Sub- and Quasi-Centurial Cycles in Solar and Geomagnetic Activity Data Series

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(Submitted on 11 January 2016. Accepted on 20 May 2016)

Abstract. The subject of this paper is the existence and stability of solar cycles with durations in the range of 20–250 years. Five types of data series are used: 1) the Zurich series (1749–2009 AD), the mean annual International sunspot number *Ri*, 2) the Group sunspot number series *Rh* (1610–1995 AD), 3) the simulated extended sunspot number from Extended time series of Solar Activity Indices (ESAI) (1090–2002 AD), 4) the simulated extended geomagnetic *aa*-index from ESAI (1099–2002 AD), 5) the Meudon filament series (1919–1991 AD). Two principally independent methods of time series analysis are used: the T-R periodogram analysis (both in standard and "scanning window" regimes) and the wavelet-analysis. The obtained results are very similar. A strong cycle with a mean duration of 55–60 years is found to exist in all series. On the other hand, a strong and stable quasi 110–120 years and ∼200-year cycles are obtained in all of these series except in the *Ri* one. The high importance of the long term solar activity dynamics for the aims of solar dynamo modeling and predictions is especially noted.
Key words: solar activity; solar cycles; methods - indices, extended solar data series

Introduction

It is usually accepted that the length of the so called "secular" (centurial) or Gleissberg solar cycle is about 7 or 8 Schwabe-Wolf's sunspot cycles, i.e. $\sim$ 80-90 years. The corresponding oscillation has been detected by different methods since the middle of 1940ies in the instrumental sunspot data series (see e.g. Gleissberg, 1944; Vitinskii, 1973) and until present days (e.g. Kane, 2008). Using tree rings ^{14}C data series (INTCAL93), Damon and Sonett (1991), Peristykh and Damon (2003) found that ∼88 year solar cycle exists and could be traced during the last 11–12000 years (post-glacial epoch, Holocene). It has been also found in these studies that such quasi-centurial oscillation is modulated by other longer bi-millennial 2000–2500 year cycle (usually called "Hallstadtzeit").

During the middle and the second half of the 20th century quasi-periodic oscillations, which are comparable with the Glessberg's sunspot cycle have been established in many solar, geophysical, climatic, and other environmental processes (see e.g. Schove 1955, 1983; Rubashev, 1963; Javaraiah et al., 2005). On the other hand, it has been also found that in many cases there is not a clear 80–90 year solar cycle (SC), but rather one or more quasi-cyclic oscillations, which are slightly shorter or longer than the "classic" Gleissberg cycle. Still in the middle of 1950ies, Schove (1955) has found that in the auroral activity during the last ∼2600 years there is not a single 78 year cycle, but rater few oscillations with sub- and quasi-centurial length in the range from 54–55 to 130 years. It has been confirmed on the base of time series analysis over the continuous part of Schove's series (last $\sim$1700 years) that except the powerful 200–210 year cycle, there are also three relatively weak, but statistically significant quasi-periodic oscillations by duration of 7, 8, and 11 SCs (1 SC = Schwabe-Wolf's cycle, $\approx$ 11.04 years), i.e. 77, 88, and 122 years, respectively (Komitov, 1997).

A powerful cycle with a length of $\sim$ 6 Schwabe-Wolf's cycles ($\sim$65 years) has been detected in the yearly numbers of Middle Latitude Aurora (MLA) in the recent and most certain part of the Catalogue of Křivský and Pejml (1988) between 1700 and 1900 AD (Komitov, 2009). This catalog covers the period 1000–1900 AD. Strong 65–67 year oscillation has been found in "Greenland" ^{10}Be series by two independent methods of time series analysis for the whole period of data sets (1423–1985 AD) (Komitov and Kaftan, 2004). Essentially weaker, but also statistically significant $\sim$65-66 year cycle has been detected by Komitov (2008) in the South Pole ^{10}Be series (Bard et al, 1997). The very impressive fact is that there is almost full coincidence of the maximums of both type $\sim$65 year cycles – these in MLA and "Greenland" ^{10}Be series during the period 1700–1900 AD (Komitov, 2009). These results are in good agreement with many other studies where $\sim$60 year cycle has been found in aurora activity and the solar related phenomena (e.g. Yu, 2002; Ogurtsov et al., 2002; Mazarella and Scafetta, 2012; Scafetta 2012).

The intrigue around this quasi–65 year cycle is even more interesting due to the fact that the same quasi-periodic oscillation has been found in large number of climatic or geodynamical parameters namely: in the Earth Northern hemisphere temperatures by instrumental observations (Thompson, 1997) since the middle of 19th century; in the World Ocean temperatures for almost the same period; in the Northern hemisphere temperatures during the last $\sim$ 300 years (Komitov 2009, 2010); in the North Atlantic Oscillation dynamics (Mazarella and Scafetta, 2012). Quasy $\sim$60 year cycle has been also detected recently in the Earth rotation velocity (the so called "Length of Day" or LOD-index) (Kaftan et al, 2016).

On the other hand, there are also a large number of cases where quasi-centurial cycles by lengths up to $\sim$1.5 times longer than the classic Gleissberg cycle has been established in solar related phenomena like the production rates of "cosmogenic" isotopes ^{10}Be and ^{14}C (Ogurtsov et al., 2002, Komitov and Kaftan, 2004), in the north-south sunspots asymmetry (Javaraiah et al., 2005; Komitov, 2010), the quasi-periodic trend by length of $\sim$117 years in Meudon solar filaments series (Duchlev, 2001), etc. There are also indirect climatic analogues, too (see Komitov et al., 2003).

Summarizing all these facts, it is clear that the near Gleissberg oscillations in a range from 5 to 12 SCs (55 to 130–140 years) are essential features of many solar and solar-related phenomena. Their study is important for the better understanding of the physical causes of their origin, including also how they could be described by the solar dynamo theory. Are they stable in time, how good they are expressed during the different epochs, i.e on the plane of long-term solar activity variations? Is there some amplitude and frequency modulation of one quasi-centurial 80–90 year cycle by other longer ones (such as the 200–210 year cycle or Hallstadtzeit)? If there is a multiplet of different oscillations with approximately constant periods, but with variable amplitudes, which of them are in the above mentioned range (55-140 years)?

Obviously, the existence of aforementioned important questions suggests that the studies made to the present time are far not enough to solve these problems. By our opinion it is necessary to analyze a large number of instrumental and historical data series of solar and geophysical indexes (the longer the better) by different methods and comparison of results.

By these reasons, the subject of this study is a more detailed analysis of the problem for existence and stability of cycles with duration in the range of 20–250 years, where the interval 5–12 SC is almost in the middle. The analysis uses different solar and geomagnetic data sets of instrumental and "historical" type. Two principally independent methods for time series analysis are used: a) the T-R periodogram analysis (standard (Komitov, 1986, 1997) and "moving window" regimes (Bonev et al., 2004)); b) wavelet analysis (Torrence and Compo, 1998; Ranucci and Sello, 2004).

1. Data and Methods

For the purposes of this study, five sets of direct and indirect data series with annual time step resolution are used:

1. The mean annual Group sunspot number (GSN) (1610–1995 AD) (ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/).
2. The international sunspot number *Ri* index series (1749–2009 AD) (ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/).
3. The simulated international sunspot (*Rsi*) number from ESAI (1090–2010 AD) (Nagovitsyn, 1997; Nagovitsyn et al., 2004) (http://www.gao.spb.ru/database/esai).
4. The simulated geomagnetic *aa*-index from the ESAI (1619–2003 AD) (Nagovitsyn, 2006) (http://www.gao.spb.ru/database/esai).
5. The Meudon filament data series (1919–1991 AD).

These data are taken from "Cartes Synoptiques de la Chromosphere Solaire et Catalogues des Filaments et des Centres d'Activity: 1919–1989", published by Observatoire de Paris, Section de Meudon, and for the period 1989–1991 from http://bass2000.obspm.fr/lastsynmap.php.

The first series (GSN) is taken as a better proxy (than the classical *Ri*-index) of the variabilities of the solar short wavelength electromagnetic radiation and the corpuscular fluxes before the middle of 19th century. Unfortunately, it ends at 1995 AD and there are no plans for its continuation at this stage (Kane, 2008).

The extended simulated sunspot series (*Rsi*) is the longest used in the present analysis – 913 yrs. The *Rsi*-series is based on the "historical" Schove's series (Schove, 1983) for the moments of extremes and magnitudes of the Schwabe-Wolf's sunspot cycles. The mean annual sunspot numbers there were derived on the basis of the Krylov-Bogolyubov's approach to the description of weakly nonlinear oscillatory processes (Nagovitsyn, 1997; Nagovitsyn et al., 2004). It is important to note that the original Schove's series is based predominantly on historical messages for auroral events and naked eye visible sunspots, i.e. on potential sources of strong flares). By this reason it could be considered as a good proxy of the solar flare activity and the number of its active centres. (Note: Since July 1, 2015 a new international sunspot activity number is used. The sunspot activity observations in the observatory Specola near Locarno, Switzerland are used as standard. The relationship of the old $R_i = R_{old}$ to the new R_{new} international sunspot index is approximately $R_{old}/R_{new} = 0.67$. See link http://sidc.be/silso/newdataset for details.)

The synthetic type of this series leads to the reasonable question about the significant errors in these data, which maybe exist there. This problem should be significant mainly for the pre-instrumental part of the *Rsi*-series, i.e. before 1610 AD. For error estimation concerning this period one should refer to Nagovitsyn (1997), who found that the error range of the corresponding series is ±30% to the mean annual *Rsi* data. However, this error should be decreased essentially by smoothing of the data over 11 yrs (see below). The reliability of the ESAI sunspots, i.e. *Rsi*-series is also tested by Nagovitsyn (1997) by comparison with other "historical" solar activity data sets (^{14}C, the naked eye visible sunspots and the last Schove's series version (Schove, 1983)).

We have provided some additional comparisons between Schove's series and *Rsi* for the epoch 1090–1610 AD. The typical uncertainty magnitudes of the 11-yr sunspot cycles in Schove's series is 25-30%, which is comparable with the same magnitudes for *Rsi* mean annual values. Both series have been also compared with standpoint of validations or violations of the "amplitudal" Gnevishev-Ohl's rule. It has been found that for all 24 pairs of even-odd 11-yr cycles during the epoch 1090–1610 AD, there is a coincidence for validation (or violation) in 19th century, i.e. for $\sim 80\%$ of the cases. In four cases out of five the corresponding pairs of cycles are very weak by amplitudes – they occurred during the Wolf's and Spoerer's minima in the 13th–14th and 15th century, respectively. Thus, one can conclude that both series are in a good agreement in almost all important aspects.

The length of the simulated *aa*-index (AA) is almost equal to GSN. The Meudon filament series is relatively short and it is analysed only with the standard T-R periodogram and wavelet analysis. That series is not long enough for studying the evolution of cycles longer than the Schwabe-Wolf's cycle. However, it is included here as a proxy of the long-lived solar magnetic field structures. Finally, the Zurich sunspot *Ri* annual series is used as an international standard for the overall sunspot activity. Two preliminary procedures have been applied to all the studied time series:

1. Removing of general nonlinear trends (polynomials of second, third or fourth degree). This procedure is used over the most of the investigated time series for removing not only the possible non-linear tendencies in the data series, but also of all potentially existing cycles with duration of $T \geq 300$ yr, which helps for better visibility of oscillations in sub-centurial, quasi-centurial and bi-centurial range. For example the existence of non-linear long term tendency is clearly visible in Fig. 1. It is downward before 17th century (Maunder minimum) and upward after that (i.e. the last ~ 300 years). The corresponding trend functions were obtained by the means of the least mean squares procedure. The best trend function expressions were determined on

the basis of the best coefficients of correlations to the corresponding time series and the Snedekor-Fisher's F-parameter.

2. A smoothing procedure by 11 points (years) over the "residuals". The effect of Schwabe-Wolf's cycle is removed and the signal of the long-term cycles is much better expressed.

We apply three types of time series analysis over these data sets.

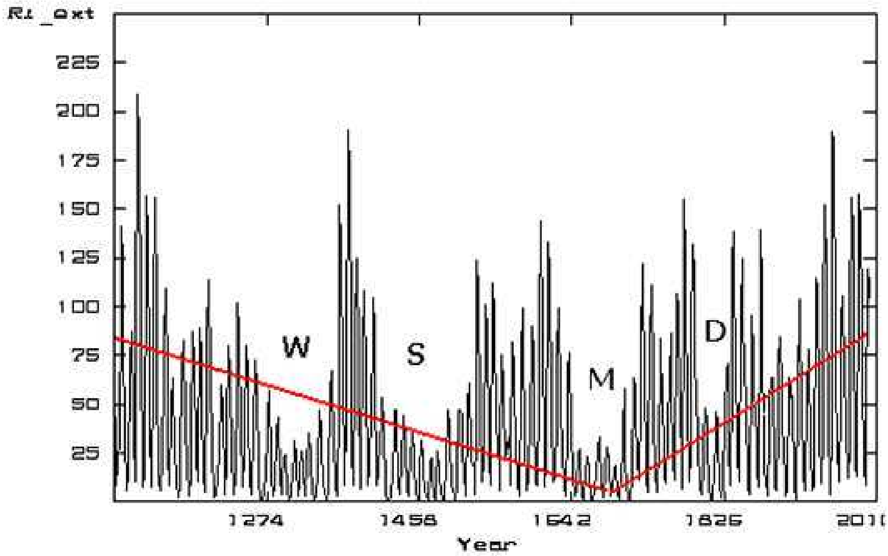

Fig. 1. Extended simulated international sunspot (Rsi) numbers from ESAI (1090–2010 AD)

A. The standard T-R periodogram procedure

The standard T-R periodogram analysis is used to search for statistically significant cycles of the whole time series (Komitov, 1986, 1997; Benson et al., 2003). This technique is very close to the algorithm, described by Scargle (1982). A more detailed description is presented in Komitov's paper (Komitov, 1997).

The idea of this method is to approximate the studied time series $F(t)$ by a minimised function $\varphi(t)$ of a simple periodic type, i.e.

$$F(t) \approx \varphi(t) = A_0 + A\cos\left(\frac{2\pi t}{T}\right) + B\sin\left(\frac{2\pi t}{T}\right), \quad (1)$$

where $t = 0, 1, 2, \ldots, n$ are the corresponding moments in time step units (the time-series step), A_0 is the mean value of $F(t)$ based on the entire time series, T is the period, which is varied in the range $[T_0, T_{max}]$ with a step ΔT. In this way, a series of p minimised functions $\varphi(t)$ is obtained, where

$p = (T_{max} - T_0)/\Delta T$. The minimal possible value of T_0 is equal to 2 steps of the time series ($\nu = 0.5$). For each one of the so obtained functions $\varphi(t)$ a coefficient R of correlation to the time series $F(t)$ is calculated. The local maxima of R point to the possible existence of cycles with duration equal to the corresponding periods T.

The spectral resolution step, used in the standard T-R periodogram calculations for the time series in this study, is $\Delta T = 0.5$ years. T is varied from 2 to 502 years, i.e. the T-R correlograms are calculated for 1000 different values of T.

B. "Moving Window" T-R Periodogram Procedure

The standard T-R periodogram procedure produces the mean parameters of the existing cycles in the time series. However, these cycles could significantly change between different parts of the time series. To study a cycle's evolution the so-called "Moving Window T-R periodogram procedure" (MWTRPP) (Bonev et al., 2004) is used. In this algorithm, a part of the time series of length P ("moving window"), where $P \leq N$, is defined. By author's opinion it is recommended that $P/N \leq 1/3$ (Komitov, 2009). At the start of the procedure the "moving window" contains the first P terms of time series $F(t)$ and the standard T-R procedure is applied to them. After that the "moving window" is shifted with a step δT, which is integer unit divisible by time series step ΔT (one or more ΔT), and the T-R procedure is repeated again. This procedure is repeated for each position of the "moving window". Using this method, a series of T-R correlograms could be obtained as columns in a two-dimensional map, which corresponds to the T values along the Y-axes, while the central or starting moments of "moving window" correspond to the X-coordinates. For building of a graphical presentation of these maps a "transitional" working parameter τ in our procedure is used. Its values are equal to the number of rows in the maps (if counted from bottom to top) and consequently they are in linear relationship with the period T according to formula $T = T_0 + \tau\Delta T$. In this way, the two-dimensional map represents the density field of the R amplitude, where the local maxima of R indicate for a possible existence of cycles with durations equal to corresponding periods T. In addition to R values, one could obtain maps of R/SR, with SR being the error of R, the amplitude $a(T) = \sqrt{A^2(T) + B^2(T)}$, and the evolution of the coefficients $A(T)$ or $B(T)$.

C. The wavelet-analysis

This technique is well known. It has been extensively applied in complex nonlinear time-series analysis, including the study of solar and stellar activity cycles (Torrence & Compo, 1998; Sello, 2003; Ranucci & Sello, 2007).

2. Results and analysis

2.1 The Zurich series (1750–2009 AD)

As shown in Fig. 2, which depicts the T-R correlogram, the main long-term cycle in the international sunspot number Ri series is approximately 95.5 years long. The corresponding peak of the correlation coefficient R exceeds its error 18.8 times. The "zero-hypothesis" probability in this case is $\ll 10^{-6}$.

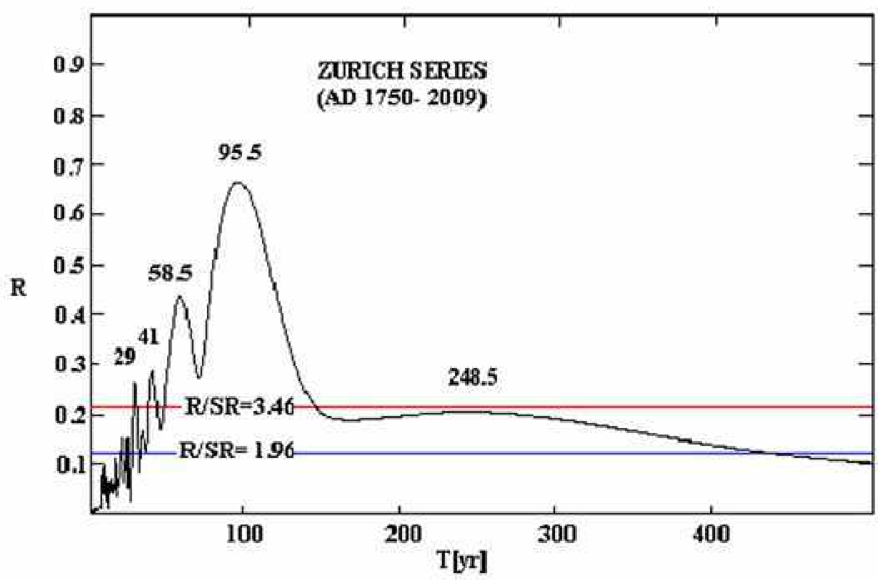

Fig. 2. T-R correlogram of the Zurich series (1750–2009 AD)

Consequently, the quasi-centurial cycle should be considered as the most important feature of the international sunspot index Ri, after the 11-year Schwabe-Wolf's cycle. The second clearly visible long-term oscillation has a period of $T = 58.5$ years. The corresponding R/SR ratio is also very high (~ 8.8) and it has significance $> 99.999\%$, i.e. the "zero-hypothesis" level is less than 0.001%. There are also traces of 41-yr and 29-yr oscillations that are weaker in amplitudes, but with high statistical significance.

The MWTRPP amplitude map of the Zurich series is shown in Fig. 3. An evolution of the quasi-century oscillation during the last 260 years is clearly visible. During the first 150 years, i.e. the second half of the 18th and 19th centuries, this cycle is noticeably shorter (~ 70 years). However, during the next decades its duration is slowly increased and in the 20th century it is already slightly longer than 100 years. The calendar centres of the moving window epochs when the quasi-centurial cycle is better expressed are near to 1825 AD (the Dalton minimum) and 1870 AD.

There is no good trace of a $\sim$ 55-year cycle during the 18th century. As shown in Fig. 3, this cycle is in a process of "separation" from the quasi century oscillation during this time. The $\sim$ 55-year cycle is well expressed in the 19th century up to 1870 AD, but after that it is decreasing both in amplitude and duration. A new increase in the amplitude is observed in the 20th century, but now the cycle length is about 40–42 years. On the other hand, a quasi 42–44-yr cycle is very well expressed in the middle of the 19th century almost simultaneously with the 55-year one. Another visible weak oscillation is the quasi 30-year cycle ($\sim$ 3 Schwabe-Wolf's cycles). Its amplitude has increased and decreased three times during the whole period

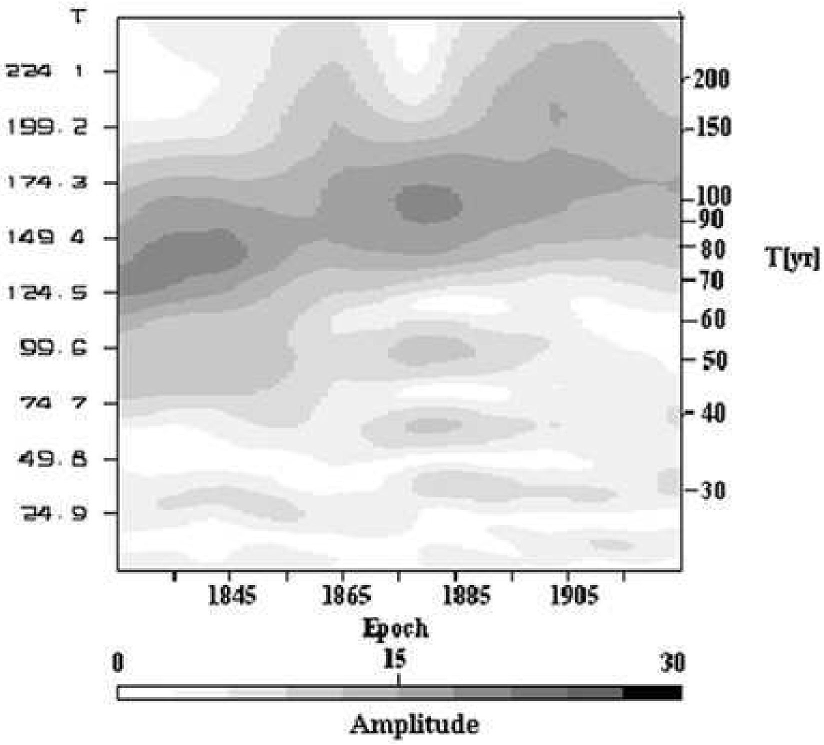

Fig. 3. MWTRPP amplitude map of the Zurich series (1750–2009 AD)

of 260 years. The last change corresponds to the recent decades. The results from the wavelet analysis (WA) are shown in Fig. 4. The strongest cycle here has a duration of $\sim$ 94 years. It is slightly shorter during the 18th century with a weak tendency of prolongation during the 19th and 20th centuries. In the recent part of the series (20th century) this cycle tends to a duration of $\sim$ 100–110 years. The second by importance cycle is the 54-year one. It is very well expressed in 18th and 19th centuries. After $\sim$ 1850 AD its amplitude is fading.

2.2 The Group sunspot number series (1610–1995 AD)

According to the results from the T-R periodogram analysis the main cycle in the GSN series during the last $\sim$ 400 years has quasi-two-century $\sim$ 202 years duration (Fig. 5) and correlation coefficient $R \sim 0.58$. The second by significance ($R = 0.52$) is the 108-year cycle. There is also a quasi 80-year oscillation. The sub-century 54-year cycle is the next significant oscillation ($R = 0.33$). There are also very weak traces of 28- and 21.5-year cycles. The ratio R/SR for the last two ones is in the range of 2.0 to 3.5.

On the basis of the MWTRPP method we found that the quasi-two-century oscillation was very powerful during the 17th and 18th centuries,

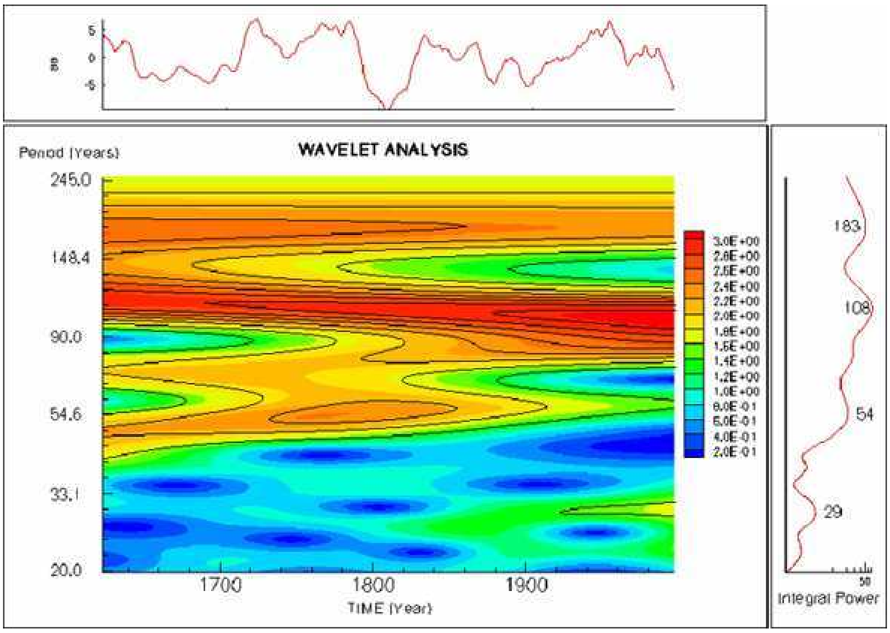

Fig. 4. WA amplitudes of the Zurich series (1750–2009 AD)

i.e. during the Maunder minimum and the first decades after that. However, since the Maunder minimum, it quickly fades and is transformed to shorter duration. Close to the Dalton minimum (1795–1830 AD) there are no more visible cyclic tendencies of a century or longer duration period. The main long-term cycle at the end of the Dalton minimum has a period of about 70–75 years. After the Dalton minimum, a slow prolongation starts for this cycle and its "actual" length during the 20th century is approximately 110 years.

A relatively stable 50–60-year cycle is clearly visible in Fig. 6 for almost the entire GSN data series. This cycle is best expressed in the 18th and 19th centuries up to 1870 AD, whereas in the 20th century it is significantly weaker. During the last decades a weak quasi 30-year cycle appeared.

According to the wavelet analysis the quasi $\sim$ 200-year cycle is most powerful in the earlier part of the GSN series (Fig. 7). Its amplitude decreases slightly in the period between 17th and 20th centuries. In contrast, a cycle with duration of $\sim$ 110 years increases in amplitude during this time. Another $\sim$ 80-year oscillation, which exists during the 17th and 18th centuries quickly converges to the 110-year one approximately after the Dalton minimum. A 54-year cycle exists in the whole series, but it is most powerful during the 18th and 19th centuries. The peak of its amplitude occurs near 1820 AD. As in the MWTRPP-map (Fig. 6) a 30–33-year cycle is well visible in the most recent part of the GSN-series.

As in the case of *Ri* (Zurich series), the wavelet amplitude spectra of

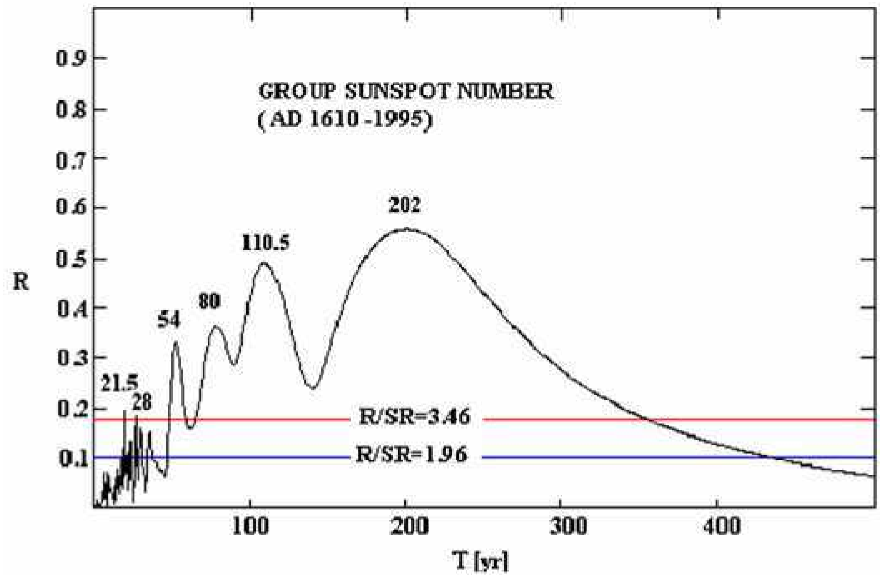

Fig. 5. T-R correlogram of the Group sunspot number (GSN) series (1610–1995 AD)

GSN (Fig. 7, right) is very similar to the corresponding T-R correlogram (Fig. 5).

2.3 The simulated *AA*-index series (1619–1999 AD)

The length of the simulated *aa*-data series from ESAI used here is almost the same as that of the GSN-series (381 yrs). The signature "*AA*" is used in this work to distinguish it from the instrumental *aa*-index. The study of this series is very interesting because of the possibility for comparison of the results of this purely "geophysically-oriented" series to the other ones, which are related to the sunspot active centres. The standard T-R spectrum is shown in Fig. 8.

According to MWTRPP the quasi 55–60-year cycle is very stable in the early and the middle parts of the series (Fig. 9) and it is slightly better expressed in the *AA* than in the GSN and *Rsi* series. On the other hand the WA results reveal that this cycle is even better expressed during the 17th century in *AA* (Fig. 10) than in GSN and *Rsi* (Figs. 7 and 15).

There is a hint of a better expressed quasi 30-year oscillation during the earlier part of the *AA*-series (Figs. 9 and 10). According to the WA the higher amplitudes of this cycle occur at the end of the 20th century (Fig. 10), while as follows from MWTRPP, the absolute amplitude peak of the quasi 30-year oscillation occurs almost a century earlier (Fig. 9).

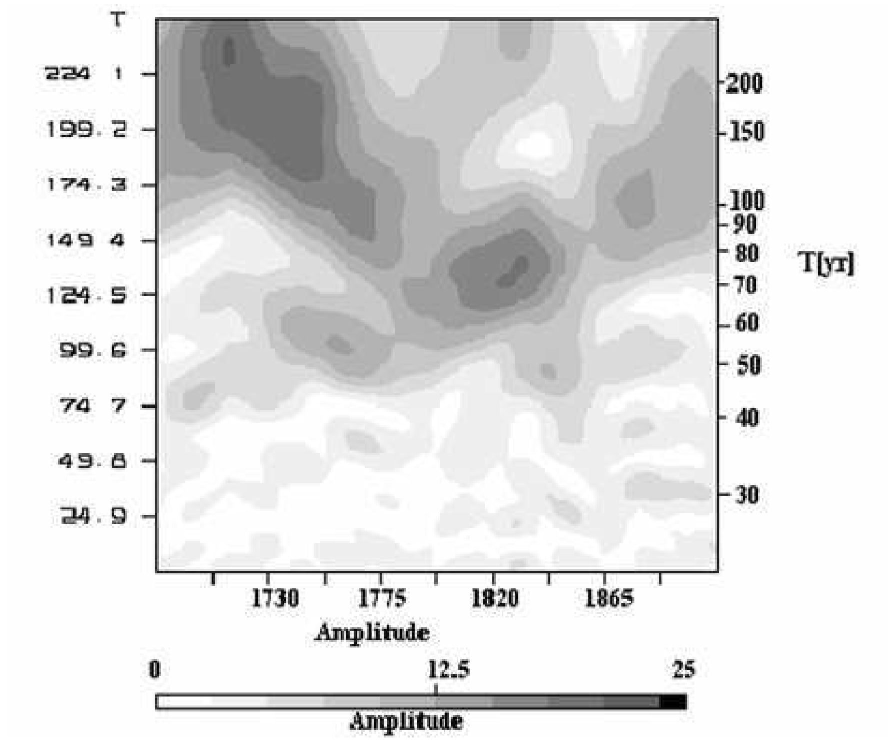

Fig. 6. The MWTRPP amplitude map of the Group sunspot number (GSN) series (1610–1995 AD)

2.4 The Meudon filament data series (1919–1991 AD)

Unlike the other data sets investigated here, the smoothed filament series of the Meudon observatory catalogue is relatively short, covering only 73 years. Hence it is not possible to effectively scan it for any cycle evolution in the multidecadial range. Therefore, we study only the mean features of the whole series mainly on the basis of the standard T-R periodogram analysis. As shown in Fig. 11, there are 66.5-, 26.5- and 17.5-year cycles. The WA-test gives, due to the shortness of this time series, only a weak 26.5-year cycle (Fig. 12).

These results are interesting due to the fact that the filaments are a relative "pure" indicator for the coronal activity phenomena. The existence of a 66.5-year cycle is an evidence that the sub-century periodicity is real for these events at least during the 20th century. The absence of a quasi 117-year cycle, which was detected earlier by Duchlev (2001), could easily be explained by the de-trending procedure.

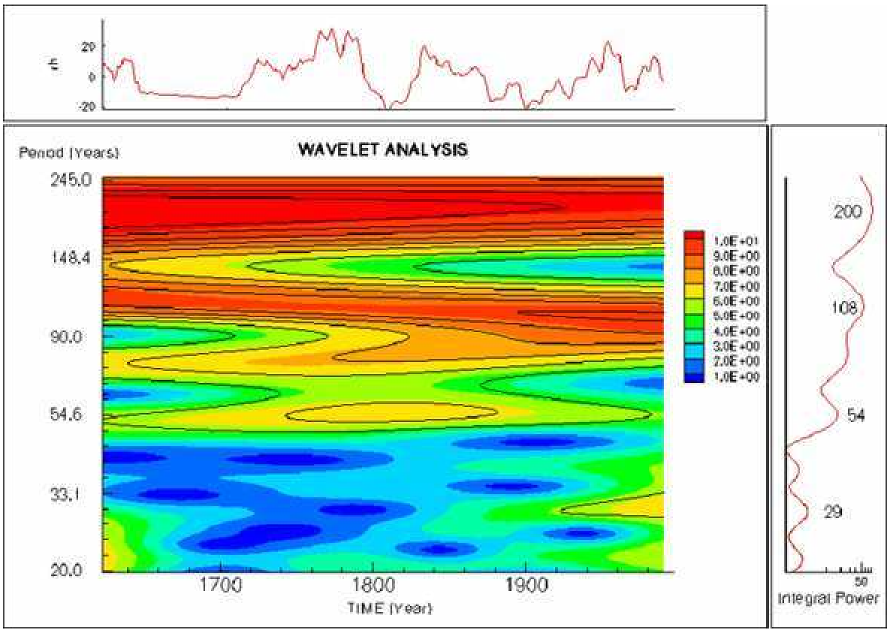

Fig. 7. WA amplitudes of the Group sunspot number (GSN) series (1610–1995 AD)

2.5 The long-term solar cycles during the last ∼ 900 years

The time series used in Section 2.1 to Section 2.5 are relatively short. They all begin near or after the supermillennial Maunder minimum (1640–1720 AD). Moreover, they are almost entirely contained within a period of a long-term upward trend of the solar activity, during the initial active phase of the quasi-bimillennial solar 2200–2400-year cycle (Hallstadtzeit) (Damon & Sonett, 1991; Dergachev, 1994; Bonev et al., 2004). As it has already been demonstrated by some of these authors (Damon & Sonett, 1991; Bonev et al., 2004) on the basis of ^{14}C data, as well as by Komitov et al. (2004) (^{14}C and Schove's series), Maunder-type minima are not only the starting phases of Hallstadtzeit cycles. Serious changes of the solar activity dynamics occur during these epochs. The amplitudes of the quasi two-century cycles (∼ 170–220 years) fade, while in their place an increase of the amplitudes of the cycles of quasi-century duration begins. Therefore it is interesting to search for the stability and evolution of the cycles from the studied range over longer than 400-year time scales. It is especially interesting to investigate how the transition from the previous Hallstadtzeit cycle to the present one affects the solar oscillations with sub-century periods (20–70 years).

For this purpose, we use the whole *Rsi* simulated data series from the ESAI. As noted in Section 1, this series starts at 1090 and ends at 2002 AD and contains annual data for 913 years. The T-R periodogram analysis is used in its both standard and MWTRPP versions. The "moving window" length P is 400 years. All other parameters are the same as those described

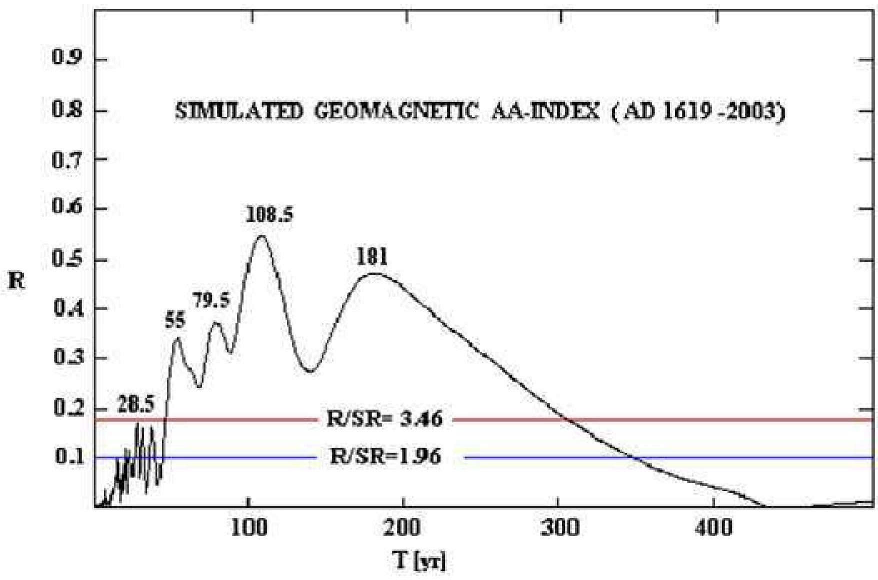

Fig. 8. T-R correlogram of the simulated *aa*-index (AA) data series (1619–2003 AD, ESAI)

in Section 1. As for the other series, the trend is removed and an 11-year smoothing procedure has been performed. The larger width of the window P provides much better conditions for the MWTRPP than in Sections 2.1–2.3 over all and especially in the range of $T \geq 150$ years. The results are shown in Figs. 13–15.

The main cyclic oscillation in the sunspot activity during the last ~ 900 yrs has duration of 204 to 209 years (Figs. 14 and 15). According to Fig. 13 there is a ~ 121.5-year cycle and an oscillation with duration of 82 years, which is very close to the so-called "Gleissberg cycle" (78 years) (Gleissberg, 1944). There is also a very well expressed 59.5-year cycle. A weaker cyclic component at $T = 54$ years is also present. Weak signatures of 41-year and 29-year cycles are found, which are near or even less than the critical level $R/SR = 3.46$.

As shown in Fig. 14 the quasi 200-year cycle is very significant before the Maunder minimum epoch. The absolute maximum of its amplitude corresponds to the calendar center of the 400-year window at ~ 1550 AD. This epoch contains both the deepest solar supercentury minima during the last 2000 years – those of Spörer (1400–1520 AD) and Maunder (1640-1720 AD). This result matches very well all other previous evidence, that during the Hallstadzeit cycles minima, the amplitude of the quasi two-century cycle tends to a maximum, while the quasi-century ones – to their absolute amplitude minima (Damon & Sonett, 1991; Bonev et al., 2004; Komitov, 2007). According to the WA results (Fig. 15) the amplitude of the ~ 200-year cycle

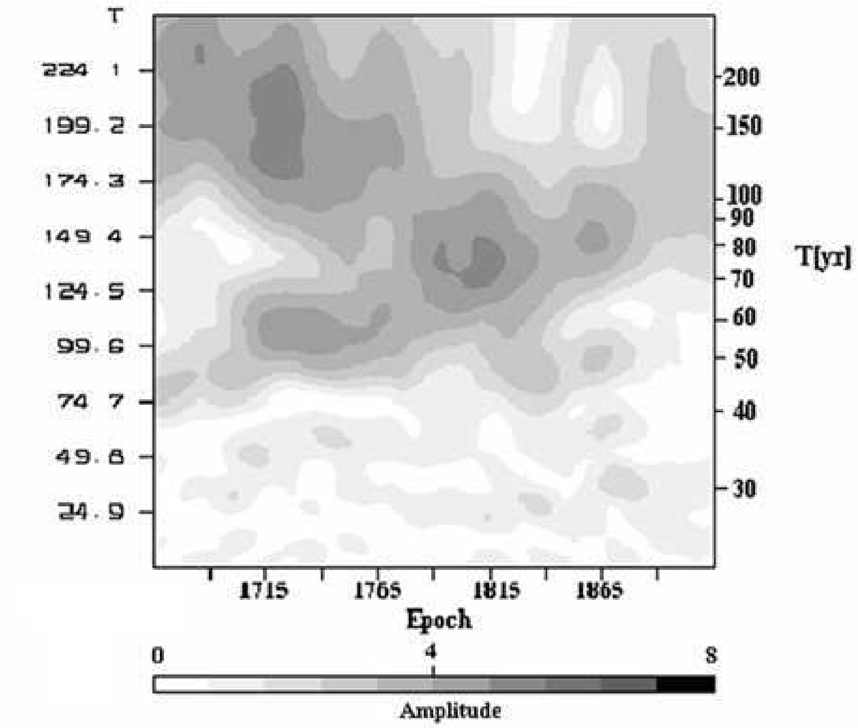

Fig. 9. The MWTRPP amplitude map of the simulated geomagnetic *aa*-index (AA) data series (1619–2003 AD, ESAI)

tends to reach its maximum near the Spörer-Maunder minimums epoch, but the fading after that is less pronounced than in Fig. 14.

The relative fading near the Maunder minimum (moving window calendar center at $\sim$ 1600 AD) of all cycles by duration 40–140 years is well pronounced in Fig. 15. However, it should be noted that in the entire period of 11th–20th centuries, three very stable oscillations in the sub-century and quasi-century range are always present, corresponding to mean periods of 55–60, $\sim$ 80 and $\sim$ 120 years. They could be followed in Fig. 14 during the entire $\sim$ 900-year period. One could conclude that there is no general mean quasi-century cycle during the last millennium, but rather a quasi century doublet (T = 80 and 110–120 years) or a triplet if the subcentury 50–60-year cycle is also considered.

The amplitude of the longest $\sim$ 120-year component reaches local maxima near the 11th, 15th, and 20th centuries. The behaviour of the $\sim$ 55–60-year cycle is also interesting. The epochs of its higher amplitudes are after the Maunder minimum, as well as in the 11th–12th centuries. Between the 12th and 17th centuries the amplitude of the 55–60-year oscillation remains significant, but not so high.

Figs. 14 and 15 illustrate that unlike the quasi-century multiplet (50–60,

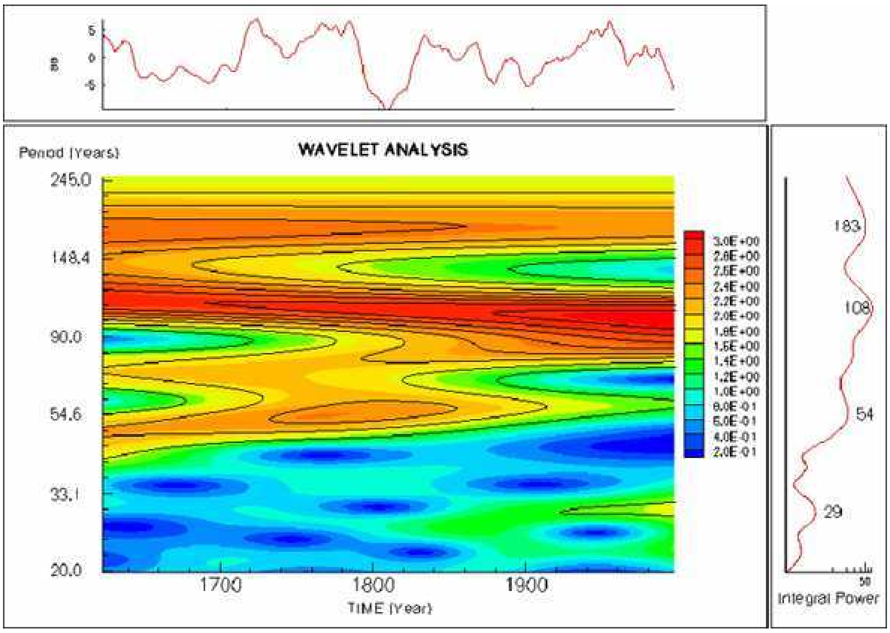

Fig. 10. WA amplitudes of the simulated geomagnetic *aa*-index (AA) data series (1619–2003 AD, ESAI)

80 and 120 years) the shorter cyclic variations are not stable on longer time scales. There are only weak traces of the 40–45-year cycles predominantly before the Maunder minimum and during the 20th century (Fig. 14). Traces of the $\sim$ 30-year ($\sim$ 3 Schwabe-Wolf's cycles) oscillation are even weaker and sporadic. As can be seen, the $\sim$ 30-year cycle is slightly better defined during the latest part of the series and mainly due to its significant increase in amplitude during the 20th century (Figs. 14 and 15).

It is interesting to note that near the Maunder minimum, significant traces of $\sim$ 35–38-year cycle are visible in Fig. 14 (the MWTRPP map). An oscillation with such duration has been detected for the period of 1932–2005 AD in the annual number of the geomagnetic storms, when the Ap-index exceeds 40 (Komitov, 2009).

3. Discussion

We could discuss the results presented in this paper and their analysis in two main aspects: 1. The long-term solar cycles and their importance for solar activity predictions in short and long time scales at all; 2. The long-term flare activity dynamics.

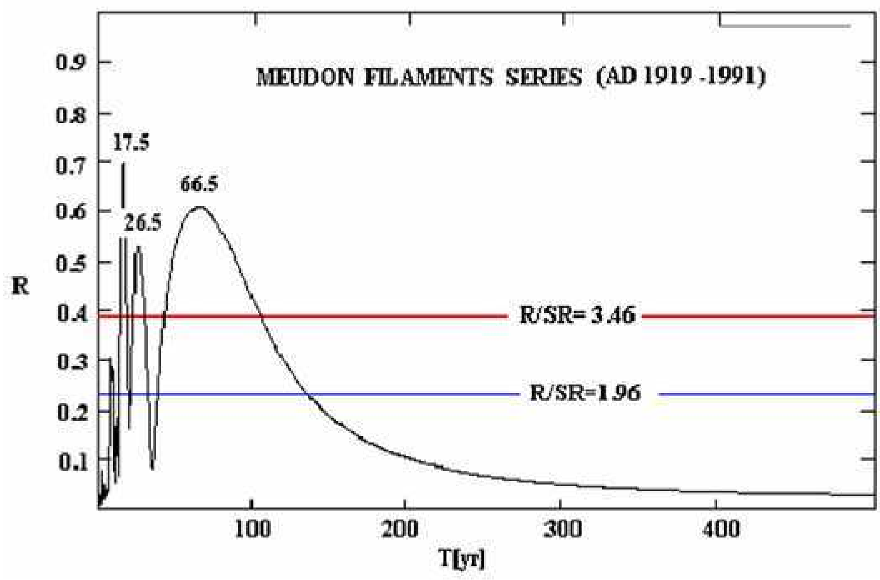

Fig. 11. T-R correlogram of the solar filaments number series from the Meudon catalogues (1919–1991 AD)

3.1 The long-term solar cycles and deep solar minima predictions

The results presented in Section 2 provide clear evidence that there are three significant and relatively stable quasi-oscillations during the last millennium with sub- and quasi-century duration: 55–60, $\sim$ 80, and 110–120 years, respectively. As shown in Section 2.1 to Section 2.3 both by the WA and the MWTRPP methods their periods are slightly variable in the shorter time scales, for example during the epoch of instrumental observations, i.e. the last $\sim$ 400 years. On the other hand, the MWTRPP test over the entire *Rsi* series shows that the reliable presence of this quasi-century multiplet covers the whole $\sim$ 900-year period if a larger smoothing window is used.

However, as illustrated by Figs. 3, 4, 6, 7, 9, 10, and 15, the ratios between the amplitudes of these three oscillations are different in the different epochs. This could explain why there are serious variations in the length of the observed quasi-century cycle in the different epochs. Gleissberg (1944) found a length of $\sim$ 78–80 years. In his study, the used data referred to sunspot cycles No 0–17, when the $\sim$ 110–120-year component is noticeably weaker. Thus, the $\sim$ 94–95-year century cycle in the whole Zurich series (Figs. 2 and 4) is only a "mean-weight" one and it is an integral effect of the more important role of the 55–80-year and the $\sim$ 80-year components in the earlier part of this series and the fading and increasing of the $\sim$ 120-year component in the recent part of the series.

On the other hand, the 50–60-year cycle is strong only in the first half of this period (before $\sim$ 1870 AD), but it is interrupted after the Gleissberg-Gnevishev minimum epoch (1898–1923 AD). However, the strengthening of

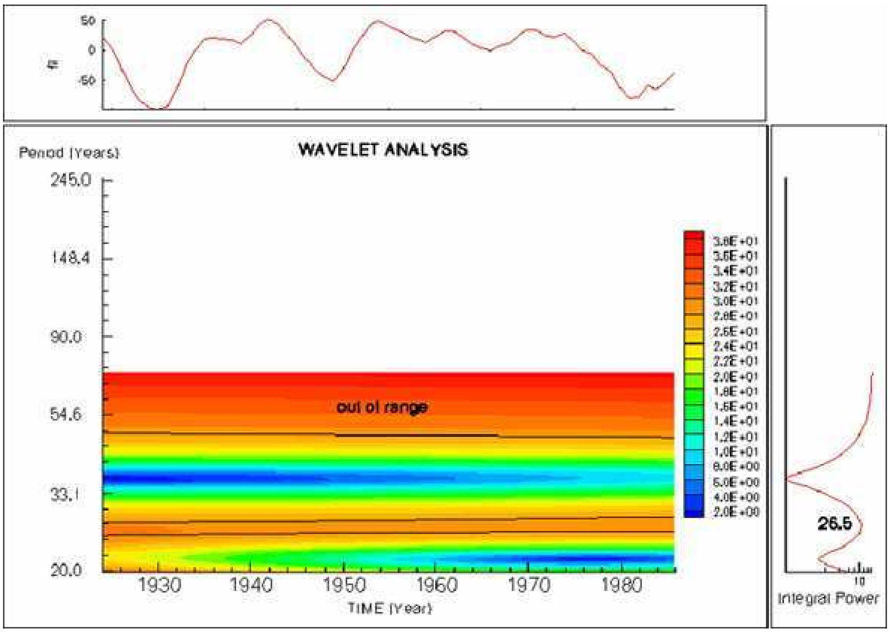

Fig. 12. Wavelet analysis of the solar filaments number series from the Meudon catalogues (1919–1991 AD)

the $\sim$ 120-year component and the weakening of the 80-year one after the Dalton minimum and during the 20th century is the cause for the delay of the next long-term minimum. That minimum began not after the end of solar cycle No 21 in 1986 AD, but about 20–22 years later, i.e. at the end of cycle No 23. The imminent long-term minimum in the first half of the 21st century, predicted by many authors (Fyodorov et al., 1996; Badalyan et al., 2001; Komitov & Bonev, 2001; Komitov & Kaftan, 2003; Solanki et al., 2004; Ogurtsov, 2005; Clilverd et al., 2006), implies a relatively low amplitude of solar cycle 24 ($Ri_{max} \sim 90$), according to Pesnell (2009), or even $\sim$ 58 (Kane, 2010).

The complicated structure of the quasi-century cycle was established by Schove (1955) on the basis of historical records of auroral events for the last $\sim$ 2000 years. In addition to the Gleissberg cycle ($\sim$ 78 year), he found also traces of longer ($\sim$ 120–130 years) or shorter (54–55 or 65 year) oscillations.

Nagovitsyn (1997) found that there should be two "fundamental" quasi-centurial cyclic components in the "extended" Wolf's series (1700–1990 AD) with duration of 80 and 115 years, respectively. This is in very good agreement with our results there. However, we found also $\sim$ 60-year cycle and that is confirmed by two quite independent methods (MWTRPA and WA).

The existence of this cycle could be explained with the fact that this cycle is in a high degree resonant correlation with the Hale 20–22-year magnetic cycle, the Schwabe-Wolf 11-year cycle, as well as with the 120-year

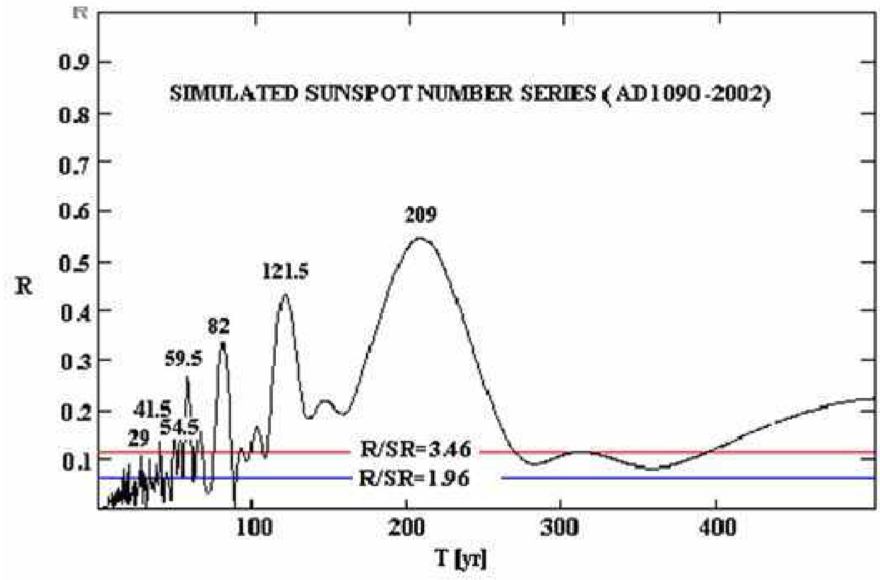

Fig. 13. T-R correlogram of the whole simulated sunspot *Rsi* number series (1090–2002 AD, ESAI)

cycle. Three Hale cycles have duration $\sim$ 60–65 years, five Schwabe-Wolf cycles are $\sim$ 50–55 years, six Schwabe-Wolf cycles are $\sim$ 60–66 years, and the 120-year cycle has duration divisible by 60. The Hale and the Schwabe-Wolf cycles are the main ones in the solar dynamo action. It is interesting, in this context, that in the long *Rsi*-series the standard T-R periodogram analysis detects two adjacent components (54.5 and 59.5 years, see Fig. 13), which indicates that all of the aforementioned resonances should play role rather than only one or two of them. However, we found it difficult to explain the stability of the 60-year cycle only on the basis of these complicated resonance correlations. Most probably there should be an additional independent source of these oscillations, which gives a contribution to the overall solar magnetic flux variability. The period of its variations is $\sim$ 55–60 years and it happens to be approximately in resonance with the 10–11-year Schwabe-Wolf, the 20–22-year Hale, and the 120-year cycles.

There are other interesting studies, which concern the existence of solar cycles with duration of quasi-3 (Ahluwalia, 1998) and quasi-5 (Du, 2006) Schwabe-Wolf cycles, i.e. 30–33 (Three-Cycle Periodicity, the so-called "TRC-rule") and $\sim$ 55 years, respectively. The existence and the stability especially of the TRC have been analysed critically by Kane (2008). He found that there have been only three sequences of Schwabe-Wolf cycles during the last 300 years, for which the TRC-rule is valid and two of them covered the Zurich cycles No 17–22. On the other hand, the periodicity of the quasi-4 Schwabe-Wolf cycle ($\sim$ 40–45 years) has been detected in the north-south asymmetry of the sunspot area by Javaraiah (2008) and Komitov (2010).

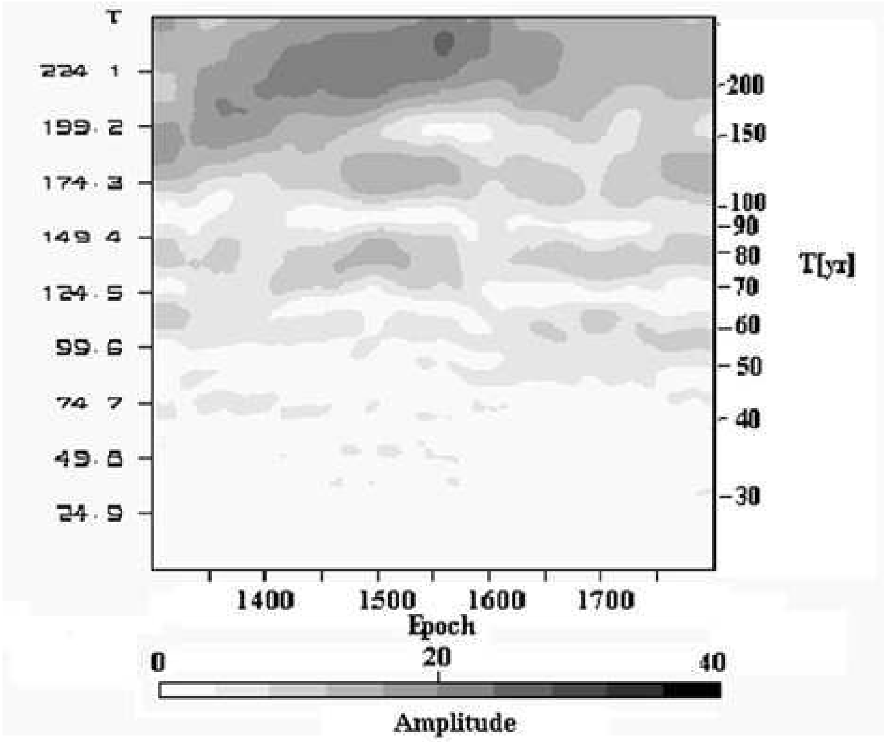

Fig. 14. The MWTRPP amplitude map of the whole simulated sunspot *Rsi* number series (1090–2002 AD, ESAI)

In the present study, we found that the $\sim$ 30 and 40–45-year oscillations are present in separate epochs in the overall sunspot and geomagnetic activity, but unlike the century multiplet components, they are very unstable and weak. However, both cycles have significantly higher amplitude in the recent epoch. The 40–45-year cycle is in resonant correspondence with the Hale cycle, while the same is not valid for the 29–30-year one.

It is noteworthy that there is no strong correspondence between the observed $\sim$ 30-year cycle and the "TRC-rule". In our study this cycle is a feature of the absolute amplitude variations of the smoothed and de-trended series of solar indices, while the "TRC-rule" describes a relative relationship between three consecutive 11-year Schwabe-Wolf's cycles.

As we already noted, the $\sim$ 200-year cycle is a main feature of solar activity on the supercentury time scale, and is clearly visible in "cosmogenic" radioisotopes data (the so-called "de Vries oscillation" in ^{14}C series) (Stuiver & Quay, 1980; Damon & Sonett, 1991; Dergachev, 1994). It is also well expressed and very stable since the end of the 3rd century in the whole continuous part of Schove's series and in the ^{14}C data (Komitov, 1997). The climate effects of "de Vries oscillation" could be tracked even 25000–

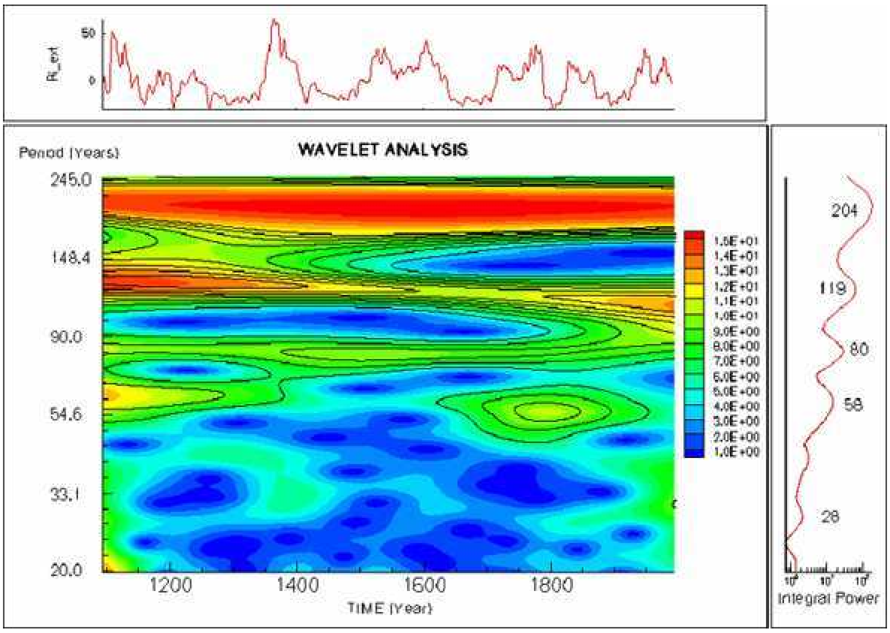

Fig. 15. WA amplitudes of the whole simulated sunspot Rsi number series (1090–2002 AD, ESAI)

50000 years in the past (Wagner et al., 2001). Unique palaeoclimatic data pointing to the $\sim$ 200-year variability were obtained through analysis of the sediments from Upper Permian Castile Formation (250 million years ago!) in the Delaware Basin in west Texas and New Mexico, USA (see Raspopov et al., 2008).

We found here that this cycle was very powerful before the Maunder minimum and was obviously weaker after that. Its decreasing amplitude during the last $\sim$ 400 years of instrumental observations is detectable, but it is quite gradual according to the WA-method. However, according to the MWTRPP procedure, this fading is abrupt. This is caused by the relatively narrow "moving window" ($P = 150$ years) we used for studying the shorter series (Figs. 3, 4, 6, 7, 9, and 10), which makes the method not precise enough for cycles of the same or longer duration. On the other hand, an additional disturbance over the $\sim$ 200-year cycle is caused by the Gleissberg-Gnevishev's minimum (1898–1923 AD). The detected dynamics of the 200-year cycle match previous results and conclusions about its high regular amplitude modulation by the Hallstadtzeit cycle (Damon & Sonett, 1991; Bonev et al., 2004; Komitov et al., 2004).

All deep solar minima with mean Schwabe-Wolf's cycle magnitudes $Ri_{max} \leq 60$ in the odd calendar centuries during the last two millennia are caused by the downward phases of the 200-year cycle. By our opinion it is obvious that the deep solar minimum between the Zurich solar cycles No

23 and 24 (2006–2009 AD) is just the start of sequent "grand" (or supercenturial) solar minimum.

The results presented in this study emphasize the important role of the long-term cycles for the solar activity dynamics. The high relative stability of the quasi-centurial multiplet ($\sim$ 60, 80 and 115–120 years), as well as of the quasi-bicenturial ($\sim$ 200 years) cycle, proves that these cycles are rather regular than stochastic phenomena. The latter is valid for the Maunder-type minima, too (the 2200–2400 Hallstadtzeit cycle), as pointed in many studies. It is very probable that some of these cycles are caused by long-term cyclic processes that operate in the deeper layers of the solar convective zone, whose physical nature is quite different from the standard solar dynamo model and the related to it 11-year and 22-year oscillations. The question for the nature of these processes (solar diameter changes, differential solar rotation variations etc. (see Sokoloff, 2004, and references therein), or how they are connected to the convective zone transport and to solar dynamo phenomena is unclear yet. It is important to take into account these long-term features of solar activity for more reliable predictions of such cycles. Their effects should be used in more precise physical models of solar variability because the present solar dynamo models give an account only the 11-year and 22-year cyclic evolution of the solar magnetic field.

It is necessary to perform an additional study especially focussed on the existence, stability and evolution of the subcenturial solar cycles. For this purpose we plan to use the long "cosmogenic" ^{14}C time series with relative high time resolution INTCAL04 (Reimer et al., 2003). It is also important to perform an independent test for the validity of the results presented there for the last millennia, based on the synthetic Rsi-series.

3.2 The long-term flare activity dynamics

It is very probable that the three distinct components of the quasi-century multiplet are connected to the long-term dynamics of different classes of active centres and sources of flare activity. On the basis of the Křivský & Pejml (1988) catalogue data Komitov (2009) found that a strong 62.5-year cycle exists in the annual numbers of middle latitude aurora (MLA) events during the 18th and 19th centuries. In this study, they also found that the peaks of this auroral cycle correspond very well to the quasi 60-year cycle maxima of ^{10}Be production rates in the "Greenland" beryllium-10 data series (Beer et al., 1998, 1990).

Three additional facts are worth mentioning here: 1. The 11-year cycle is weak in the T-R spectra of MLA annual number (Komitov, 2009), while the 62.5-year one is very strong (the corresponding peak in the correlation coefficient R is $\sim$ 0.65). This result was found without the application of any smoothing procedure.; 2. There are closely coinciding peaks both in the MLA and the ^{10}Be production rates near 1725–1735 AD, $\sim$ 1800–1805 AD, and 1865–1870 AD (there is a slight delay in ^{10}Be in the range of 3–4 years, which could be particulary explained by the "resident time" (Komitov, 2009).; 3. The annual number of MLA events has drastically decreased after 1870 AD (Fig. 16) when the 50–60-year cycle in Ri, GSN, AA, and Rsi is weaker (Figs. 3, 4, 6, 7, 9, and 10).

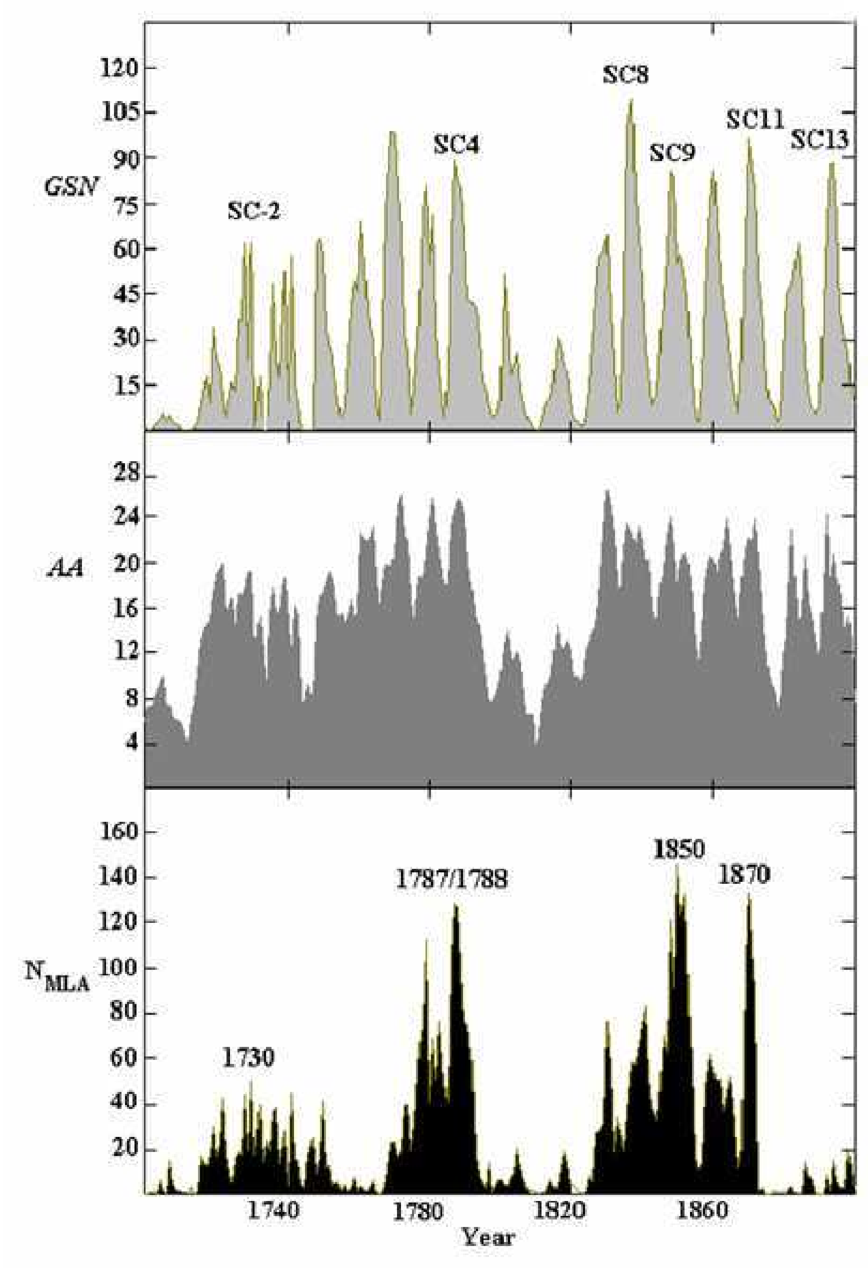

Fig. 16. The sunspot number (GSN), geomagnetic (AA) and MLA activity(N_{MLA}) (1700–1900 AD).

The strong quasi-60-year cycle in MLA events annual numbers (N_{MLA}) during the period 1700–1900 AD is clearly illustrated in Fig. 16 (the lowest panel). Three local main maxima at 1730, 1787–88, and 1850 AD are discernible. There is also another significant maximum at 1870 AD. The N_{MLA} dynamics is quite different from those of the AA-index (the middle panel) and GSN (the upper panel). All these MLA maxima are related to corresponding sunspot Schwabe-Wolf's cycles ones (in 1730 AD to SC-2, in 1787–88 AD to SC 4, in 1850 AD to SC 9, and in 1870 AD to SC 11). However, as it is shown, in the short-time structure of N_{MLA} series the 11-year cycle is not clear at all. It is seriously damaged by many other low amplitude variations on order of a few years. The long-term changes of GSN and N_{MLA} are also not well matched. For example there is a general increase in the sunspot cycle amplitudes between 1730 and 1770 AD, while the MLA activity decreases during this period. Another period of large differences occurs between 1870 and 1900 AD. There is a relatively high sunspot maximum No 13 in 1890, but the MLA events are very rare during this period. The most powerful sunspot cycle during the 19th century is SC 8, but the highest MLA activity is during the maximum of the next SC 9. So, there is no strong relationship between the sunspot activity and the MLA events. It is possible that during strong sunspot Schwabe-Wolf's cycles the corresponding MLA activity could be low and high MLA activity could occur during weak sunspot cycles. The coefficient of linear correlation between GSN and N_{MLA} is $r = +0.57$ for the epoch 1700–1900 AD. It follows from the Snedekor-Fisher's F-test (in this case $F = 1.48$, the ratio between the total and residual variances) that only 32% of the MLA activity variations could be directly related to the overall sunspot activity changes. The situation is almost identical if the geomagnetic activity (AA) and the N_{MLA} are compared. In this case we have $r = +0.61$. Consequently, there is not a very close relationship between the geomagnetic activity and MLA either. It should also be noted that $r = +0.73$ ($F = 2.17$) between GSN and AA during the same epoch (1700–1900 AD).

The MLA phenomena are usually associated with strong solar flare events, which also cause coronal mass ejections (CME). Thus, the aforementioned three facts imply that the 50–60-year cycle is a feature of these active solar centers, which are the typical sources of strong flares and CMEs. The coincidence of the local peaks in the 60-year cycles of ^{10}Be and the MLA during the 18th and 19th centuries suggests that a significant fraction of the ^{10}Be atoms could be produced in the stratosphere by highly energetic particles from strong solar flares (Usoskin et al., 2006). This could make the relationship between the sunspot activity and ^{10}Be production rates much more complicated on century or subcentury time scales compared to the case of only a pure Forbush-effect (Komitov, 2009).

There could be another point of view on the possible relation of the 50–60-year cycle to the sources of the strongest solar flares. It concerns the historical data for giant naked eye visible sunspots and sunspot groups. As shown by Vaquero et al. (2010), there are three peaks of the 50 years smoothing annual numbers of the giant sunspots during the 18th and 19th centuries, near 1735 AD, 1805 AD, and 1870 AD. These three peaks correspond well to local maxima of the MLA annual number (Fig. 16) and the

"Greenland" ^{10}Be concentrations data. The strong downward trend in the annual number of giant sunspots after 1875 AD coincides well both with the corresponding downward tendency of MLA (Komitov, 2009) and the amplitude of the 55–60-year oscillation (Figs. 3, 4, 6, 7, 9, and 10 in Section 2). Thus, it is very probable that the 55–60-year cycle in the sunspot activity is connected directly to the active regions, which are the sources of strongest solar flares, corresponding to the X-ray classes M and X.

Vaquero et al. (2010) shows that there is no significant correlation between the GSN, i.e. the overall sunspot activity, and the visible by naked eye sunspots at all. On the contrary, there is a significant anticorrelation or non-correlation over large fractions of this period. It is also evident from Figs. 14 and 15, that the amplitude changes of the 55–60-year cycle are the most independent relative to the two other components of the quasi-century multiplet. Let us also take into account the very small amplitude of Schwabe-Wolf's cycle in the MLA events (Komitov, 2009). By combining and comparing all these facts, we arrive at the conclusion that these active centres, which are related to the most powerful flare classes and are subject to the 55–60-year cycle are relatively independent of the overall sunspot activity dynamics. Thus, the weak correlation between X and M class flares and the overall sunspot activity Ri during the last 35–40 years (Table 1) is a consequence of the same weak relationship on long time scales.

Table 1. Coefficients of linear correlation between the monthly values of 6 solar and solar-modulated indices for the period 1980–2009

Indices	Ri	F_{107}	N_{flux}	N_C	N_M	N_X
Ri	1	0.975	-0.798	0.847	0.719	0.449
$F_{10.7}$	0.975	1	-0.796	0.856	0.759	0.487
N_{flux}	-0.798	-0.796	1	-0.724	-0.615	-0.464
N_C	0.847	0.856	-0.724	1	0.672	0.415
N_M	0.719	0.759	-0.615	0.672	1	0.770
N_X	0.449	0.487	-0.464	0.415	0.770	1

Ri – the international sunspot number
F_{107} – solar radio flux at $f = 2800$ MHz ($\lambda = 10.7$ cm)
N_{flux} – GCR neutron flux (Moscow)
N_C**,** N_M**, and** N_X – monthly numbers of X-ray flares of C, M, and X classes by GOES satellite data

4. Conclusions

By applying the essentially independent methods of the T-R periodogram analysis, both in the standard and in the "moving window" regimes, and the wavelet analysis on five different solar and geomagnetic data sets a detailed analysis of the problem of the existence and stability of cycles with duration in the range of 20–250 years was made. On the basis of the results obtained in this study, the following main conclusions can be made:

1. The quasi century cycle in sunspot and geomagnetic activity has a complicated multiplet structure, which is clearly detected in different types of instrumental and simulated indirect data on supercentury time scales ($\sim$ 400 years or longer). There are three important and relatively stable components of this multiplet with durations of 55–60, $\sim$ 80, and $\sim$ 120 years. Most probably they are related to different types of solar active centers and sunspot groups. The Gleissberg's $\sim$ 78–80-year cycle is only one of these components.

2. There is a tendency during the last $\sim$ 150 years towards increasing the amplitude of the quasi 120-year cyclic component and fading of the $\sim$ 80-year one. There is an indication that a $\sim$ 400-year cyclic amplitude modulation of the 120-year component exists.

3. It is very probable that the quasi 50–60-year cycle is related to the sources of the most powerful solar flare events (X-ray classes M and even more X) and CMEs. They are relatively independent from the overall sunspot activity dynamics and the 11-year Schwabe-Wolf's cycle. Their relative participation in the overall solar and geomagnetic indices is low.

4. The quasi 40–45-year and 30-year cycles in the solar and the geomagnetic activity indices are unstable on long time scales and their amplitudes are low. A visible strengthening of the quasi 30-year periodicity during the last few decades is observed.

5. Most of the detected solar oscillations with duration $\geq$ 50 years are relatively stable and rather regular than stochastic phenomena. This is related to both the quasi centurial triplet (60, 80, and 120-year) and the $\sim$ 200-year cycle. Their existence, as well as their temporal evolution, should be taken into account in solar dynamo models and long-term solar activity predictions. It is very important in this course to understand the physical nature of these oscillations.

Acknowledgements

The authors are grateful to Prof. Yu. Nagovitsyn for his help with compiling the data from ESAI (http://www.gao.spb.ru/database/esai). The authors are thankful to the National Geophysical Data Center, Boulder, Colorado, U.S.A. for providing the annual Group sunspots number and international sunspot number data via ftp://ftp.ngdc.noaa.gov/STP, and the Meudon Observatory, Paris, France for providing the filament number data via http://bass2000.obspm.fr. The authors are thankful also to Prof. I. Usoskin for important comments, concerning some of the results and their analysis.

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