Structural Function Analysis of Selected Luminous Blue Variables in the Local Group

Gantcho Gantchev[1], Antoniya Valcheva[1], Evgeni Ovcharov[1] & Petko Nedialkov[1]
[1] Department of Astronomy, University of Sofia, 5 J. Bourchier blvd., Sofia 1164
g_gantchev@phys.uni-sofia.bg
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Abstract. We compiled historical observations spanning ∼100 yr period for a dozen of the best studied LBVs in the Local Group. We constructed structure functions for their light curves and calculated two parameters that describe the LBVs' behavior: structure function slope and characteristic time scale. The sensitivity of these parameters to the variability behavior of the stars was tested with a number of photometric data sets. The slope of the structure function may anti-correlate with the time scale. Our preliminary analysis hints that the time scale of the LBVs may be used to extend the period–luminosity relation, combining classical Cepheids and LBVs, and using the LBVs as an extragalactic distance indicator.
Key words: Luminous Blue Variables, structure function, time scale

1 Introduction

Luminous blue variables (LBVs) are rare hot massive stars undergoing sporadic violent eruptions and mass-loss events on time scales of years and decades (Humphreys & Davidson, 1994). They are bright and massive (typically $M_{bol} \leq -9.6$ mag, $\geq 50\,M_{\odot}$, respectively), and the most luminous of them are found close to the Eddington limit. LBVs represent a short phase (∼40 000 yr) in massive stars' evolution characterized with a strong mass-loss ($\dot{M} \sim 0.3$–$0.5\,M_{\odot}\,\mathrm{yr}^{-1}$ during the eruptions). They are now considered to be transition objects from early O stars towards Wolf-Rayet (WR) stars (Meynet, Eggenberger & Maeder, 2011). However, recent theoretical work (Groh et al., 2013) has shown that less massive (20-25 $M_{\odot}$) rotating stars can also undergo an LBV phase after the Red supergiant (RSG) stage and before exploding as a supernova.

LBVs can increase their brightness during major eruptions – occurring once in a few centuries – by more than 3 mag. The ejected mass exceeds one solar mass and may reach up to 10 $M_{\odot}$, as estimated for η Car and P Cygni. Smaller "normal" eruptions, observed in the famous S Dor and AG Car, cause variations of 1–2 mag on time scales of years to a few decades. The effective temperatures at minimum light, or the quiescent stage are T_{eff}=10 000–30 000 K. This state usually lasts several years and is followed by visual brightening of the stars within a few months. At that time a slowly expanding (100–200 km s^{-1}) optically thick "pseudo-photosphere" is formed. The objects redden and reach the Humphreys-Davidson limit while their mass loss accelerates. The apparent temperature decreases to ∼7 000–8 000 K and the brightness maximum is shifted from the UV to the visible wavelengths. After the star has lost enough mass in such an eruption, it returns to its quiescent "hot" state (Humphreys & Davidson, 1994 and references therein). Most often, LBVs exhibit micro-variations of 0.1–0.2 mag with a period of a day to a few dozens of days. Wolf (1989) recorded variations of $\sim 23.5^d$ between 1983 and 1985, shortening to $\sim 14.3^d$ between 1986 and 1987 for the well-studied LBV R71. Van Genderen et al. (1998) report similar findings for AG Car (P$\sim 10^d$) and HR Car (P$\sim 20^d$).

The brightness variations of LBVs provide important constraints to the last evolutionary stages of massive stars during their instability phase. The first objects of this class – η Car and P Cyg in the Milky way – have been observed since the seventeenth century and even earlier, but they were not recognized as objects belonging to the same type until the mid-1960s. Actually, Hubble & Sandage (1953) identified five LBVs in M31 and M33, while looking for photometric variability on archival plates 40 yr ago. Subsequent work brought this number to eight (see Parker, 1993).

The LBV studies usualy meet following obstacles: 1) the short duration of the LBV phase limits the total number of objects, and 2) the lack of large photometric variations on reasonably short time scales which could be easy to cover with homogeneous observations complicates the discovery of new LBVs. Recent spectroscopic surveys have helped to alleviate these problems selecting UV-bright objects (Massey et al., 1996), He-emission objects (Corral & Herrero, 2003), or H-emission stars (Massey et al., 2007 and references therein). The latter work estimated the combined LBVs population of M31 and M33 to several hundred objects, much larger than the $\sim$60 known or suspected LBVs in these galaxies.

The main purpose of this paper is to characterize the variability of several LBVs in M 31, M 33, LMC and Milky Way via the structure function analyses (Hughes et al., 1992) of a long series of photometric data with the primary goal to use this approach for future identification of new LBV candidates. Section 2 describes our method and the LBV sample; results are given in Section 3 and they are discussed in Section 4; Section 5 lists our conclusions.

2 Structure functions: definition and data

The structure function describes the tendency of a source to change its observables as a funciton of the time between two measurements. The structure function analysis tends to be less sensitive to the homogeneity of the observational coverage than other methods utilizing time series of data (e.g., the Fourier transform method) and for that reason it is often used to study the variability of quasars where the time scales are extremely long and can easily exceed years (e.g. Hughes et al., 1992; Hook et al., 1994). Following the original work of Hughes et al. (1994), Ovcharov et al. (2008) have defined the structure function of a photometric time sequence as:

$$S(\tau) = \langle [m(t) - m(t+\tau)]^2 \rangle. \tag{1}$$

Where $m(t)$ is the magnitude at a time t and τ is the time interval or "lag" between the two measurements. The time lags are binned, and the angle brackets express average over measurements within the same time lag bin. Usually, the structure function is parametrized in term of its slope

$$b \ = \ d \ log \, S \ / \ d \ log \, \tau. \tag{2}$$

As shown in Fig. 1 of Hughes, Aller & Aller (1992), the general behavior of a structure function which accounts also for the measurement noise can be characterized by three regimens, clearly:

(i) a plateau at τ much shorter than the characteristic variability time scales, with a level determined by the measurements errors;

(ii) linear part of correlated behavior at τ comparable with the typical variability time scales, where the structure function increases with τ as a power-law function;

(iii) a flattening at τ longer than the variability time scales, caused by random disturbances, although a periodic variability may lead to minima at integer times the period.

The slope of the structure function and the characteristic time scale at which it is plateauing are two of the parameters derived by Hughes et al. (1992) for a sample of ∼50 BL Lac objects and quasars. They explained the shape of $S(\tau)$ with shocked jets, more or less correlated with the objects' magnetic fields.

The photometric variability of LBVs, caused by changes in mass loss rate during light minima and maxima, might be another process suitable to be studied via the structure function analyses. To test this possibility we selected a sample of 10 Local group LBVs with well known photometric light curves covering time spans from a dozen to a hundreds of years. Since most of the observations were carried out before the CCD era and in order to minimize the calibration problems, only historical light curves have been considered in this paper. Modern archival light curves will be investigated in a separate work. The photographic m_{pg} magnitudes from Hubble & Sandage (1953) and Rosino & Bianchini (1973) were transformed into the standard B-band system of Sharov (1990) by adding a constant offsets, determined from the overlapping regions of the light curves, as follows: of 0.56 mag for AF And, 0.60 mag for Var 15 and 0.42 mag for Var A-1. The LBV in our sample and some of the data sets' properties are given in Table 1. The light curve of AF And, spanning 73 years, is shown in Fig. 1 as an example.

Table 1. The LBV sample: summary of observations and the derived structure function parameters. The references for the observation data are: *a* – Hubble & Sandage (1953), *b* – Rosino & Bianchini (1973), *c* – Sharov (1990), *d* – Fernández-Lajús (2009), *e* – Lamers et al. (1998).

LBV ID	Host galaxy	Time coverage	Nr. observations	Maximum light [mag]	Band	Reference	Time scale [yr]	Slope
AF And	M31	1917–1989	394	15.4	B	a, b, c	2.3 ± 0.5	0.87 ± 0.10
AE And	M31	1968–1989	119	16.4	B	c	>31.6 ± 9.1	0.58 ± 0.05
Var A-1	M31	1954–1989	233	16.2	B	b, c	>31.6 ± 9.1	0.73 ± 0.03
Var 15	M31	1954–1989	227	17.2	B	b, c	>17.8 ± 5.1	0.55 ± 0.03
Var A	M33	1920–1954	164	15.6	B	a, b	1.1 ± 0.3	1.12 ± 0.08
Var B	M33	1919–1973	205	14.6	B	a, b	14.1 ± 4.6	0.58 ± 0.04
Var C	M33	1920–1989	139	15.1	B	a, b, c	5.1 ± 1.4	1.27 ± 0.13
Var 2	M33	1915–1989	219	15.3	B	a, b, c	6.5 ± 1.6	1.06 ± 0.04
η Car	Milky Way	1823–2009	1447	–1.2	V	d	40.8 ± 23.5	1.34 ± 0.08
S Dor	LMC	1874–1993	176	8.1	V	e	0.4 ± 0.2	1.53 ± 0.12

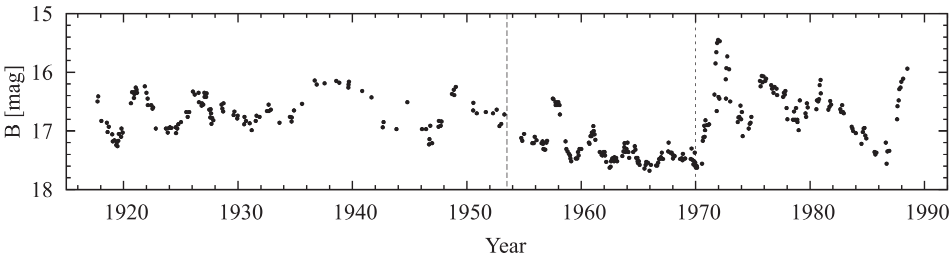

Fig. 1. Homogenized light curve of AF And from 1917 to 1989. The vertical dashed lines separate roughly the three curve data sets used to construct it – Hubble & Sandage (1953), Rosino & Bianchini (1973) and Sharov (1990), from left to right.

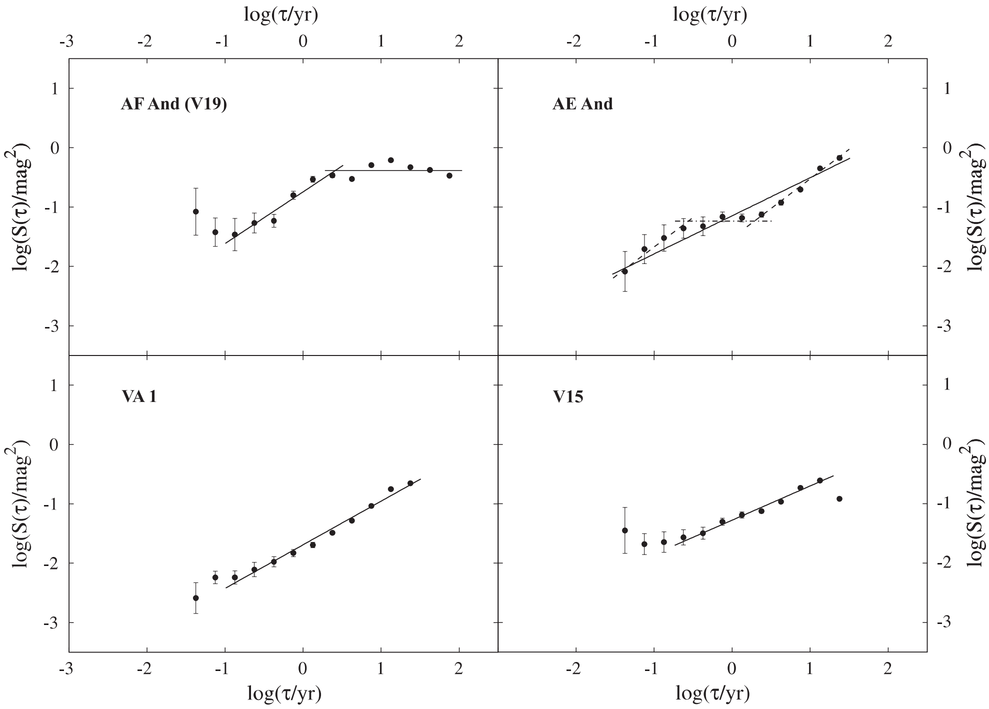

Fig. 2. Structure functions for 4 LBVs in the M31 galaxy are shown. For each variable, the slopes of structure functions are derived from the inclined lines. In the case of AF And (upper left panel), the higher plateau is represented by a horizontal line.

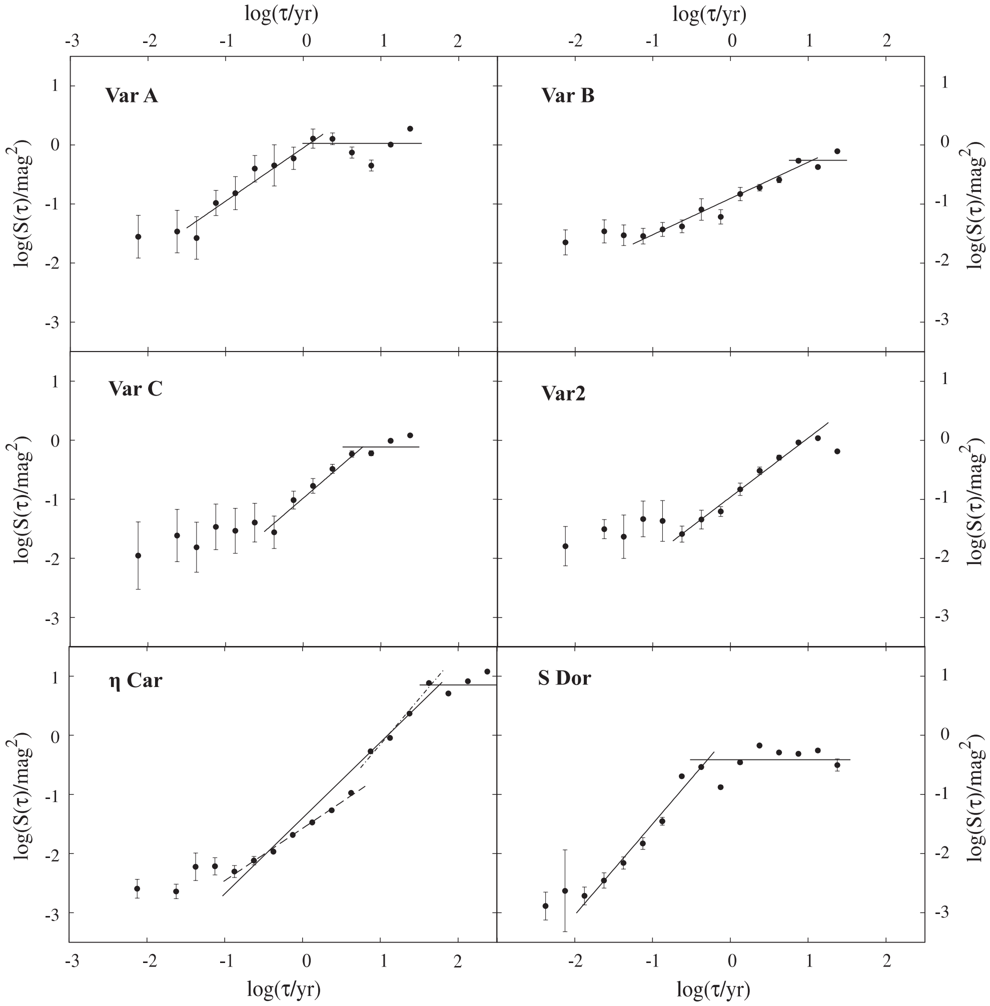

Fig. 3. The same as Fig. 2 but for the 4 LBVs in M33, η Car, and S Dor.

3 Structure functions: slopes and time scales

We have calculated the structure functions for our targets using the expression given in Eq. 1 with the same logarithmic time lag bin equal to 0.25 dex. This value is by a factor of five larger for our sample than the one used by Hughes et al. (1992), but is suitable for our case because: we have on average the same number data-points as Hughes et al., but we aim at obtaining comparable results for all LBVs, especially those with really poor coverage.

The structure function slopes were determined from fitting a first-order polynomial to the data-points located between the time lags where the function is increasing linearly. The characteristic time scale is determined by the intersect of a horizontal line $S(\tau) = const$ running through at least 3 plateau points and the fitting line used to derive the slope. All structure function values following its first decrease are considered to belong to the plateau.

The structure functions of the LBVs in M31 are shown in Fig. 2. Slopes have been derived for AE And because of the hint for a plateau at intermediate time lags. The result might be interpreted as a signature of two distinct processes driving the stellar variability (Hughes et al., 1992). Notably, AE And is the only LBV in M31 that exhibits plateaus at relatively small time scales of ~1 yr with amplitudes ~1 mag corresponding to the so called "normal" variations rather than the sporadic mass loss events. The characteristic time, however, can be derived only for AF And, because among the remaining LBVs in M 31 only V 15 displays a hint for a plateau at long time lags. In those cases we set only a lower limit of that parameter. The structure function for the LBVs in M33 and the Milky Way one (Fig. 3) have a shape similar to the expected one. Thus allows a straightforward derivation of their time scales and slopes, except for Var 2 for which only the minimum time scale is determined. Like AE And, S Dor starts to exhibit a plateau at time scales of ~1 yr. Because the high plateau is ill-defined with a single possible point it is much better fitted with two different slopes, instead of with one. All derived parameters for the objects in our sample are listed in Table 1.

4 Discussion

4.1 Stability of the LBV structure function derived results

As the derived structure function parameters might depend on the type of variability (e.g. quiescent or eruptive stage of the LBV), the passband, the time span covered by the data sets, etc (Humphreys & Davidson, 1994), we investigated the stability of our results using AF And as a test example, because this star has the longest and most uniformly covered light curve among the extra-galactic LBVs (Fig. 1). The fits of the structure function from each data set are shown on the left panel of Fig. 4. All slopes and time scales agree within 1σ level, except for the structure function for the Sharov's (1990) data set: first, it lies above the other structure functions on all time scales, indicating higher amplitude variability during the monitored period; second, it seems to peak at $log(t) = -1.3$ or $\sim 20^d$ which can be related to the monthly variations at a few tenths of magnitude level (similar peaks are seen in the structure functions of Var 2 and η Car – Fig. 3).

Further, to verify the accuracy of the derived parameters, we split the Sharov's (1990) data set into two separate subsets, taking the first and the second half of the observations and recalculating the structure function parameters (Table 2 and Fig. 4, right). The derived slopes and the time scales agree at 2σ level. The same test for the other LBVs in our sample leads to similar results.

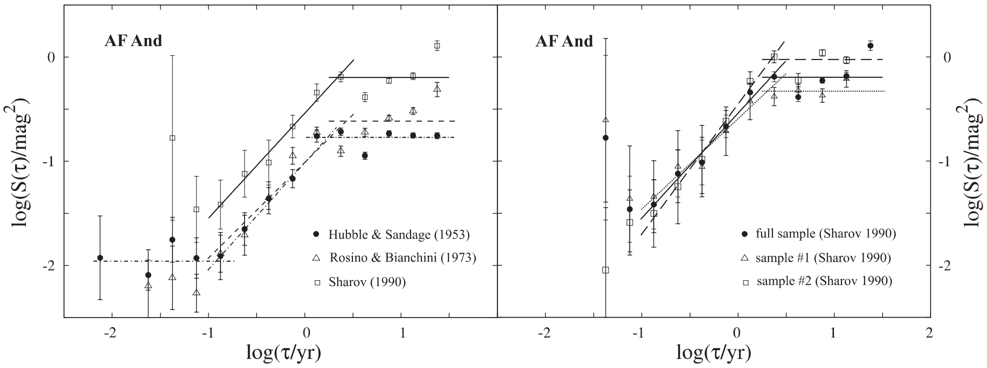

Fig. 4. Left: Structure functions of AF And based on the data sets of Hubble & Sandage (1953), Rosino & Bianchini (1973) and Sharov (1990) (solid dots, open triangles and squares, respectively). **Right:** Structure functions of the same LBV based on the full data set and two independent by time subsets of Sharov (1990) (solid circles, open triangles and open squares, respectively).

Table 2. AF And structure functions parameters derived from different data sets: a – Hubble & Sandage (1953), b – Rosino & Bianchini (1973), c – Sharov (1990). The last data set was divided into two subsets and analysed independently (c^1 and c^2; see the text).

Time scale [yr]	Slope	Reference
2.29 ± 0.47	0.87 ± 0.10	a, b, c
1.17 ± 0.24	0.92 ± 0.08	a
1.75 ± 0.69	1.03 ± 0.13	b
1.52 ± 0.46	1.01 ± 0.06	c
1.52 ± 0.32	1.25 ± 0.05	c^1
1.35 ± 0.28	0.87 ± 0.13	c^2

4.2 LBVs as distance indicators

Ivanov (1989) suggested that the semi-periodic behavior of LBVs in M31 might place these objects on an extension to the period–luminosity relation for Cepheids if the regular Cepheid periods are replaced by the LBV's characteristic time scales derived via structure function analyses. If this extended period-luminosity relation holds for LBVs, their high intrinsic luminosity might make the semi-periodic LBVs useful extragalactic distance indicators. To check this possibility we performed structure function analysis for classical Cepheids with extremely long periods and built a combined Cepheid and LBV diagram. Note that in case of periodic behavior the structure function is equivalent to the well known phase dispersion minimization (PDM) technique (Lafler & Kinman, 1965).

The result from the structure function analysis for the Milky way classical Cepheid S Vul (period P$\sim$68^d; Heiser (1996)), with 326 V-band observations covering several periods, is shown in Fig. 5. The linear part is well defined, and has a slope of 1.52±0.06. The estimated time scale is $16{\pm}2^d$ and it is characterizing the rising time of $\sim 17^d$ of the S Vul light curve. However, the minima of the function at $\sim n{\times}$P (where n is an integer number) of the Cepheid's period P are apparent only at smallest bin size, which underlines the advantage of having a rich observational data set. The minima disappear if the bins are too wide, or if the dataset is too sparse (which is the case for the Berdnikov & Ivanov (1986) data set). We reached a similar conclusion from the structure function analysis of AF And – we see a shallow minimum at time lags 3–5 yr.

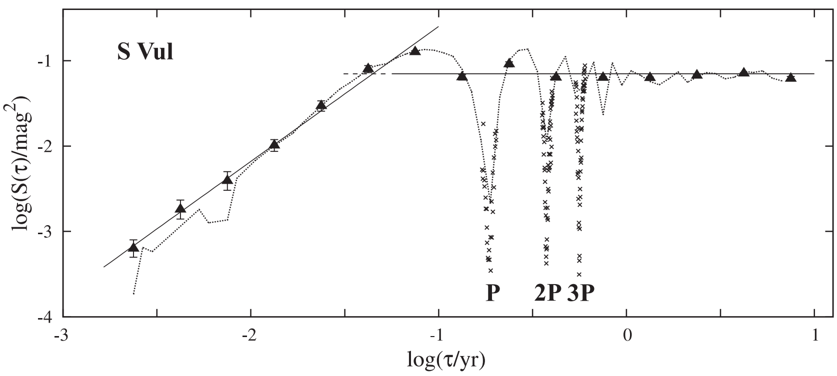

Fig. 5. Structure function analysis of the Milky Way Cepheid S Vul (data from Heiser, 1996) for three logarithmic bin sizes: 0.25 dex (filled triangles), 0.05 dex (dotted line) and 0.001 dex (×'s). Only in the last case the minima at the multiples of the period $P{=}68.20^d$ reaches the level of the photometric accuracy.

Fig. 6 shows the slopes versus time scales for the objects we analyze here. To show that QSO+BLLacs may be a source of contamination for LBV samples build using the structure function we marked the locus occupied by the QSO+BL Lac sample of Hughes et al. (1992). We remind the reader that the slopes and the time scales for QSO+BL Lac are derived from data sets different from those for the LBVs, and may contain biases that need further investigation, preferably with a comparable observational data sets. Six (AF And in M 31, Var A, Var C and Var 2 in M 33, η Car in the Milky Way, and S Dor in LMC) of the ten LBVs fall within the locus of active galactic nuclei. The average slopes of the two samples are indistinguishable: ~1.0±0.4 for the LBVs falls well withing the QSOs/BL Lacs, but slopes and logarithmic time scales of the LBVs occupy somewhat wider parametric space than those of QSOs/BL Lacs.

There is a group of 4 LBVs (3 in M 31 and Var B in M 33) that have systematically flatter slopes at long time scales. Unfortunately, for three of them we only can set lower limits on the characteristic time scales, but they hint at an anti-correlation between the slope and the time scale. If we treat the lower limits as measurements, the correlation coefficient module is

0.31, but if we exclude these, it drops down to 0.07. More data are needed to investigate this possibility.

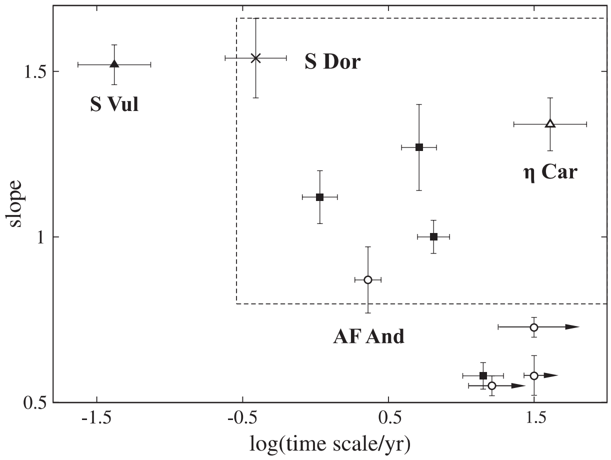

Fig. 6. Slopes versus time scales for the objects analyzed in this paper. The LBVs in M31 are plotted with open circles, in M33 – with solid squares, in LMC – with $\times$'s, in the Milky Way – with open triangles, and the Cepheid S Vul – with a solid triangle. The locus of QSO and BL Lac objects defined by Hughes et al. (1992) is marked dashed line.

The contribution of Var A to our conclusions have to be treated with caution because finally, the Humphreys & Weis (2014) reclassified this object as a post-red supergiant, evolving into a WR star. The parameters we derive have typical values close to the mean of the sample, except for the a prominent minimum at $t = 7.5$ yr, that can be related to a cyclic process rather than to a spontaneous eruption.

5 Conclusion

The results outlined in this paper suggest that the structure function analysis might be a useful tool for interpreting the variability of LBVs in the Local group on various times scales – from several days to nearly a century. In many cases we were able to recover from historical data the expected typical shape of the structure function: the two plateaus at the shortest and at the longest time lags, and the power-law portion in between them.

We derived structure function slopes for all ten LBVs and characteristic time scales for seven of them; we set lower limits for time scales of the remaining three. There are indications that these two parameters may be anti-correlated, but more data are needed to verify this result.

The structure functions of some LBVs in our sample deviate from its "universal" shape: AE And with plateau at middle-range time lags, Var A and S Dor with a prominent minimum after the longest correlation time scale and η Car for which the correlation range is best fitted with two different power-law slopes. In some cases the LBV's monthly variations may influence the structure function at the corresponding short time lags.

We use a well-studied long-period Cepheid S Vul to demonstrate that the structure function of a periodical variable shows a series of minima at long time scales, corresponding to the multiples of its period. The superposition of these minima can result in a flattening of the structure function.

Most importantly, it seems that if the LBV's characteristic time scales derived via structure function analyses are used instead of the regular Cepheid periods, then the LBVs can be placed on an extension to the period–luminosity relation. This has the potential to turn the LBVs into useful extragalactic distance indicators. This possibility needs a revisit with better data sets.

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