Dark matter and rotation curves of spiral galaxies

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Abstract. We suggest that nonbaryonic dark matter need not be taken into account to explain the observed rapid rotation of spiral galaxies. The main reason is a special form of the gravitational potential of a flat disk which guarantees large orbital velocities of stars at the galaxy edge. In particular, we prove that a star orbiting a central mass point along a circular trajectory of radius R has a smaller speed than if it were to orbit a flat disk of radius R and the same mass with an arbitrary rotationally symmetric density distribution.

Key words: red dwarf, dark matter, spiral galaxy, orbital velocity, gravitational potential, Shell Theorem

1. Introduction

The term *dark matter* first appeared in Jan Oort's paper (Oort, 1932, p. 285). One year latter Fritz Zwicky used this term in (Zwicky, 1933, p. 125) to explain large velocities of several galaxies in the Coma cluster A1656. With the help of classical Newtonian mechanics he derived a very simple relation for the virial mass of the cluster which gave him much larger values than could be accounted for by just the observed luminous mass. However, his detailed calculation in (Zwicky, 1937) is based on many simplifications. For instance, he assumed that galaxies are distributed uniformly, that the Virial Theorem holds exactly, and that gravitation has an infinite speed of propagation. He substituted a spacetime curved by more than one thousand galaxies by Euclidean space. He replaced galaxies of diameter about 10^{10} au by mass points. Such approximations do not allow one to consider angular momenta of rotating galaxies that surely contribute to the total angular momentum. Tidal forces among galaxies were not included as well. Further simplifications are listed in (Křížek, Křížek, Somer, 2014). Moreover, Zwicky's observational data were not relevant, since he largely underestimated the number of stars in galaxies and their distances from the Earth. For present data the discrepancy between the virial mass and luminosity mass is not so obvious. We showed that the nonuniformity of galaxy distribution, relativistic effects of high velocities, gravitational redshift, the selflensing effect, the decreasing Hubble parameter, dark energy, and some other phenomena can essentially reduce the virial mass and thus nonbaryonic dark matter may not exist.

The existence of nonbaryonic dark matter was also postulated from the Friedmann equation. However, this equation was derived by means of excessive extrapolations as we will explain in the last section, see also (Křížek, Somer, 2014). Therefore, we will investigate other methods which look for dark matter and which are independent of the Friedmann equation. This paper can be regarded as a natural continuation of our previous work (Křížek, Křížek, Somer, 2014) on this topic.

Vera Rubin's greatest discovery was the fact that spiral galaxies have "flat" rotation curves (Rubin et al., 1962). On that basis, in the 70's of the last century she developed her own theory of rotation curves of galaxies. From the high orbital speed of stars she concluded that galaxies should contain much more nonluminous than luminous matter to be kept together by gravity — see e.g. her review articles (Rubin, 1983) and (Rubin, 2003) on dark matter.

In this article we shall look more closely at her hypothesis. Consider a test particle of mass m (typically this will be a star) and let $M \gg m$ be the mass of some body generating the central force field. Assume that the test particle revolves about the center along a circular orbit with radius r and speed v. Then from Newton's law of gravitation and the relation for centripetal force Rubin easily obtained that, see (Rubin, 1983),

$$G\frac{Mm}{r^2} = \frac{mv^2}{r}, \quad \text{i.e.} \quad v = \sqrt{\frac{GM}{r}}. \tag{1}$$

The velocity v of a particle on a circular orbit is thus proportional to $r^{-1/2}$. Such orbits are called *Keplerian* (see Fig. 1).

Rubin states that, see (Rubin et al., 1962, p. 491),

the stellar curve does not decrease as is expected for Keplerian orbits.

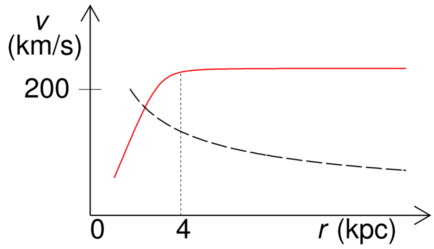


Fig. 1. The dashed line shows the rate of decrease of velocities for Keplerian orbits that depend on the distance r from the center of a spiral galaxy. The solid line shows an idealized rotation curve whose shape was derived by Rubin by means of a variety of measurements.

To explain this discrepancy, it is important to realize that spiral galaxies do not have a central force field except within a close neighborhood of the center, where e.g. in the Milky Way stars S1, S2, ... orbit the central black hole according to Kepler's laws with velocities up to 7000 km/s. The mass of this black hole is roughly 3.5 million solar masses, which is less than 0.01 % of the total mass of our Galaxy (cf. (5)). In the Solar system, on the contrary, 99.85 % of the mass is concentrated at the Sun. The planets barely interact gravitationally among themselves and their movements are determined mainly by the central force of the Sun. On the other hand, trajectories of stars in a galactic disk are substantially influenced mainly by neighboring stars, because the central bulge contains only about 10 % of all mass of a galaxy. Let us recall the following famous statement:

Newton's First Theorem. *If the density distribution of a ball of mass M is spherically symmetric, then the size of the force between the ball and a point mass m, that lies outside the interior of the ball, is given by the left-hand side of* (1), *where r is the distance between the point and the center of the ball.*

In Section 2, we outline why the force of a disk-shaped galaxy acting on a test particle is much larger than it would be if its whole mass were to be concentrated at one central point (cf. Newton's First Theorem). Therefore, the speed v of stars on circular orbits in a spiral galaxy should be higher than for Keplerian orbits (see Fig. 1).

In nearby spiral galaxies Rubin found, see (Rubin, 2003) and also (Rubin et al., 1980, p. 480), that all stars of these galaxies move at almost the same constant speed of order $v \approx 200$ km/s for $r > r_0$, where $r_0 > 0$ approximately corresponds to the radius of the central bulge and is typically equal to a few kpc (see Fig. 1). On the other hand, she observed that for $r \leq r_0$ the inside of the spiral galaxy (including a possible bar) rotates with roughly constant angular speed in a manner like that of a DVD record, i.e., the speeds of these stars are approximately linearly proportional to their distance from the center (see Fig. 1). An exception is a very close neighborhood of the central black hole. In Section 3, we show that large orbital velocities of stars (cf. (4) and (6)) can also be explained by presently measured mass distribution.

The average thickness of the disk (except for the bulge) of spiral galaxies varies from 300 pc to 1 kpc. It is therefore about 30 to 100 times thinner than the diameter of the visible part of the galaxy. This is easily seen when galaxies are observed edge-on. Moreover, the gas and dust are mainly found close to the plane of symmetry of the disk (Pohen et al., 2010). Consequently, in Section 4 we will treat the disk just as a two-dimensional body, which is obviously a better approximation than a central mass point. The gravitational field of spiral galaxies will therefore be approximated by the gravitational field of a flat disk with rotationally symmetric mass density distribution.

In Section 5, we include the bulge and halo. Finally, Section 6 is devoted to discussion on the existence of nonbaryonic dark matter.

2. Forces acting on a flat disk

The equation on the right of (1) provides only a rough estimate for expressing orbital velocities of stars in a spiral galaxy. Let us therefore show now that a test particle (star) orbiting a ball of radius r with arbitrary spherically symmetric mass density distribution has a lower speed than when orbiting a disk of the same radius r and the same mass. In doing so, we will consider a special distribution of the density of the disk, which arises as projection of the original ball perpendicularly to the horizontal xy plane of the disk.

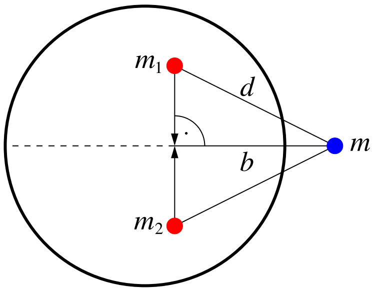


Fig. 2. A ball with symmetrically distributed mass with respect to the horizontal plane acts on a test particle by a smaller force than the mass projected perpendicularly to the horizontal plane of the disk — dashed.

To be convinced of this assertion, just consider two arbitrary mass points with masses $m_1 = m_2$ located inside a ball placed symmetrically with respect to the horizontal xy plane (see Fig. 2). Then the total force F of both mass points acting on the test particle of mass m, will be less than the force $\overline{F}$ of both mass points projected perpendicularly to the disk and acting on m. Let d be the distance between m_1 and m. Denoting by b its orthogonal projection on the horizontal xy plane, we find that

$$F = G\frac{2m_1 m}{d^2} \cdot \frac{b}{d} \quad \text{and} \quad \overline{F} = G\frac{2m_1 m}{b^2}.$$

Thus we see that the ratio of forces $\overline{F}$ and F is equal to the third power of the fraction d/b, namely,

$$\overline{F} = \left(\frac{d}{b}\right)^3 F \geq F. \tag{2}$$

By (1) this cubic nonlinearity causes a greater attractive gravitational force by the disk than by the ball, and thus also a higher orbital speed around the disk. A more detailed estimate will be given in Section 4. An analytical expression of the gravitational influence of the entire disk on an outer test particle leads to elliptic integrals (Binney, Tremaine, 1987, p. 73).

3. Orbital velocity around a spherically symmetric body

In this section we introduce a rough conservative estimate for the orbital velocities of stars for the case in which all baryonic matter (i.e. mainly protons and neutrons) of the Milky Way is replaced by a ball with spherically symmetric mass density distribution. In the next section we will focus on a flat disk with arbitrary rotationally symmetric mass density distribution.

The radius of the visible part of the disk of our Galaxy is estimated by

$$r_{\mathrm{G}} = 16 \text{ kpc} = 4.938 \cdot 10^{20} \text{ m}. \tag{3}$$

Our Sun has the mass

$$M_\odot = 2 \cdot 10^{30} \text{ kg}$$

and orbits the center of the Milky Way with the speed (cf. Fig. 1)

$$v_\odot = 230 \text{ km/s} \tag{4}$$

on a path of radius $r_\odot = 8.3$ kpc, i.e. it is found about halfway out from the center of the Galaxy, where the density of stars is relatively small. Note that most sources give the speed of the Sun $v_\odot$ as being in the range of 220 to 240 km/s. Stars orbiting the center of our Galaxy at a distance $r > r_0 \approx 3$ kpc should have a speed similar to $v_\odot$ due to the expected flat rotation curve (see Fig. 1).

Denote by $\mathcal{M}(r)$ the *mass of baryonic matter* within the ball of radius r and center placed at the center of gravity of our Galaxy. To estimate $\mathcal{M}(r_G)$ for r_G given by (3) we will use the distribution of stars from Table 1, see e.g. (Mikulášek, Krtička, 2005, p. 394). It is based on Hipparcos' data taken from our close neighborhood up to a distance of several hundreds parsecs. The Harvard Spectral Classification shows a similar relative representation of stars that will be further improved by data from the Gaia satellite. Gaia is able to look at the center of our Galaxy and in the opposite direction also at its boundary. However, the accuracy of measurements depends essentially on the magnitude and extinction. The mass distributions of stars in disk galaxies (initial mass function) have been studied extensively also by other authors. Let us mention e.g. the seminal works (Kroupa, 2001, 2002) and (Chabrier, 2003).

Table 1. Distribution of stars in our Galaxy according to their spectral classes. The second line shows the corresponding mass of a typical star in units of the solar mass $M_\odot$. The third line indicates the number of stars of a particular spectral class divided by 10^9. The last line presents the calculated mass of all the stars in a particular spectral class in billions of solar masses. The last column corresponds to white dwarfs (WD).

Spectral class	O	B	A	F	G	K	M	WD
Mass in $M_\odot$	25	5	1.7	1.2	0.9	0.5	0.25	0.7
Number in billions	10^{-5}	0.3	3	12	26	52	**270**	35
Product	≈ 0	1.5	5.1	14.4	23.4	26	**67.5**	24.5

From the third line of Table 1, we see that our Galaxy contains approximately 400 billion stars. While at the end of the last century it was thought that red dwarfs of spectral class M form only 3 % of the total number of stars, see (Binney, Merrifield, 1998, p. 93), at present it is estimated from Table 1 that they are in the vast majority — about 70 %. To support this statement it should be noted that among the 20 nearest stars to our Sun, 13 red dwarfs are currently known. Note that the mass of a red dwarf ranges from $0.08M_\odot$ to $0.45M_\odot$. From the last line of Table 1 it is evident that the spectral class M contributes the most to the total mass of our Galaxy of all the spectral classes. The mass of the most common star is about $0.4M_\odot$. Rubin, of course, could not know about the existence of so many red dwarfs in this smallest weight category. This growth is due to the continual improvements of the sensitivity of space telescopes. In this way, the estimated mass of the baryonic matter in our Galaxy has considerably increased. Summing up the numbers in the last row of Table 1, we get

$$\mathcal{M}(r_{\rm G}) \geq 162.4 \cdot 10^9 M_\odot = 3.25 \cdot 10^{41} \text{ kg}.$$

The amount of stars in the left part of Table 1 is so small, because they live very briefly. On the other hand, there may exist many superdense compact remnants left by these stars in the Galaxy. Unfortunately, we cannot so far reliably determine the contribution to $M(r_{\rm G})$ from black holes, neutron stars, infrared dwarfs, exoplanets, etc., whose luminosity is small. The three new spectral classes for small cold stars include: L (red-brown dwarfs), T (brown dwarfs), and Y (black dwarfs). Their total mass is probably also non-negligible. For instance, in 2013 Kevin Luhman discovered a pair of brown dwarfs only 6.5 ly from the Sun. Another brown dwarf WISE J085510.83-071442.5 is located 7.2 ly from us.

According to (Mikulášek, Krtička, 2005, p. 393), the mass of the baryonic matter of all the stars in the Galaxy is about

$$175 \cdot 10^9 M_\odot = 3.5 \cdot 10^{41} \text{ kg},$$

including further stars of the luminosity classes I–IV (i.e. supergiants, giants, and subgiants). The disk and bulge contains also a large amount of non-luminous baryonic matter in the form of dust, gas, and plasma (Pohlen, 2010). It is well known that atomic hydrogen contributes substantially to the observed rotation curves. Radio observations of many spiral show that it extends far beyond the stellar disk.

In (Mikulášek, Krtička, 2005, p. 353), the amount of interstellar matter (without hypothetical nonbaryonic dark matter) is estimated at about 10% of the total mass of the Milky Way's stars. Sparse non-luminous baryonic matter is also spread in a spherical galactic halo, as can be determined from radio waves of 21 cm, corresponding to spin flip in the hydrogen atom, see (Rubin et at., 1980, p. 485). Therefore, for the total mass of baryonic matter inside the considered ball of radius $r_{\rm G}$ we have a lower estimate

$$\mathcal{M}(r_{\rm G}) \geq 3.85 \cdot 10^{41} \text{ kg}. \tag{5}$$

By astronomical tables (Lang, 2006, p. 127) the total mass of the Galaxy is $M_{\rm G} = 10^{12} M_\odot = 2 \cdot 10^{42}$ kg. Another source (Irrgang et al., 2014) even reports a three times greater value amounting to 200 kpc from the center.

According to (Mikulášek, Krtička, 2005), the mass density distribution $\rho = \rho(r)$ beyond the visible edge decreases faster than r^{-2}; otherwise the integral $\int_{r_{\rm G}}^\infty \rho(r) 4\pi r^2 \mathrm{d}r$ would diverge. However, the Shell Theorem indicates that this matter (including possible nonbaryonic dark matter) has no effect on the movement of stars, if the mass distribution is spherically symmetric. By Newton's First Theorem, we may concentrate all baryonic matter inside the ball of radius $r_{\rm G}$ at one central point. Then from relations (1), (3), and

(5) we find that the orbital velocity of stars on the radius r_G of the visible disk is

$$v = \sqrt{\frac{G\mathcal{M}(r_{\rm G})}{r_{\rm G}}} \geq \sqrt{\frac{6.674 \cdot 10^{-11} \cdot 3.85 \cdot 10^{41}}{4.938 \cdot 10^{20}}} = 228 \cdot 10^3 \text{ m/s}, \qquad (6)$$

This value is indeed comparable to the measured speed (4). Although relation (6) is only approximate, to postulate the existence of 5–6 times more nonbaryonic dark matter than baryonic matter, see e.g. (Bosma, 2003) and (Planck, 2014), to hold the Galaxy together by gravity seems to be somewhat overestimated due to (2). Now we will elaborate it on more details.

4. Orbital velocity around a flat disk

Of course, one can raise the objection that relationship (6) was derived just in the case of a central force for a given mass point (that is equivalent to a ball with a spherically symmetric mass density distribution) which may lead to a large modeling error. In this section we will therefore approximate the gravitational field of a spiral galaxy by the gravitational field of a flat disk with rotationally symmetric mass density distribution.

Theorem 2. *A particle orbiting a central mass point along a circular trajectory of radius R has a smaller speed than if it were to orbit a flat disk of radius R and the same mass with an arbitrary rotationally symmetric density distribution.*

P r o o f . A greater attractive force has to be balanced by a larger orbital speed if the testing particle should stay on a circular trajectory. Therefore, we only need to compare the force of the central mass point with the force of a disk of the same mass. Under the assumptions of Theorem 2 the areal density of the disk $\rho = \rho(r) \geq 0$ depends only on the distance from its center. First, we will investigate the gravitational influence of a fixed one-dimensional homogeneous ring of radius $r \in (0, R)$ on a test particle of mass m, whose distance from the center is R. The total mass of the ring equals $M = 2\pi r\rho$, where ρ is the length density (i.e. one-dimensional mass density). Concentrating the mass of the ring at its center, the corresponding force acting on a test particle is equal to

$$\overline{F} = G\frac{2\pi r\rho m}{R^2}. \qquad (7)$$

Our goal will be to show that $\overline{F}$ is smaller than the force of the ring acting on the test particle. The statement of Theorem 2 will then follow by integration along r.

In polar coordinates (r, φ), consider two equal length elements of the ring

$$\mathrm{d}l = r\,\mathrm{d}\varphi \qquad (8)$$

located symmetrically with respect to the horizontal axis at a distance s from the test particle as shown in Fig. 3. Then according to the law of cosines, we have

$$s^2 = r^2 + R^2 - 2rR\cos\varphi \qquad (9)$$

and the force with which this pair acts on the test particle equals

$$\mathrm{d}F = G\frac{2\mathrm{d}l\,\rho m}{s^2}\cos\alpha. \tag{10}$$

From the law of sines $r\sin\varphi = s\sin\alpha$ it follows that

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \frac{1}{s}\sqrt{s^2 - r^2\sin^2\varphi}. \tag{11}$$

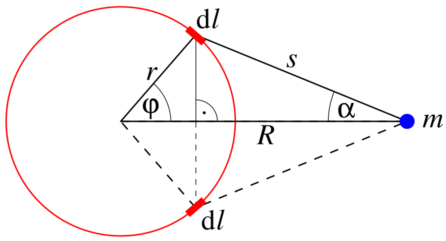


Fig. 3. A homogeneous ring acts gravitationally on an outer particle by a larger force than if the total mass were to be concentrated at the center of the ring.

Without loss of generality we may further assume that the gravitational constant $G = 1$, $R = 1$, $m = 1$, and that the length density of the ring is $\rho = 1$. Then for $r \in (0, 1)$ and $\varphi \in [0, \pi]$, by substituting (8), (9), and (11) into (10), we get

$$\mathrm{d}F = \frac{2\,\mathrm{d}l}{s^2}\frac{1}{s}\sqrt{r^2+1-2r\cos\varphi - r^2\sin^2\varphi} = \frac{2r\,\mathrm{d}\varphi}{s^3}\sqrt{(1-r\cos\varphi)^2}$$
$$= 2r\frac{1-r\cos\varphi}{(r^2+1-2r\cos\varphi)^{3/2}}\mathrm{d}\varphi,$$

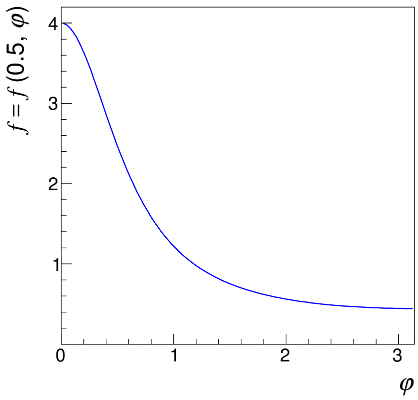
because $1 > r\cos\varphi$. Thus the total gravitational force of the ring of radius r that acts on the test particle is

$$F(r) = 2r\int_0^\pi f(r,\varphi)\mathrm{d}\varphi = 2r\int_0^\pi \frac{1-r\cos\varphi}{(r^2+1-2r\cos\varphi)^{3/2}}\mathrm{d}\varphi, \tag{12}$$

where for a fixed $r \in (0, 1)$ the integrated function

$$\varphi \mapsto f(r,\varphi) = \frac{1-r\cos\varphi}{(r^2+1-2r\cos\varphi)^{3/2}}$$

is positive, continuous, and decreasing. Since the values at the endpoints $f(r, 0) = (1-r)^{-2}$ and $f(r,\pi) = (1+r)^{-2}$ are finite numbers, the integral in (12) is finite as well (see Fig. 4).



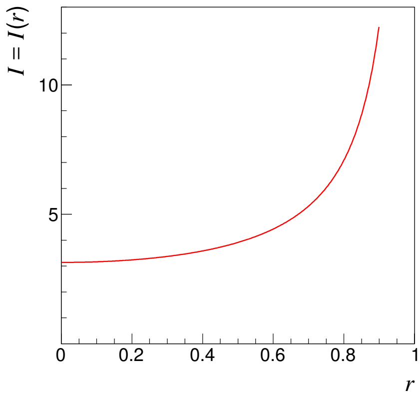


Fig. 4. On the left there is a graph of the integrated function (13) for $r = 0.5$ on the interval $[0, \pi]$. On the right there are numerically calculated values of the integral $I(r)$ for $r \in [0, 1)$.

The integral

$$I(r) = \int_0^\pi \frac{1 - r\cos\varphi}{(r^2 + 1 - 2r\cos\varphi)^{3/2}} \mathrm{d}\varphi \tag{13}$$

appearing in equation (12) unfortunately has no known analytical expression for $r \in (0, 1)$. However, we can find that $I = I(r)$ is an increasing function and may analytically evaluate its limits. The function I is even strictly convex and $\dot{I}(0) = 0$. For $r = 0$ we see that the integrated function is equal to one, and thus (see Fig. 4)

$$I(0) = \pi. \tag{14}$$

Consider now the point $r = 1$. By the Taylor expansion we get

$$\cos\varphi = 1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} - \frac{\varphi^6}{6!} + \cdots \geq 1 - \frac{\varphi^2}{2}.$$

Hence,

$$\varphi^2 \geq 2 - 2\cos\varphi, \tag{15}$$

and therefore we obtain (see Fig. 4)

$$2I(1) = \int_0^\pi \frac{2 - 2\cos\varphi}{(2 - 2\cos\varphi)^{3/2}} \mathrm{d}\varphi = \int_0^\pi \frac{\mathrm{d}\varphi}{\sqrt{2 - 2\cos\varphi}} \geq \int_0^\pi \frac{\mathrm{d}\varphi}{\varphi} = \infty, \tag{16}$$

that is

$$I(1) = \infty. \tag{17}$$

A similar trick with the lower estimate can be used to replace the cosine function in (13) by quadratic polynomials in φ, which can already be calculated analytically and leads to the required inequality

$$F(r) = 2rI(r) > \overline{F} = 2rI(0) \quad \text{for } r \in (0, 1], \tag{18}$$

where the forces are defined in (12) and (7) for $G = 1$, $R = 1$, $m = 1$, and $\rho = 1$. Q.E.D.

Note that for a spherically symmetric mass distribution of the halo we may neglect by the Shell Theorem the influence of dark and baryonic matter outside a ball that contains the galactic disc. For simplicity, assume that the areal mass density $\sigma = \sigma(r)$ of the galactic disk decreases as r^{-1}. Then the total mass of the disk inside the circle of radius r is

$$\mathcal{M}(r) = 2\pi \int_0^r \sigma(s) s \,\mathrm{d}s = Cr,$$

where s is the Jacobian of the polar coordinates and $C > 0$ is a constant of proportionality. Substituting $\mathcal{M}(r)$ into (cf. (1) and (6))

$$v = \sqrt{\frac{G\mathcal{M}(r)}{r}},$$

we find that v is constant. This is, of course, only a very rough estimate, but it suggests why the real velocities are almost constant for $r > r_0$.

5. Orbital velocity around a galaxy with bulge and halo

The gravitational force of a galaxy acting on its star is approximately the sum of the gravitational effects of the bulge, flat disk, and halo for $r \le r_\mathrm{G}$ if the outside of the galaxy is spherically symmetric. The bulge of spiral galaxies is usually spherically symmetric. For instance, the neighboring Andromeda galaxy M31 has a clear bulge making up to 20–25 % of its radius (see Remark below). The gravitational influence of the spherical bulge on outer stars can be approximated by the central force of a mass point, into which the whole mass of the bulge is concentrated. By the Shell Theorem we may neglect the gravitational influence of the halo $r > r_\mathrm{G}$.

Theorem 2 can be modified to the case of a ring with inner radius 20–25 % of R and outer radius R, since the mass density function $\rho = \rho(r) \ge 0$ is arbitrary. The gravitational force of the ring on the test particle on the outer edge of the ring is again larger than when the total mass is concentrated at the center. The velocity of stars at the edge of the Galaxy at distance R from the center is therefore larger than in (6).

Remark. The observed orbital velocity of stars which are not close to the center of M31 is again about 230 km/s according to (Rubin, 2003, p. 7). The radius of M31 is $r_\mathrm{A} \approx 2r_\mathrm{G}$ (see (3)) and the total mass is estimated to be $M_\mathrm{A} \approx 3M_\mathrm{G}$. Then by (6) we even get a bigger disagreement with the postulation of nonbaryonic dark matter in M31 than for our Galaxy. Namely, the calculated velocity by (6) will be $\sqrt{3/2}$ times larger than the observed orbital velocity. In this case, not a single gram of nobaryonic dark matter is needed, since we even have a surplus of luminous matter.

The fact that rotation curves of spiral galaxies are flat does not imply that there must exist some nonbaryonic dark matter concentrated around galaxies. It follows that the gravitational force between a two-dimensional homogeneous sphere (shell) and a mass point lying on it is finite. On the other hand, the force of a one-dimensional ring acting on a fixed point, which lies on it, is infinite according to (12)–(17), since the function $f = f(r, \varphi)$ from (12) has a singularity for $r \to 1$ and $\varphi \to 0$. Thus we see that there is a substantial difference between the two-dimensional and three-dimensional model. From the two-dimensional model it is also evident why the stars at the edge of the spiral galaxy orbit so rapidly. Therefore, we should not exchange a gravitational field of a galaxy with the gravitational field of a central mass point. The discrepancy in large velocities of stars observed by Rubin thus may have a natural explanation.

6. Discussion

In previous sections as well as in (Křížek, Křížek, Somer, 2014, 2015) we introduced several arguments showing that the amount of nonbaryonic dark matter seems to be considerably overestimated. It is very probable that Newton's law of gravitation on galactic or even cosmological scales approximates reality only very roughly, since it assumes an infinite speed of gravitational interaction.

Several modifications of Newtonian theory, e.g. MOND = Modified Newtonian Dynamics (McGaugh, 2008), (Milgrom, 1983), (Sanders, McGaugh, 2002) and its relativistic generalization TeVeS = Tensor-Vector-Scalar (Bekenstein, 2004) are at present being developed and studied. Effects that are attributed to nonbaryonic dark matter are explained by a different form of the law of gravitation, see also (Arbab, 2015) and (Brownstein, Moffat, 2007). However, note that MOND assumes an infinite speed of gravity, which surely contributes to a modeling error.

Missing dark matter in the local universe is demonstrated in (Karachentsev, 2012). Furthermore, in (Kroupa et al., 2010), (Kroupa et al., 2012), and (Kroupa, 2015) several trustworthy arguments are presented that point to the absence of nonbaryonic dark matter around our Galaxy. In (Pawlowski et al., 2014) and (Pawlowski et al., 2015) it is claimed that dwarf galaxies orbiting the Milky Way are in conflict with the spherical distribution of dominant nonbaryonic dark matter, since they are in almost one plane. A number of other papers (Banhatti, 2008), (Feng, Gallo, 2014), (Feng, Gallo, 2015), (Gallo, Feng, 2010), (Jałocha et al., 2008), (Kroupa, 2012), (Moni Bidin et al., 2012), (Nicholson, 2007), (Sikora et al., 2012), and (Wu, Kroupa, 2015) also confirm that on scales of galactic disks, Newton's theory of gravitation is still a fairly good approximation of reality and it is not necessary to modify it, or to assume the existence of nonbaryonic dark matter.

The observed oscillations of stars perpendicularly to the galactic plane can be explained by classical Newtonian mechanics without nonbaryonic dark matter (Flynn, Fuchs, 1994, p. 477) and (Moni Bidin et al., 2012). In other words, nonbaryonic dark matter may be referred to as a modeling error resulting from an incorrect cosmological model and misinterpretation of measured data on extremely large scales.

The influence of nonbaryonic dark matter in the Solar system has not been observed (Moni Bidin et al., 2012), even though our Sun is a large gravitational attractor. Thus it seems that nonbaryonic dark matter, if it exists, is not able to dissipate its inner energy, and therefore cannot be concentrated in the Sun's neighborhood.

On the other hand, Douglas Clowe in his paper *A direct empirical proof of the existence of nonbaryonic dark matter* proposes an example of the collision of two galaxy clusters MACS J0025.4-1222, where the intergalactic gas is stopped, while the galaxies continue in an unchanged direction together with nonbaryonic dark matter which is "detected" by gravitational lensing. However, we are not able to measure tangential components of the velocities of these clusters to prove that the collision really happened. Moreover, there are several strange circumstances:

1. The clusters MACS J0025.4-1222 from (Clowe et al., 2006) have almost the same size and they lie on one line together with clouds of nonbaryonic dark matter (as also the Bullet cluster or the Musketball cluster). This is very unlikely from a statistical point of view. The clusters should have different sizes and their positions together with gas should not lie on one line, since their initial velocities were not in one line, in general.

2. Due to the large density of galaxies, tidal tails and the effect of dynamical friction should be observed among galaxies, but they are not.

3. The proposed (not measured) infall velocity $v \approx 3000$–4500 km/s for this collision is at least 1 % of the speed of light and has the opposite sign to the overall expansion speed of the universe. How could these two galaxy clusters get such unlikely large velocities and thus also kinetic energy proportional to v^2 in an isotropic and homogeneous universe, where the local peculiar speed of galaxies is usually only several hundreds km/s?

4. The regions with hypothetical nonbaryonic dark matter are artificially colored on the basis of some numerical, not exactly explained simulations based on gravitational lensing.

Now we present another argument against nonbaryonic dark matter. Note that the Milky Way has a diameter of the order of 10^{10} astronomical units. The size of our universe is at least five orders of magnitude larger. Hence, the Friedman equation (Friedman, 1922) was derived under a considerably unjustified extrapolation ignoring the modeling error (Křížek, Somer, 2014). So it probably does not describe reality well. The validity of Einstein's equations is "tested" on much smaller scales. This seems to be the main misconception of current cosmology.

Nowadays there is a large discussion on what nonbaryonic dark matter is. The discrepancy of some model with reality does not mean that nonbaryonic dark matter really exists, since the model can be wrong. Therefore, direct proofs of the existence of nonbaryonic dark matter are being sought. For this purpose many sophisticated detectors (CDMS, DAMA/LIBRA, ADMX, ...) were constructed, but for the time being no particle of nonbaryonic dark matter has been detected. Also the Large Hadron Collider in CERN has not found any signs of new physics that could explain nonbaryonic dark matter.

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