of the VFMs are considered independently. An influence of YE on evolution of the asteroid orbits is considered as well and thus the cases YN and YY are considered. Let us take the k-bin of the asteroids' size, which comprises n_k asteroids. The error bar of any of the values Λa_k, Δe_k, and Δ $\sin(i_k - i_v)$ in the k-bin is inversely proportional to the square root of the number of asteroids in this bin $n_k^{-1/2}$. To compare relative weights of data in different bins the value of constant of proportionality is not necessary and, therefore, it is assumed arbitrary for each of the cases considered below.

Evolution of semimajor axes. The mean value of Δa_k in this bin presented in dimensionless form is given by the formula

$$\Delta a_k = \frac{1}{a_v n_k} \sum_{i=1}^{n_k} \Delta a_i \tag{3}$$

Table 2. Distribution of VFMs in the 100 m width bins of the radius. Data are for the initial stage and the final stage, after 1 Gy. Moreover, they are for the cases YN and YY. Note the important influence of YE on removing small asteroids from the family. The columns are: →1: Bin center in kilometers. The width of each bin is 0.1 km: →2, 6, 10: Initial numbers of VFMs in the bins: →3, 7, 11: Final numbers, at 1 Gy forward, of VFMs in the bins; YN. →4, 8, 12: Final numbers, at 1 Gy forward, of VFMs in the bins; YY. →5, 9, 13: The ratios of figures in the appropriate rows of the columns 4 and 3, 8 and 7, 12 and 11, respectively that are the ratios YY/YN in %. These ratios are illustrated in Fig. 5. The values of errors are listed as well.

Bin center km	Left group of VFMs $a < a_v$				Right group of VFMs $a \geq a_v$				All VFMs			
1	2 Initial	3 Final YN	4 Final YY	5=4/3, % YY/YN Fig. 5left	6 Initial	7 Final YN	8 Final YY	9=8/7, % YY/YN Fig.5mid	10 Initial	11 Final YN	12 Final YY	13=12/11, % YY/YN Fig.5right
0.25	187	163	121	74.2±17	49	72	96	133±22	236	235	217	92.3±13
0.35	1341	1284	999	77.8±6.0	714	761	812	106.7±7.1	2055	2045	1811	88.6±4.6
0.45	2060	2043	1725	84.4±4.6	1596	1591	1501	94.3±5.1	3656	3634	3226	88.8±3.4
0.55	1560	1560	1422	91.2±5.2	1368	1355	1138	84.0±5.7	2928	2915	2560	87.8±3.8
0.65	1011	1014	943	93.0±6.4	911	904	736	81.4±7.0	1922	1918	1679	87.5±4.7
0.75	880	876	778	88.8±7.0	733	730	635	87.0±7.7	1613	1606	1413	88.0±5.2
0.85	638	653	598	91.6±8.0	625	607	529	87.1±8.4	1263	1260	1127	89.4±5.8
0.95	395	415	387	93.3±10	357	334	295	88.3±11	752	749	682	91.1±7.5
1.05	331	322	325	101±11	275	283	228	80.6±13	606	605	553	91.4±8.3
1.15	282	282	266	94.3±12	246	242	214	88.4±13	528	524	480	91.6±8.9
1.25	207	202	191	94.6±14	173	177	152	85.9±16	380	379	343	90.5±9.7
1.35	88	93	83	89.2±21	88	83	76	92±22	176	176	159	90.3±15
1.45	163	159	142	89.3±16	117	121	118	98±18	280	280	260	92.9±12
1.55	68	69	52	75.4±26	56	55	65	118±26	124	124	117	94.4±18
1.65	105	113	98	86.7±20	90	82	82	100±22	195	195	180	92.3±14
1.75	52	52	49	94.2±28	45	45	34	75±32	97	97	83	85.6±21
1.85	33	33	32	97.0±35	33	33	28	85±36	66	66	60	90.9±25
1.95	34	34	28	82.4±36	23	23	26	113±40	57	57	54	94.7±27
2.05	27	26	23	88.5±40	20	21	22	105±43	47	47	45	95.7±29
2.15	5	5	14	280±71	21	21	11	52±52	26	26	25	96.2±40
2.25	10	10	14	140±58	14	14	10	71±58	24	24	24	100±41
2.35	15	17	15	88.2±50	9	7	8	114±73	24	24	23	95.8±41
2.45	12	11	8	72.7±66	8	9	10	111±65	20	20	18	90.0±46
≥2.55	33	32	43	134±33	56	57	45	79±28	89	89	88	98.9±21
All	9537	9468	8356	88.26±2.12	7627	7627	6871	90.09±2.35	17164	17095	15227	89.1±1.6

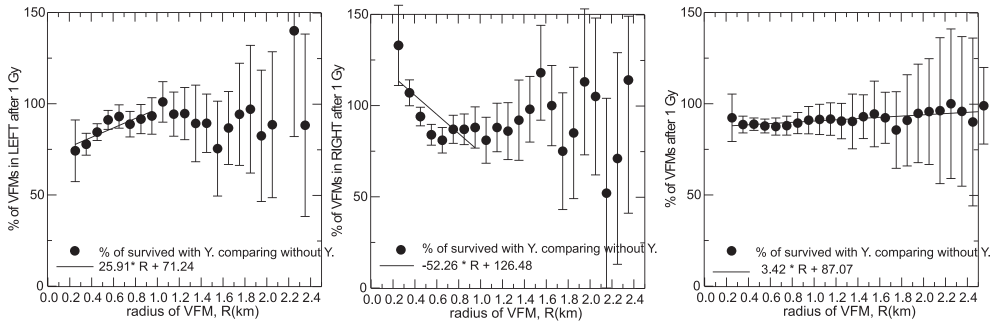

Fig. 5. The ratio YY/YN that is the ratio of number of the VFMs survived after 1 Gy with or without YE versus the radii of asteroids. The ratios and the error bars are from columns 5, 9 and 13 in Table 2. The error increases when radius increases since the number of asteroids in the bins considerably decreases. Regression straight lined segments YY/YN versus asteroids' radii that are the percent of surviving VFMs p_{surv} with/without YE are drawn. The segments concern these intervals of radii where the relation p_{surv} versus R seems to be linear. The panels and the equations for the straight lined segments are: (left) For the LG of VFMs, p_{surv} = 25.91R + 71.24, correlation coefficient is 0.758; (middle) for the RG of VFMs, p_{surv} = -52.26R + 126.48, correlation coefficient is 0.551; (right) all VFMs p_{surv} = 3.42R + 87.07, correlation coefficient is 0.371. Here the radii R are in kilometers.

where $\Delta a_i = a_{i,final} - a_{i,initial}$ and a_v=2.3619 au.

Here a_v is the present-day major semiaxis of Vesta. Subscripts initial and final are related to the present state and to the final state that is to the state after 1 Gy, respectively. Formula (3) illustrates in Fig. 6 for all four cases: Left Group LG without Yarkovsky Effect YN, Right Group RG without Yarkovsky Effect YN, Left Group LG with Yarkovsky Effect YY, and Right Group RG with Yarkovsky Effect YY. The centers of the size-bins spread over the range from $R = 0.25$ km to $R = 2.25$ km. For detailed description and comments see Fig. 6 caption.

Evolution of eccentricity. The mean value of change of eccentricity of the VFMs belonging to the k-bin of size is

$$\Delta e_k = \frac{1}{n_k} \sum_{i=1}^{n_k} \Delta e_i \qquad (4)$$

where $\Delta e_i = e_{i,final} - e_{i,initial}$.

Formula (4) is illustrated in Fig. 7.

Evolution of orbital inclination. The mean value of change of sinus of orbital inclination relative to inclination of Vesta of the VFMs belonging to the k-bin of size is

$$\Delta sin(i_k - i_v) = \frac{1}{n_k} \sum_{i=1}^{n_k} \Delta sin(i_i - i_v) \qquad (5)$$

where $\Delta sin(i_i - i_v) = sin(i_{i,final} - i_v) - sin(i_{i,initial} - i_v)$.

Formula (5) is illustrated in Fig. 8.

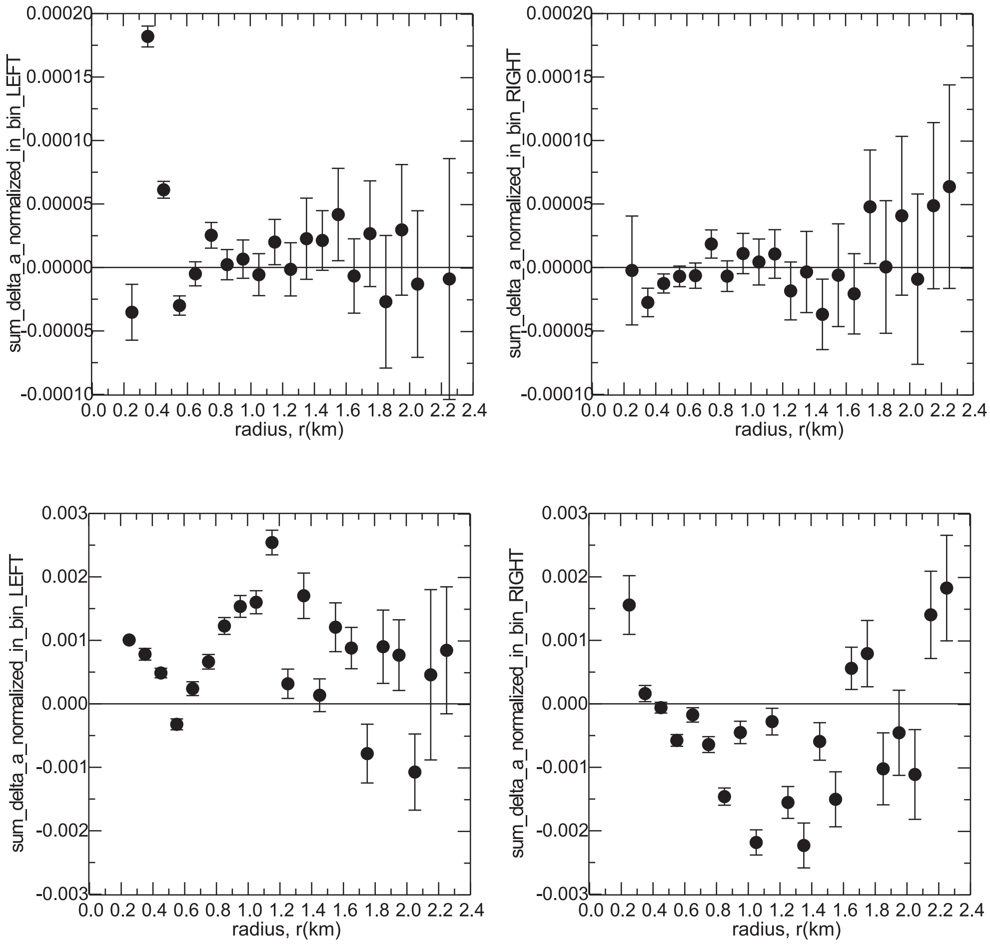

Fig. 6. Evolution of the mean values of semimajor axis of VFMs during the time interval 1 Gy versus their radii according to Eq. (3). The case YN: Plates in the top row illustrate evolution of semimajor axes of the LG and RG of the VFMs. Assumed error bars are equal to $\pm(3\times10^{-4}n_k^{-1/2})$. The case YY: Plates in the bottom row illustrate evolution of semimajor axes of the LG and RG of the VFMs for. Assumed error bars are equal to $\pm(3\times10^{-3}n_k^{-1/2})$. Comments: For LG a mean *increase* of semimajor axes is of the order of $0.001a_v$/1Gy≈3.5× 10^5 km/1Gy. For RG semimajor axes decrease with the same rate. For the case YN (upper plates) the mean drift of VFMs is at least 10 times slower. The trend (LG semiaxis increasing / RG semiaxis decreasing) leads to very slow shifting of VFMs toward the orbit of Vesta. However this mechanism of VF densifications is certainly orders of magnitude less important than diffusive mechanism originated from planetary perturbations, see Table 1 and Fig. 2. General comment: A R-depending trend is not visible.

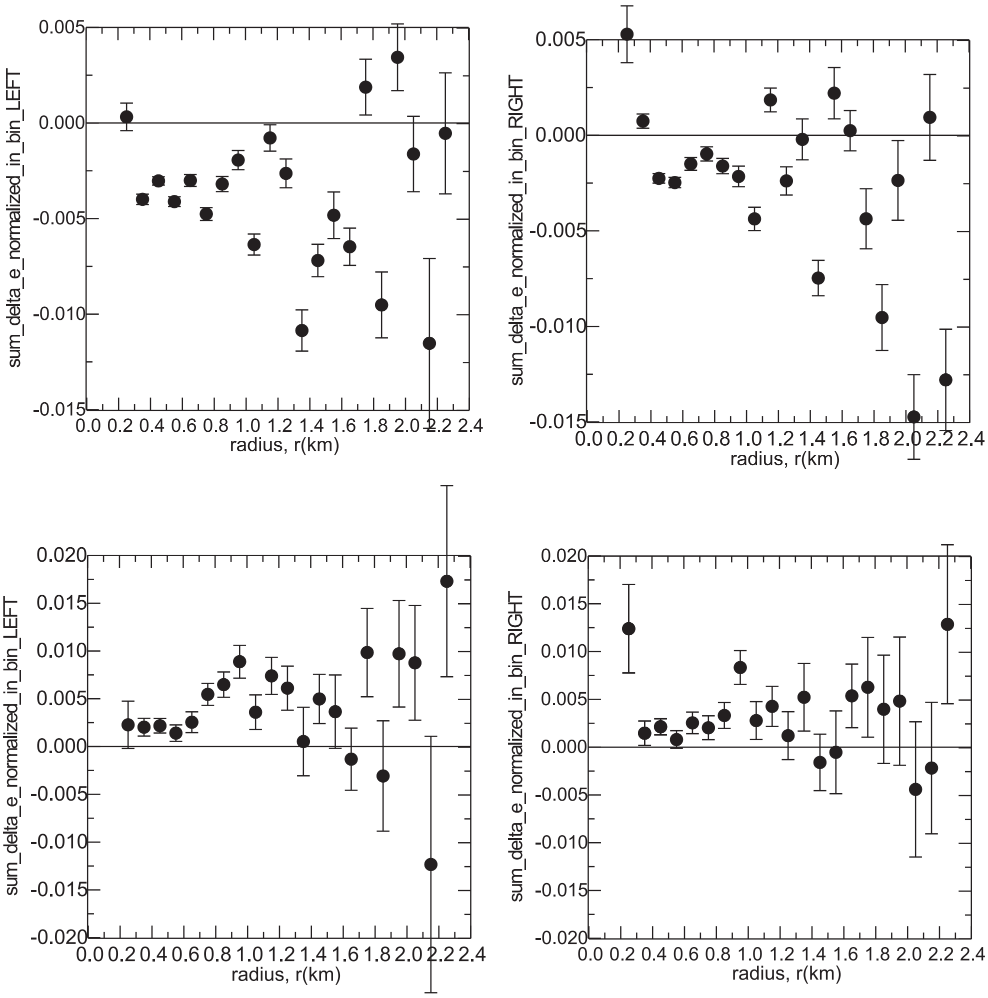

Fig. 7. Evolution of the mean values of eccentricity of the VFMs orbits during the time interval 1 Gy versus their radii according to Eq. (5). Top plates are for the case YN. Assumed error bars are equal to $\pm(10^{-2}n_k^{-1/2})$. Comment: Eccentricity changing rate varies from $\sim$(0.00/1Gy) to $\sim$ (-0.05/1Gy) when radii of asteroids increase. Bottom plates are for the case YY. Assumed error bars are equal to $\pm(3\times10^{-2}n_k^{-1/2})$. Comment: Eccentricity changing rate is $\sim$(+0.05/1Gy) and its dependence versus asteroids radii is not visible.

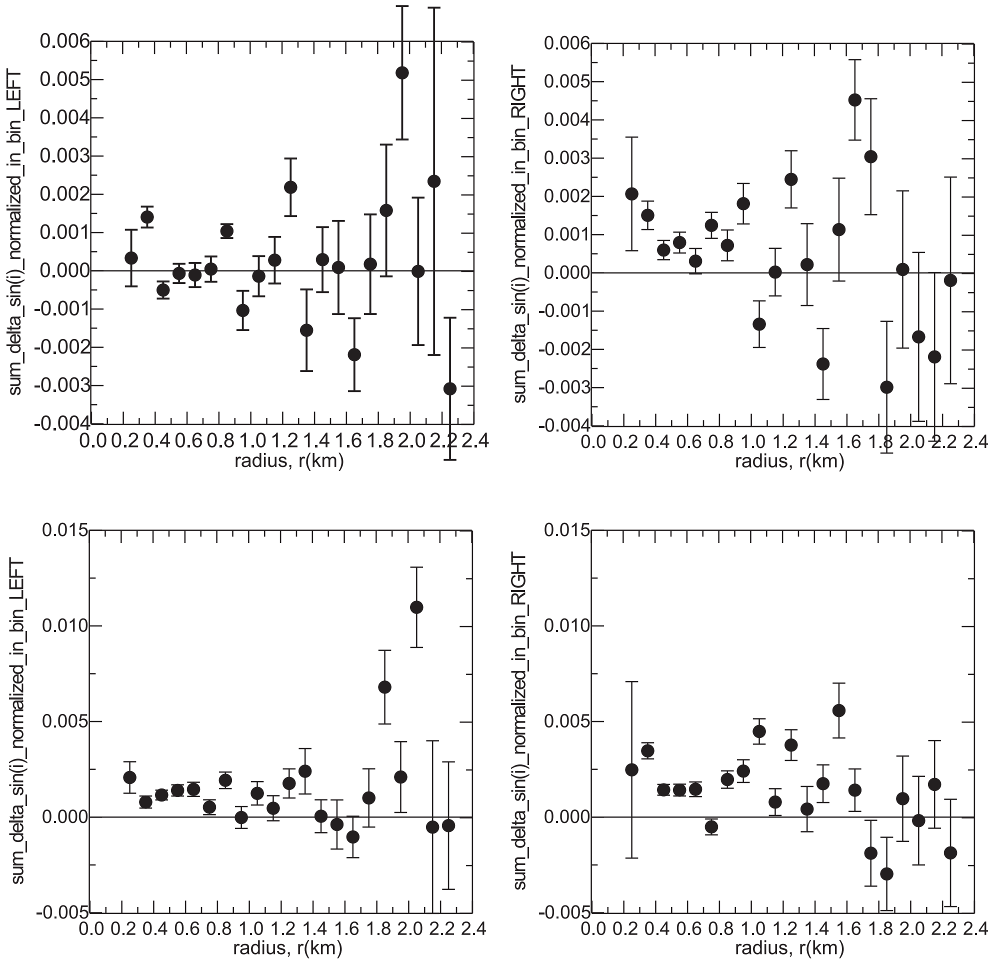

Fig. 8. Evolution of the mean values of orbital inclination of VFMs during the time interval 1 Gy versus their radii according to Eq. (6). Top plates are for the case YN. Assumed error bars are equal to $\pm(10^{-2}n_k^{-1/2})$. Comments: Evolution of the mean values of orbital inclination, asymmetry LG/RG, and dependence on the radius are not visible. Bottom plates are for the case YY. Assumed error bars are equal to $\pm(3 \times 10^{-2}n_k^{-1/2})$. Comments: Asymmetry LG/RG and dependence on the radius are not visible. However, evolution of the mean values of orbital inclination is equal to $\sim$0.11/1Gy for both the LG and RG.

Conclusion

The calculations demonstrate clearly that the orbital evolution of the whole ensemble of VFMs differs for the cases YN and YY. In particular:

(a)A dispersion of the family is almost invisible for the case YN but it is equal to about 10%/1Gy by number as well as by mass for the case YY (Table 2, Fig. 2).

(b) Asteroids to be ejected from the VF are, initially, mostly in the vicinity of the MMRs in the case YN (Fig. 3 - left and middle panels). In contrast, for the case YY, the asteroids are ejecting almost uniformly from the whole area that they occupied initially. However, in the vicinities of the MMRs indistinct maxima are slightly marked; maximum 3:1J is stronger than 7:2J (Fig. 4 - left and middle panels).

(c) The ratio YY/YN indicates that the role of YE is slightly stronger for the smaller asteroids than for the larger ones (Fig. 5 right panel).

(d). In the Tables 1 and 2 as well as in some plots the left/right (that is the Sun inward/outward) asymmetry of ejection from the VF is marked.

(e) Dependence of orbital evolution versus asteroid size is rather marginal (case YN; Fig. 7 - top panels) or invisible (cases YN and YY; Fig. 8).

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