

Theoretical Model of Non-Conservative Mass Transfer with Uniform Mass Accretion Rate in Contact Binary Stars

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Abstract. In contact binaries mass transfer is usually non-conservative which ends into loss of mass as well as angular momentum in the system. In the present work we have presented a new mathematical model of the non-conservative mass transfer with a uniform mass accretion rate in a contact binary system with lower angular momentum. The model has been developed under the consideration of reverse mass transfer which may occur simultaneously with the original mass transfer as a result of the large scale circulations encircling the entire donor and a significant portion of the gainer. These circulations in contact binaries with lower angular momentum are caused by the overflow of the critical equipotential surface by both components of the binary system making the governing system more intricate and uncertain.

Key words: Contact binary; donor star ; gainer star; non-conservative mass transfer, reverse mass transfer

1 Introduction

There exists a large number of binary systems in which the component pairs of stars are with very small separations and with orbital periods nearly less than 10 years such that mass can transfer from one star to another. This mass transfer can change the structure of both the stars causing subsequent evolution in the system as a whole (Podsiadlowski 2001). These paired systems are known as contact binaries. Thus mass transfer is a regular event in contact binary stars. This mass transfer may occur in two ways. First type is conservative in which total mass and angular momentum remains unchanged. The second type is non-conservative where some mass is lost during its journey from donor to gainer. Non-conservative mass transfer is of two types: with uniform mass accretion rate (Podsiadlowski et al. 1992; Sepinsky et al. 2006; Van Rensbergen et al. 2010; Davis et al. 2013) and with non-uniform mass accretion rate with respect to time (Stepien and Kiraga 2013; Izzard et al. 2013; Gharami et al. 2015).

In contact binaries reverse mass transfer may occur simultaneously with this regular mass transfer making a significant change in the overall dynamics and evolution (Longair 1994; Nelson and Eggleton 2000; Stepien 2009; Stepien and Kiraga 2013). In case of contact binaries with lower angular momentum the overflow of the critical equipotential surface by both components drives large scale circulations surrounding the entire donor and a huge fraction the gainer. A portion of the mass transported by the donor to

the gainer returns back to the donor by this circulation with the mass flux of the order of 10^{-3} to $10^{-4} M_{\odot}$ per year (Stepien 2009).

In this paper we have proposed a theoretical model of non-conservative mass transfer with uniform mass exchange rate with respect to time in contact binaries with lower angular momentum under the consideration of reverse mass transfer following the argument of Stepien (Stepien 2009). We have developed a numerical model for the presently proposed theory.

2 Theory

We offer the following theoretical model of non-conservative mass transfer with uniform mass accretion rate with respect to time in contact binaries with lower angular momentum taking into account the reverse mass transfer originated as a result of large scale circulations encircling the entire donor and a major portion of the gainer star. We here assume that M_1 is the mass of the gainer and M_2 is the mass of the donor at any time t .

$$\dot{M}_1 = \beta_0 \dot{M}_2^{(o)} \quad (1)$$

$$\dot{M}_2^{(o)} = \frac{A}{\tau} M_2 \quad (2)$$

$$\dot{M}_2 = -\dot{M}_2^{(o)} + \dot{M}_2^{(i)} \quad (3)$$

$$\dot{M}_2^{(i)} = \gamma(1 - \beta_0) \dot{M}_2^{(o)} \quad (4)$$

where β_0 characterizes the mass accretion process by the gainer and γ portrays the process of formation of reverse mass jet directed to the donor. (2) comes from the Bernoulli's law applied in the mass flow through the inner Lagrangian point considering adiabatic index $\gamma = \frac{5}{3}$ assuming the component stars being with convective envelopes. A is a numerical constant lying between 1 and 2 and τ is the entire timescale during which the mass transfer is taking place (Pols 2012). $\dot{M}_2^{(o)}$ and $\dot{M}_2^{(i)}$ indicate respectively the total outgoing jet of mass as a result of non-conservative mass transfer from donor to gainer and total incoming mass flow directed towards the donor due to the reverse mass transfer in time t .

Using (2) and (4) in (3) we get,

$$\dot{M}_2 = -[1 - \gamma(1 - \beta_0)] \frac{A}{\tau} M_2 \quad (5)$$

Integrating equation (5) with the initial condition that at $t = 0$, $M_2 = M_{2,0}$, we get

$$M_2 = M_{2,0} e^{-\frac{A[1-\gamma(1-\beta_0)]t}{\tau}} \quad (6)$$

Again using (2) in (1) and on integration with the initial condition that at $t = 0$, $M_1 = M_{1,0}$ we have

$$M_1 = M_{1,0} + \frac{\beta_0 M_{2,0}}{1 - \gamma(1 - \beta_0)} \left[1 - e^{-\frac{A[1-\gamma(1-\beta_0)]t}{\tau}} \right] \quad (7)$$

Again using (6) in (2) we get,

$$\dot{M}_2^{(o)} = \frac{A}{\tau} M_{2,0} e^{-\frac{A[1-\gamma(1-\beta_0)]t}{\tau}} \quad (8)$$

Integrating (8) with the initial condition $M_2^{(o)} = M_{2,0}^{(o)}$ at $t = 0$, we get

$$M_2^{(o)} = M_{2,0}^{(o)} + \frac{M_{2,0}}{1 - \gamma(1 - \beta_0)} \left[1 - e^{-\frac{A[1-\gamma(1-\beta_0)]t}{\tau}} \right] \quad (9)$$

Again using (2), we get from (4)

$$\dot{M}_2^{(i)} = \gamma(1 - \beta_0) \frac{A}{\tau} M_2 \quad (10)$$

Integrating equation (10) with the initial condition at $t = 0$, $M_{2,0}^{(i)} = 0$ (understandably there should not be any initial jet of reverse flow at the very beginning instant of mass transfer) we get,

$$M_2^{(i)} = \frac{\gamma(1 - \beta_0)}{1 - \gamma(1 - \beta_0)} M_{2,0} \left[1 - e^{-\frac{A[1-\gamma(1-\beta_0)]t}{\tau}} \right] \quad (11)$$

3 Results:

Here we produce a numerical example taking the initial masses of the gainer and donor as $M_{1,0} = 9 \times 10^{31}$ (g) and $M_{2,0} = 4 \times 10^{32}$ (g) respectively. For the present calculation we take the values of the parameters as $A = 1.5$, $\gamma = 0.1$ and $\beta_0 = 0.4$. The time scale of the mass exchange is taken as $\tau = 10^4$ (years) and the outgoing mass from the donor at initial instant i.e. at time $t = 0$ is taken as $M_{2,0}^{(o)} = 1.0 \times 10^{29}$ (g). Also $M_{2,0}^{(i)} = 0$ i.e. at the initial instant no incoming mass jet is taken into account towards the donor. The orbital period of the system is considered to be less than 100 days so that the gainer can fill the Roche lobe during its expansion (Monzoori 2011). Here we have produced the graphs for mass incoming to the donor vs. time (years) (Figure 1), mass outgoing from the donor vs. time (years) (Figure 2), mass of the donor vs. time (years) (Figure 3) and mass of the gainer vs. time (years) (Figure 4). Figure 1, 2 and 4 show increasing profiles while Figure 2 for obvious reason exhibits decreasing profile.

4 Discussion:

The present work proposes a theoretical model of non-conservative mass transfer with uniform mass accretion rate with respect to time in contact binaries with lower angular momentum under the consideration of reverse

mass transfer based on the proposal of Stepien (2009) which points out a possible large scale circulation generated by the overflow of the critical equipotential surface by both components as the possible driving force of this reverse mass transfer. Stepien and Kiraga (2013) proposed an alternative cause of reverse mass transfer in contact binaries. They proposed that as the altitude of the equatorial bulge quantifies to a certain percentage of the stellar radius above the surface hence when the radius of the accretor approaches the size of the Roche lobe the apex of the bulge protrudes beyond the inner critical surface. As a result, part of the matter flows above the Roche lobe and comes back to the donor. In future, we may work on this aspect of reverse mass transfer. Moreover, the present work focuses only on the issue of non-conservative mass transfer with uniform accretion rate. Future work may be carried out in the direction of non-uniform mass accretion rate. There is also a provision to study the effect of magnetic field in reverse mass transfer. Theoretical study in future in this direction should bring some interesting results.

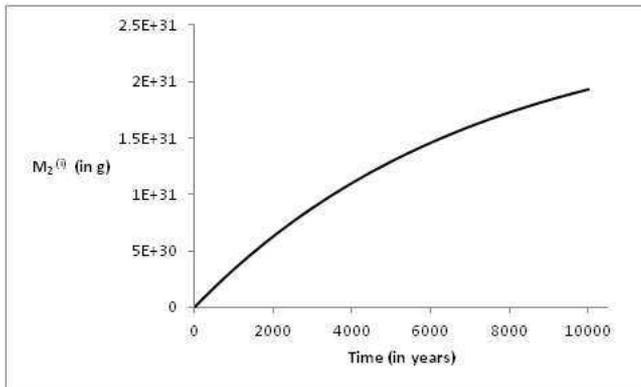


Fig. 1. Mass incoming to the donor.

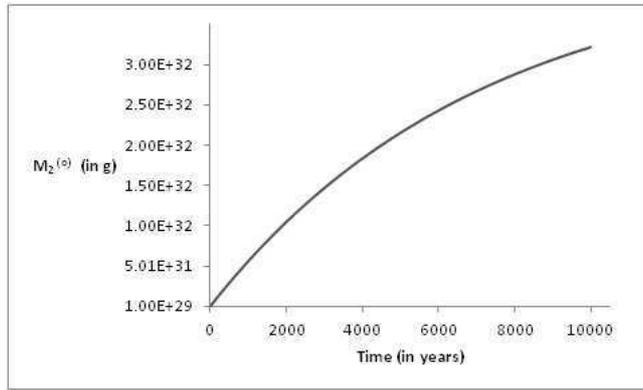


Fig. 2. Mass outgoing from the donor.

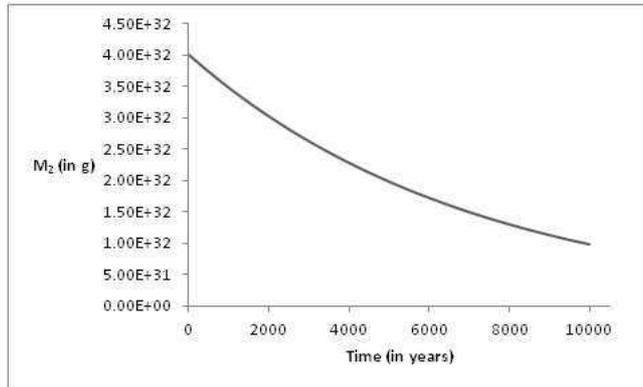


Fig. 3. Mass of the donor.

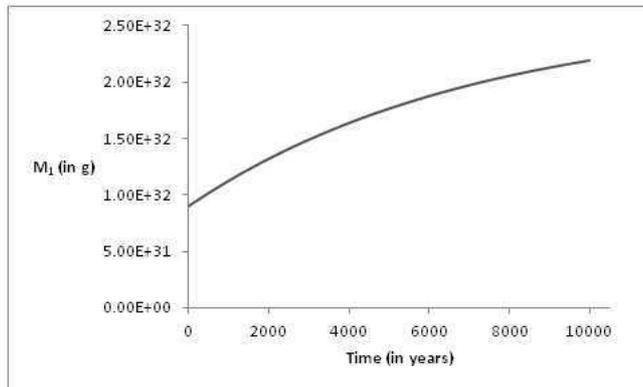


Fig. 4. Mass of the gainer.

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