Analysis of the calendar C. Ptolemy "Phases of the fixed stars"

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Abstract. The article examines the paper of the Alexandrian astronomer Claudius Ptolemy's "Phases of the fixed stars." By analyzing the calendar dates heliacal risings and settings of stars shows that these dates are the result of calculations, which are based on photometric system of the "Almagest" star catalogue.
Key words: Phases of the fixed stars, Ptolemy

1. Introduction

The famous paper of the Alexandrian astronomer Claudius Ptolemy (1816) "Phases of the fixed stars" is a calendar of weather predictions based on visibility of stars. The first part of this study has been lost. We have the second part where an information is given almost for every day of the civil calendar it is informed about the event of rising or setting of a star in a certain climate. In ancient times, there was a geographical division of the Earth according to climates. Each climate corresponds to a particular maximum of the day length. For example, a 14-hour climate corresponds to the territory (or latitude) where the maximum duration of the daylight is 14 hours. The appearance of each star is accompanied by a description of related weather phenomena that occur on the specified day in several chosen climates.

The similar stellar calendars were necessary to identify the main events of agricultural year. In Greek astronomy, the first attempt to define conformity between visibility of the stars and nature events belongs to Hesiod (2001). According to Hesiod the days of the solstices, the morning rising and evening setting of Pleiades and the evening rising of Arcturus determined five fixed points of the year. With their help, the year was divided into seasons which defined the time of sowing, reaping, harvesting grapes and navigation. Evketemon's parapegm consisted of several dozen events of risings and settings of stars, which were followed by the prediction of weather phenomena typical of the given time (Van der Waerden, 1974). Evktemon's parapegm did not reach us in its original form, but it was reconstructed by Rehm (1913) on the basis of Greek sources (Kurtik, 2001). In the annex to the Gemin's paper "Introduction to the (celestial) phenomena" the excerpts of parapegms from different authors were collected. The problem of analysis of these data is that the data were copied many times. As a result, they contain random errors and targeted corrections, when the copyist of the text thought that he eliminates the inaccuracy. In the "Natural History" by Pliny there is a compilation of hundreds of observations about the rising and setting of stars for different countries and associated with these events, agricultural advice (Pliny, 2009). By the number of predictions and the geographic scope Ptolemy's study "Phases of the fixed stars" is the most comprehensive of all the Greek works.

Let's note that in the literature, "Phases of the fixed stars" is called "Phases" to reduce the Greek name "$\Phi\alpha\sigma\varepsilon\iota\sigma$ $\alpha\pi\lambda\alpha\nu\omega\nu$ $\alpha\sigma\tau\varepsilon\pi\omega\nu$". In the future, we will stick to this terminology.

The most detailed investigations of "Phases" belongs to Vogt (1920) and Grasshoff (1993). These studies are based on originally incorporated opposing views on the contents of "Phases". Vogt believed that, in general, "Phases" is the result of observations. Indeed, in the Book VIII of the Almagest Ptolemy argues that to determine the arc of visibility of each star it is necessary to hold separate observations. It is unlikely that Ptolemy himself conducted observations on different parallels; however, he could use observations of other astronomers. For example, in the "Natural History" by Pliny reports of stellar visibility for Italy (Rome), Greece (Attica+Beotia), Egypt and Assyria are presented. Pliny was not an astronomer and it is unlikely that he made observations himself. It is believed he borrowed the real observations from other people. Some descriptions of observations were distorted and in these cases we find large errors. Thus, Vogt's point of view has a base.

Grasshoff takes the opposite point of view. He assumes that the events of the stellar visibility which are described in the calendar of Ptolemy are resulted from calculation. Taking into account Ptolemy's methods, which he used in the Almagest and possibilities which are consistent with his historical era, Grasshoff offers a simple linear model. This model describes the dependence of the arc of visibility γ on the difference between the azimuths $\Delta\theta$ for a given stellar magnitude.

$$\gamma_{scopic} = \gamma(360^0 - \Delta\theta)/360^0$$

In Morelon's study (Morelon, 1981) there is a fragment of text of Arab astronomer Thabit ibn Qurra, who quotes Ptolemy's method of calculating the arc of visibility. Grasshoff shows that the model used by Ptolemy is fully consistent with its reconstruction.

But it does not imply that the "Phases" wholly or partly were calculated by the offered model. In fig. 8 of this work Grasshoff provides the dependence of the arc of visibility on the difference between the azimuths for the stars of the first magnitude. In this figure every event of stellar visibility corresponds to a certain point. All the points are grouped in three clusters which correspond to $\theta_1 = -180^0 \pm 45^0$, $\theta_2 = 0^0 \pm 45^0$ and $\theta_3 = 180^0 \pm 45^0$. The first and the third clusters correspond to acronychal risings and cosmic settings, and the second one to heliacal risings and settings. According to Grasshoff, the average value of arc of visibility for the first-magnitude star is about 7.5^0 for acronychal and cosmic events. However, the author does not provide the method by which these values were calculated according the data of "Phases".

The main purpose of this investigation is to determine whether the contents of "Phases" is a result of actual observations or events of stellar visibility were calculated.

2. The contents of "Phases of fixed stars"

Visibility of the stars. "Phases" describes the beginning and the end of thestellar visibility. In general case we differentiate four events. The first morning visibility is the event when star comes out from behind the Sun and becomes visible in the rays of dawn. The last evening visibility is the event, when star is moving closer to the Sun and grows invisible in the glow of sunset. The first event is called heliacal rising and the second one - heliacal setting. The first evening visibility or acronychal rising is the event when star has the first visibility during twilight after sunset. The last morning visibility or cosmic set is case when the star has the last visibility during morning sunrise.

In the Almagest Ptolemy provided ecliptic coordinates and the value of stellar magnitude for each star. Therefore, we can compare the brightness of a star in the Ptolemaic system with its brilliance in the filter "V" of Johnson's system, which is similar to the human vision during twilight. The comparison showed that sometimes Ptolemy's values of stellar magnitude are close to the values of Johnson's system, but in some cases the difference is significant. Ptolemy's errors have a random and systematic character, which is a consequence of an unfortunate choice of stars-standards.

Let's note that the moments of acronychal rising and cosmic setting are determined less accurately than the moments of heliacal sunrises and sunsets. In the latter case, we fix a specific event of the beginning or finishing of visibility, which is uniquely determined. The moment of cosmic setting (and similarly acronychal rising) is determined with greater uncertainty because the star can be observed before the beginning of the event and after it. In this case, to find the date of cosmic setting the observer has to evaluate the brightness of twilight sky and use it as a standard for subsequent observations. The standard brightness of the dawn is chosen subjectively, so it is difficult to obtain the moment of the cosmic setting from direct observation.

Stars. In "Phases" Ptolemy mentions 33 stars, if we take into account the stars, which are mentioned at least only once. Among them, 29 stars have univocal identification, and in 4 cases the identification is doubtful, see Annex 1. The identified stars are either of the brightest stars on the sky, or ones of the most brilliant stars of the constellations. The reliability of identification is provided by Ptolemy's verbal description, which is beyond any doubt. In this list the only faint star is the "star in the knee of Sagittarius" with magnitude $m_V = 3.96^m$. However, in this case one verbal description is quite enough.

The list of stars that Ptolemy calls by proper names is of interest. They are: Ear (α Vir), Arcturus (α Boo), Antares (α Sco), Goat (α Aur), Canopus (α Car), Dog (α CMa), Procyon (α CMi). For some reason this list does not contain bright such stars as Regulus (α Leo), Vega (α Lyr) and Altair (α Aql) which names are mentioned in the "Almagest".

Let's consider the list of unidentified stars. The first in this list is "*the brightest star in Aquarius*" (month 12, day 27). However, in Aquarius, there are no bright stars at all. The brightest stars are β Aqr and α Aqr which have third magnitude $m_V = 2.91^m$ and $m_V = 2.96^m$. So we can select any star from this pair. Taking into account that the star with such verbal description is mentioned in the "Phases" only once, it is impossible to find preference for one of these stars by modeling the conditions of visibility. Usually, a star with reliable identification is used about 6-10 times, because events of rising and setting are described for different climates. Therefore, we can assume that the stars mentioned at least only once are described incorrectly.

Equally there are doubts about identification of the star called as "*the last star in Taurus*". In "Phases" it was referred only once (month 2, day 6) and its visibility relates to the event of cosmic setting. According to the "Almagest" star catalogue, the brightest and the last in longitude star in Taurus is ζ Tau with magnitude $m_V = 3.0^m$. In the Almagest ζ Tau was described as "*the star at the end of southern horn*" and this is the last star in the figure of Taurus. In Taurus there are five stars with greater longitude, which are not included in its figure. Besides, all these stars have magnitude fainter than $m_V = 4 - 5^m$, so ζ Tau is the best variant of identification. But it is impossible to prove its reliably on the basis of a single event.

"*The star in the head of a Lion*" is mentioned in "Phases" but according to the "Almagest" star catalogue there are two stars with such description: ε Leo, $m_V = 2.98^m$ and μ Leo, $m_V = 3.88^m$. The first one is brighter, so it should be preferred. However, such identification is not obvious. Visibility of this star is mentioned in two events, however, in one case, the climate is not specified. As result we cannot use this report to evaluate the best choice from this pair. The second event (Epagomens, day 3) gives better fit to the second star μ Leo, but any evaluation based on a single observation is not reliable. Therefore, we cannot uniquely identify this star.

According to the "Almagest" catalogue "*the last star in the River*" has the first magnitude. However, there is not any star at all at the coordinates specified by Ptolemy. According to coordinates the nearest star is θ Eri or Acamar. Acamar is at 2.5^0 from the star №805 of the "Almagest" star catalogue and it has the magnitude 3.24^m. In the absence of other candidates, the star catalogue №805 "Almagest" was identified with θ Eri by Knobel and Peters (1915). At the same time, they assumed that the author made misprint in the stellar magnitude and instead letter "$\varDelta$" he wrote the letter "A". In "Phases" there are 9 events where visibility of this star is described. It allows to estimate its magnitude based on mathematical modeling.

Climates. In total, the "Phases" contains about 400 events in which 9 climates are mentioned. If in any message the information about the climate was not specified, all events related to this message were not considered in future analysis. Fortunately, there are few messages relating to unknown climates. In Annex 2, the distribution of events by climates is given, fig 1. According to these data the following conclusions can be inferred.

First, 12.5, 16 and 16.5-hour climates are represented by single events. Also, the 13-hour climate has a relatively low volume. Perhaps the Calendar was not very important for these extreme southern and northern territories. Second, the largest number of messages are related to the parallels of the island of Rhodes and Siena, but not to the climate of Alexandria, where Ptolemy conducted observations. Third, most of the events relate to acronychal risings and cosmic settings.

Weather events and their authors. Ptolemy accompanies each message about rising or setting star with a variety of weather phenomena with reference to different authors. The authors are Hipparchus, Democritus, Dosifej, Eudoxus, Kalipa, Conon, Meton, Metrodorus, Philip, Caesar, Evktemon and "Egyptians". The first 11 authors are specific historical characters. Ptolemy refers the term "Egyptian" to a group of Greek authors, who observed in Lower Egypt during the Hellenistic period. The examples of weather events are: "equinoctial wind blows from the east", "thunder and rain", "storm at sea", "cold air", and etc.

3. A photometric system of the "Phases"

So Ptolemy describes about 400 events that take place at 9 climates during a year. It looks extremely doubtful, that he visited all climates and observed all the phenomena by himself. Let's consider two the most likely possibilities. Ptolemy either compiled "Phases" based on his own and borrowed observations, or calculated all the events of stellar risings and settings based on theoretical model. The latter possibility was technically realizable, since for such calculation it is necessary to have a star catalogue and model of stellar visibility during twilight. It's known that Ptolemy had the "Almagest" star catalogue, and in the 8th book of the "Almagest", the concept of "arcus of visionis" was introduced. It allows determining the moments of stellar visibility as a function of its magnitude and position in the sky.

Let's consider the dependence of the arc of visibility γ on visual stellar magnitude in Johnson's system m_V and in the Ptolemy's system m_P. If the "Phases" are based on real observations, we can expect that the highest value of correlation coefficient K, coefficient of determination R^2 and the lowest value of $p - V$ will provide the dependence $\gamma(m_V)$. (The value $p - V$ gives an evaluation of the significance of the regression equation.) Otherwise, if the moments of visible stars were calculated, the dependence $\gamma(m_P)$ will have the best characteristics. In order to make a distinction between $\gamma(m_V)$ and $\gamma(m_P)$ more significant we excluded from the analysis the stars which have approximately the same magnitude $|m_V - m_P| \leq 0.35^m$ in Johnson's and Ptolemy's systems. To improve the quality of estimation of the regression parameters we combined the sets of morning and evening events by introducing a dummy variable. Acronychal and cosmic events are defined with worse precision, so we considered them separately.

The results of calculations are presented in Appendix 3. Comparison of characteristics of models shows that, on average, they describe the data "Phases" for the photometric system of Johnson and the system of Ptolemy equally well. In this case, regression analysis can give preference to neither of photometric systems. If we substitute stars of the first and second magnitudes in Ptolemaic model $\gamma_T(m_P) = 9.8{+}2.2{\cdot}m_P$ we obtain $\gamma(1) = 12.0^0$ and $\gamma(2) = 14.4^0$. (Index "T" means that this regression equation was obtained based on the total set of observations. Values γ_R and γ_S were calculated based on events of first morning and last evening visibility.) The first value is close enough to the Ptolemaic arcs of visibility of Saturn and Mars (Toomer, 1998). However, Ptolemy does not report visual magnitude for these planets, so we can only assume that he attributed the brightness of Mars and Saturn to the first magnitude.

In Johnson's system the slopes of the regressions $\gamma_R(m_V)$ and $\gamma_S(m_V)$ converge well with each other, but there is a difference in the linear term (constant). The constant $\gamma_T(m_V)$ in the equation corresponds to the phenomenon of setting. To get the equation of the rising it is necessary to add a constant $D_V = 1.1^0$. In the Ptolemaic system, the opposite situation is realized. The shift differs insignificantly from zero $D_P = 0^0$, but the slopes of regressions do not match. This is explained by errors.

The value of the slope coefficient is determined by the characteristics of human vision, so it should be a constant for the set of morning and evening observations. Conversely, the constant term of regression equation can be different for the morning and evening observations. In this case, the arc of vision for a morning observation is greater than for an evening one for the same star. This can be explained by the fact that in a morning observation the first appearance of a star is recorded. In the evening observation we register the date of the termination of stellar visibility. So, the last evening appearance occurred the day before. During the day the Sun passes over the ecliptic about 1^0, what leads to decreasing a value of the arc of vision γ_S in comparison with the previous day. In addition, there are objective factors which have influence on visibility conditions. On the one hand, the morning atmosphere is usually cleaner than the evening one, so in the morning more favorable conditions are realized. On the other hand, during the evening visibility, the place of the stellar appearance is well known from previous observations, but in the morning observations the appearance of a star is known approximately.

In appendix 3, fig. 2 the dependencies of arc of vision γ on the difference of azimuths of the rising(or setting) star θ^* and the Sun θ_S for the stars of the first and second magnitude Ptolemy's catalogue are defined.

$$\gamma(1) = 12.36 - 0.090 \cdot \mid \Delta\theta \mid$$
$$\gamma(2) = 15.49 - 0.096 \cdot \mid \Delta\theta \mid$$

In the regression equations, constants correspond to the arcs of visibility for the stars of the first and second magnitude at the time when the star is directly above the Sun, $\Delta\theta = 0$. It provides the next estimates $\gamma(1) = 12.4^0$ and $\gamma(2) = 15.5^0$. These results are in a good agreement with previously obtained values $\gamma(1) = 12.0^0$, $\gamma(2) = 14.4^0$. Heteroscedasticity leads to biased estimates of regression coefficients and inefficiency of estimates. Therefore, the evaluation of the arcs of visibility γ is more accurate for values $|\Delta\theta| \leq 30^0$.

In the Canon of Mas'ud (Biruni, 1976), the famous Khorezmian scientist Biruni provided information about stellar arcs of visibility from the lost study of Ptolemy, "a book about rising of stars and storms."

Determining the value of reduction [arc of visibility] Ptolemy, and [some] of his predecessors had this in mind. They found that for the stars of the first magnitude is - two fifths of the zodiac sign, and for the second magnitude - half of the zodiac sign and similarly for the other quantities. Therefore, Ptolemy in his "book about rising of stars and storms," says that he learnt [the information] about the stars that the ancients called invisible, such as the stars of Arrow, Dolphin and Pleiades. [Canon of Mas'ud, Part 2.]

Thus, the arc of visibility is 2/5 part of the zodiac sign, or $\gamma = 12^0$ for the stars of first magnitude, and 1/2 part of zodiac sign or $\gamma = 15^0$ for the stars of second magnitude. These values are with high accuracy correspond to the previously evaluated values.

It might be hypothesized that this coincidence is not accidental and "Phases" is a fragment of the "Book about rising of stars and storms". It follows from the title of study of lost Ptolemy's study that it was associated with the phenomenon of stellar visibility and weather events. "Phases" are devoted to the same subject. Secondly, arcs of visibility for stars of the first and second magnitude values which we meet in the "Phases" and "Book about rising of stars and storms" are the same. Both of these facts are in favor of this assumption.

Variables m and $\Delta\theta$ are separated, so both derived equations $\gamma(m)$ and $\gamma(|\Delta\theta)|$ can be combined into a single equation

$$\gamma(m;|\Delta\theta|) = 9.8 + 2.2 \cdot m - 0.093\cdot \mid \Delta\theta \mid,$$

and $\gamma_R = \gamma_S$. On average, this equation describes the events of "Phases" satisfactorily. However, the proposed model is one of the possible models, because we do not know what kind of explaining variables was used by Ptolemy.

As a result, we can give preference to neither of the photometric systems, so on the basis of this analysis it cannot be contended that the content of "Phases" was obtained by observations or not. Let's note that fig. 2 contains the points corresponding to the arcs of visibility $\gamma \leq 8 \div 8.5$. With these values γ even the stars of zero magnitude cannot be observed; however, the corresponding points have been used in the construction of regression. Obviously, the presence of such data in "Phases" is the result of some systematic errors. The arcs of visibility for these unobserved stars were formally determined by finding the optimum.

4. The problem of acronychal rises and cosmic settings

As we briefly mentioned above, the events of acronychal rises and cosmic settings are determined from observations less precisely in comparison with the heliacal phenomena. The main reason for this is as follows. At the heliacal rising a star appears for the first time in the morning sky, and a period of its visibility begins. At the heliacal setting a star hides in the rays of sunset and a period of its invisibility starts. The moments of these events are fixed with high precision, because the arcs of visibility are determined at the beginning or finishing of stellar visibility. In the case of acronychal rising (or cosmic setting) a star can be observed both before and after reaching the minimum value of the arc of visibility. Due to this reason, it is very difficult to obtain the arc of visibility from observations.

Appendix 4, in fig. 3 shows an example of the calculation of functions $f(h_{Sun}, H)$, which characterizes limiting visibility conditions for acronychal rising (or cosmic setting) of Arcturus and Spica. Algorithm for computing the function is as follows. Let's take some value of stellar altitude H and for a given value of the extinction coefficient k we calculate the total absorption in the direction of the star Δm. Taking into account atmospheric absorption, the apparent magnitude of the star will be $m' = m_V + \Delta m$. Then, this value should be substituted in the right-hand side of equations (3b) or (4b) of the study (Belokrylov et al., 2011). The result is a minimal value of the Sun depression below the horizon, h_{Sun}, which is necessary for visual observation of the star under the given conditions. Repeating this procedure for different values of H, we construct a function $f(h_{Sun}, H)$, which connects the Sun depression below the horizon h_{Sun} and the altitude of the star H.

Star remains visible everywhere on this line or beneath it. The range of values $H \sim 0.5^0 \div 3.0^0$ corresponds to the case when a low-altitude star can be observed at value of the Sun depression below the horizon, $h_{Sun} < -7^0$. At this moment it is possible to observe the stars up to $\sim 4^m$ in the zenith. At values $H \sim 3^0 \div 7^0$ the star is high enough above the horizon, and it is much less attenuated by the atmospheric absorption. This case corresponds to values of the Sun depression $h_{Sun} = -3.5^0 \div -4.0^0$ and greater brightness of the sunset. Stellar visibility becomes in both cases and from the point of view of the observer a criterion of the event can be the brightness of the dawn, which corresponds to the acronychal rising. However, the brightness of the dawn is a very subjective evaluation criterion. A similar situation occurs in the case of cosmic setting.

From mathematical point of view, the problem of finding an arc of visibility for acronychal rising (cosmic setting) can be solved if we consider the function $\Gamma = |h_{Sun} + H|$, fig. 4. We shall assume that the arc of visibility is achieved in the time t_0, when the function $\Gamma(H(t))$ takes a minimum value: $\gamma = min(\Gamma(H(t)))$. For Arcturus ($k = 0.20$), the dependence $\gamma_A = \Gamma(H) \approx 9^0$ has a minimum value in the range $1.5^0 \leq H \leq 4.0^0$. Thus, the acronychal arc of visibility for Arcturus is 9^0. We calculated arcs of visibility for acronychal rising (and cosmic setting) for Arcturus and Spica for different extinction coefficients: $\gamma_A(\alpha Boo; k = 0.25) \approx \gamma_A(\alpha Vir; k = 0.20) \approx 10^0$ and $\gamma_A(\alpha Vir; k = 0.25) \approx 11^0$. Note should be taken that the acronychal arcs are smaller than the heliacal arcs only by about two degrees. For example, $\gamma_H(\alpha Vir; k = 0.20) \approx 12^0$ and $\gamma_H(\alpha Vir; k = 0.25) \approx 13^0$. Therefore, if Ptolemy did spot an acronychal arc as half of the heliacal arc, then it was wrong.

In Fig. 5 the tracks corresponding to the evening visibility of Spica on 01, 10 and 20 April 2013 for the latitude of Alexandria, and the extinction coefficient $k = 0.20$ are shown. It follows from calculations that the minimum of function $\Gamma(H)$ is reached on 09-10 April. However, the star remains visible in the evening, both before and after reaching the optimum. Therefore, the determination of acronychal arc of visibility from the direct observations is a difficult task.

It is likely that the lack of clear criteria for the stellar visibility for acronychal rising and cosmic setting leads to very large differences in the evaluation of this phenomenon. As an example, let's consider Pliny's testimony (Book XVIII) about cosmic setting of the Pleiades:

Let us see, however, for example, a single disagreement among the authors who live in one country, but contradict to each other. Hesiod says that the Pleiades begin to rise in the morning, as soon as the autumnal equinox is over. Thales assumes that it happens on the 25^{th} *day after the equinox, Anaximander - on* 31^{st} *day, Evktemon - on* 44^{th} *day, Eudoxus on the* 48^{th}*. (Pliny, pp. 353, 213.)*

This quote shows, that there are three clusters of dates for the cosmic setting of the Pleiades: Hesiod (0 days after the equinox), Thales and Anaximander (25-31 days after the equinox), Evktemon and Eudoxus (44-48 days after the equinox). Such a large difference among estimates can not be explained by the effect of precession, because the influence of precession is very slight on the interval of 100-300 years. The variation of the extinction coefficient can provide a shift of event up to a few days. But Pleiades consist of faint stars, which cannot be registered on low altitudes at high air mass, so the influence on a shift of extinction coefficient is small. Hesiod's evaluation differs from most of all other assessments, so it's possible to assume some error in the original text. The difference in the estimates of other authors is from 4 to 23 days. This error can be explained in two ways.

Greek authors determined the date of the cosmic setting of the Pleiades, using different criteria of evaluation. We can add some small errors due to the variation of extinction coefficients. The second way to explain such difference is that, as Biruni wrote (see quote above), nobody watched faint stars such as the Pleiades, and their risings and settings were determined by the observation of other stars. Perhaps this approach applies to the majority of Greek observations, but it is not fair for all cases. Definition of the moment of setting of the Pleiades by using various stars, inevitably leads to different time estimates. The difference will increase even more if we add the error due to the ambiguity of the definition of the conditions of cosmic setting.

Let's return to the analysis of the acronychal and cosmic arcs of vision, which can be evaluated from "Phases". According to Grasshoff (1993) (Fig. 8) the arc of visibility lies in the range of $6 - 11$ degrees for stars of the first value range with the average value about $\gamma_A \approx 7.5^0$. Let's note that acronychal (or cosmic) arc of visibility cannot be less than 10 degrees for the star of the first magnitude in a very clear atmosphere $k = 0.20$. Therefore, the estimates obtained by Grasshoff raise serious doubt.

We carried out the verification of "Phases's" contents using procedures described above. Acronychal risings and cosmic setting of α Leo, α Vir and α Tau were considered. In the Ptolemaic photometric system these stars belong to the first magnitude. In addition, they are located near the ecliptic, so at acronychal and cosmic events they have a difference of azimuths with the Sun close to the 180 degrees. For each of the stars it is necessary to define function and calculate a track for the given date, similar to Fig. 5. The intersection point of function and track correspond to a value of acronychal (or cosmic) arc of visibility laid down in the "Phases". According to our calculations, in most cases, the calculated arc of visibility for each of the considered stars does not exceed the optimum value by more than one degree, i.e. $\gamma_{calc} \in [\gamma_A; \gamma_A + 1]$. Thus, in the "Phases", the moments of acronychal risings and cosmic settings are defined quite realistically.

5. Markers of the "Almagest"

In case the "Phases" is really the result of the calculations made by Ptolemy on the basis of the star catalogue "Almagest" the "Phases" has to inherit the error brilliance of individual stars in the catalog. Therefore, the detection of such errors would be a proof that the "Phases" is a by-product of the star catalogue "Almagest". Let's consider some examples.

1. "Star in the knee of Sagittarius." According to the Almagest, "the star at the knee of Sagittarius" has a magnitude $m_P = 2 - 3$ (in Ptolemaic system). This value corresponds to a star of the second magnitude, and the stars with such brilliance are represented in "Phases" in a large number of events. Based on the coordinates and the verbal description, "the star at the knee of Sagittarius" is uniquely identified with α Sgr which has magnitude $m_V = 3.97^m$. Data processing of "Phases" shows that the maximum arc of visibility for α Sgr is $\gamma_H = 12^0$. According to above mentioned estimates, this value is acceptable for a star of the 2nd magnitude with the difference of azimuths between the star and the Sun about $|\Delta\theta| = 30^0 \div 40^0$. However, in fact, α Sgr is the star of the 4th magnitude, and its arc visibility is not less than 20^0 (Belokrylov et al., 2011). Consequently, all the descriptions of α Sgr visibility were obtained by Ptolemy not as a result of real observations but were calculated on the basis of the "Almagest" stellar magnitude.

2. "The star is called Dog." "The star, called Dog" corresponds to α CMa or Sirius. In Johnson's system, Sirius has magnitude $m_V = -1.46^m$, and in the system of Ptolemy Sirius is the star of the first magnitude $m_P = 1$. So, the difference of magnitude is very noticeable and equal $\Delta m = 2.46^m$. Therefore, if the descriptions of visibility of Sirius are the result of observations, the arc visibility of Sirius has to be significantly lower than the arc of visibility that corresponds to a star of the first magnitude. Otherwise, Sirius's arc visibility will not differ from other stars of first magnitude in Ptolemy's system. For comparison, it is necessary to take stars which have the same difference of azimuths $|\Delta\theta|$ with a solar azimuth, Appendix 5.

In Johnson system, we could not find stars of 1st magnitude, which have the same value of parameter $|\Delta\theta|$ as Sirius. Therefore, the stars of zero magnitude were chosen. A comparison of the arc of visibility of Sirius during heliacal rising with three heliacal arches visibility of Rigel (β Ori) shows that $\gamma(\alpha CMa) \leq \gamma(\beta Ori)$, Table 4a. This implies that the arc of visibility of Sirius does not exceed the arc of visibility of Rigel, hence these stars should have the same magnitude. Table 4b shows the results of comparison for heliacal setting of Sirius. It should be noted that the observations of Arcturus and Sirius corresponding to the arcs of of visibility $\gamma = 3.5^0$ and $\gamma = 5.5^0$ are wrong so they were excluded from consideration. Perhaps the arc of visibility $\gamma(\beta Ori) = 7^0$ is too small, although this is less important for analysis. The presented data show that the average arc of visibility of Sirius does not exceed the value of the similar arcs of visibility of Arcturus and Rigel. It should be noted that in some observations Sirius has more favorable observing conditions, because $|\Delta\theta|(\alpha CMa)$ exceeds $|\Delta\theta|(\beta Ori)$. Therefore Sirius, Arcturus and Rigel should have roughly the same magnitude that we observe in the star catalogue of the "Almagest".

3. "The star named Antares" and "bright stars in the Claws". The bright stars in the Claws are the stars α Lib with magnitude $m_V = 2.75^m$ (southern claw) and β Lib with magnitude $m_V = 2.61^m$ (northern claw). Antares or α Sco is the star with magnitude $m_V = 0.96^m$. The difference of magnitude of these stars and Antares is about $\Delta m = 1.6^m$ however, according to the star catalogue of "Almagest", all these stars have 2nd magnitude. The difference of brilliance is so large that it can be used for verification.

Evaluations of the arcs of visibility which correspond to the events of heliacal rising and setting for α Sco, α and β Lib are shown in the Table 4. Two events for Antares, which correspond to values of arcs $\gamma = 4^0$ and $\gamma = 8^0$ are erroneous. Estimated value of arc visibility for the star of the 1st magnitude is $\gamma_{calc} \sim 10^0$. Taking into account a color amendment for Antares, the value of arc can be reduced to $\gamma \approx 9^0$ (Belokrylov et al., 2011), which is not enough to observe the star.

Especially these values of arcs visibility cannot match the stars of the 2nd magnitude from the "Almagest" star catalogue. Therefore, the descriptions corresponding to these arcs of visibility are erroneous. It was estimated for a set of other observations, that the arc of Antares corresponds to arcs of α Lib and β Lib. The equality of brilliance follows from the equality of the arcs of visibility. It leads again to the "Almagest" star catalog.

4. "The last star in the River." According to the "Almagest" catalogue, "the last star in the River" has the first magnitude. However, there are no stars with the coordinates provided by Ptolemy and θ Eri or Acamar is the nearest suitable star to the given coordinates. Acamar is located about three degrees from the star №805 of the "Almagest" catalogue, and it has visual magnitude $m_V = 3.2^m$. In the absence of other candidates, the star №805 of the "Almagest" was identified by Peters and Nobel with θ Eri (Peters and Knobel, 1915). At the same time, they assumed that the author made an error in the stellar magnitude and instead of Greek letter "Δ" he wrote the letter "A". The calculated value of the arc of visibility does not exceed $\gamma = 7^0$, which can correspond to the star of 1st magnitude at $|\Delta\theta| > 70^0$. Moreover, the presence of star of the first magnitude is more expected than the presence of the fourth magnitude, as all the other stars of the first magnitude are presented in the "Phases".

From this the following chronology of events can be established. At first, Ptolemy made an error with the star №805 from the star catalogue of "Almagest". There is probability that, according to Peters and Knobel, he wrote a wrong letter. But, we assume, that he included star of the first magnitude in catalogue, because he knew that such star exists from some reports. When Ptolemy made the "Phases", the star №805 was included in the set of the stars according to the formal criteria of the brilliance. Next, using stellar magnitude and coordinates, he calculated the moments for its visibility and put them together with other data. In this case, Ptolemy did not make a verification of stellar visibility, because in principle, he could not check all the events for each climate.

Thus, in the "Phases" we found four unique errors of the "Almagest" star catalog. Demonstrated errors are no exception. This can be confirmed by analyzing the events of visibility which correspond to the stars which brilliance in Johnson's system and in Ptolemy's system differ most of all. However, we believe that the found evidence is sufficient. Consequently, Ptolemy's calendar "Phases" is a by-product in relation to the star catalogue "Almagest". That is, the phenomena described in the "Phases" are the results of the calculations, which are based on data from the "Almagest" star catalog. The computational origin of the content of "Phases" can be confirmed by the values of arcs of visibility with $\gamma \leq 8^0$, Fig. 2. At such values of the parameter γ it's impossible to observe stars of zero and first magnitudes. However, everything falls into place, if we accept the assumption that the content of the "Phases" has computational origin.

6. Conclusion

The analysis showed that the events of the stellar rising and setting described in "Phases" are the results of the calculations, which are based on a photometric system of the "Almagest" star catalog. The arc of visibility can be described by the regression equation for $|\Delta\theta| < 90^0$:

$$\gamma(m_P; |\Delta\theta|) = 9.8 + 2.2 \cdot m_P - 0.093 \cdot |\Delta\theta|.$$

It is reasonable to assume that Ptolemy used the stellar magnitude m_P and the difference azimuth between the Sun and the star $|\Delta\theta|$ as explanatory variables. However, we cannot claim that his model contained only the two variables that the relationship was linear and it did not contain cross-terms.

We found the values of the arcs of visibility for stars of the 1st and 2nd magnitude match with the similar values from the lost Ptolemy's study the "Book about rising stars and storms". Taking into consideration that both works examine the connection of star visibility with weather events one can assume that the "Phases" is an applied part from the "Book about rising stars and storms". It is impossible to prove this hypothesis rigorously by using the available data.

Verification of Ptolemy's values of the arcs of visibility which correspond to acronychal risings and cosmic settings showed a good accordance with our calculations. For the stars of the first magnitude heliacal and acronychal arcs differ by 1-2 degrees, so the ratio $\gamma_A = \gamma_C = \gamma_H/2$ is incorrect. According to our estimates, the data of the "Phases" correspond to the values of arcs $\gamma_A = \gamma_C = 10.5^0 \div 11.5^0$ for the stars of the first magnitude, so the model proposed by Grasshoff does not match the contents of the "Phases".

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Appendix 1. The stars which were mentioned in the "Phases".

	The star	**Description of position**
1	α Leo	The bright star in the heart of Lion.
2	β Leo	The star in the tail of Lion.
3	α Vir	The star is called Spica.
4	α Boo	The star is called Arcturus.
5	α Lib	The bright star in the south Claw.
6	β Lib	The bright star in the northern Claw.
7	α Cen	The star in the right leg of Centaur.
8	α Sco	The star is called Antares.
9	α CrB	The bright star in Corona Borealis.
10	α Sgr	The bright star at the knee of Sagittarius.
11	α Lyr	The bright star in Lyra.
12	α Cyg	The bright star in Bird.
13	α Aql	The bright star in Eagle.
14	α PsA	The bright star in South Fish.
15	α And	Common star in Horse and Andromeda.
16	α Tau	The bright star in Hyades.
17	α Per	The bright star in Pursues.
18	β Ori	Common star in Orion foot and in River.
19	γ Ori	The star in the west shoulder of Orion.
20	ε Ori	The middle star in Orion's belt.
21	α Ori	The star in the east shoulder of Orion.
22	α Aur	The star is called Goat.
23	β Aur	The star in the east shoulder of Charioteer.
24	α Gem	The star in the head of west Twin.
25	β Gem	The star in the head of east Twin.
26	α Car	The star is called Canopus.
27	α CMa	The Dog.
28	α CMi	The star is called Procyon.
29	α Hya	The bright star in Hydra.
30	β Aqr	*The bright star in Aquarius.**
31	ζ Tau	*The last star in Taurus.**
32	ε/μ Leo	*The star in the head of Lion.**
33	α/θ Eri	*The last star in River.**

Table 1. List of the stars which were mentioned in the "Phases" is presented. Last four stars have a poor identification.

Appendix 2. The climates which were mentioned in the "Phases".

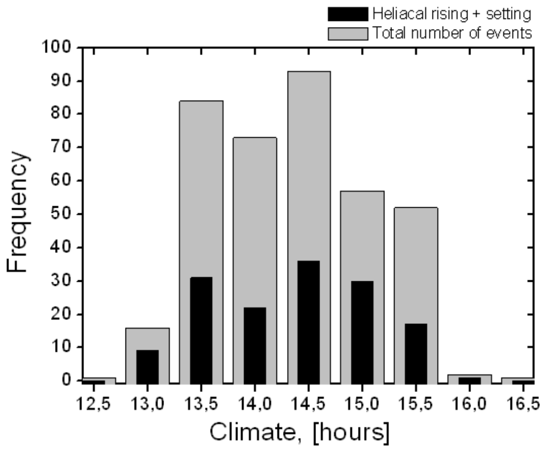

Fig. 1. The distribution of the number of messages on maximum duration of daytime. The events of the first morning and the last evening visibility are marked by black columns. Gray columns correspond to the whole set of events.

N	Maximum duration of the day	Geographic location	Coordinates
1	12.5^h	Avalitian bay	$8^0 25'$
2	13^h	island Meroe	$16^0 27'$
3	13.5^h	Siena	$23^0 51'$
4	14^h	country's of lower Egypt	$30^0 22'$
5	14.5^h	island Rhodes	36^0
6	15^h	Hellespont	$40^0 56'$
7	15.5^h	mid of sea	$45^0 51'$
8	16^h	mouth of Boristhen	$48^0 30'$
9	16.5^h	south of Britain	$51^0 30'$

Table 2. List of the climates.

Appendix 3. The reconstructed models of the arcs of visibility.

Photom. system	Number of observation	Regression equation	Evaluation of model quality
m_P	41	$\gamma_R = 9.4 + 2.5 \cdot m$	$K = 0.54; R^2 = 29$ $p - V < 10^{-3}$
m_P	46	$\gamma_S = 9.9 + 1.8 \cdot m$	$K = 0.46; R^2 = 21$ $p - V \approx 10^{-3}$
m_P	87	$\gamma_T = 9.8 + 2.2 \cdot m$ $D_P = 0$	$K = 0.46; R^2 = 20$ $p - V < 10^{-4}$
m_V	41	$\gamma_R = 12.1 + 1.4 \cdot m$	$K = 0.47; R^2 = 22$ $p - V \approx 10^{-3}$
m_V	46	$\gamma_S = 10.6 + 1.2 \cdot m$	$K = 0.52; R^2 = 27$ $p - V < 10^{-3}$
m_V	87	$\gamma_T = 10.7 + 1.2 \cdot m$ $D_V = 1.1$	$K = 0.48; R^2 = 22$ $p - V < 10^{-4}$

Table 3.

The models of the arcs of visibility $\gamma_R(m)$, $\gamma_S(m)$, $\gamma_T(m)$ depending on stellar magnitude in Johnson's m_V and Ptolemy's m_P photometric systems are presented. Models $\gamma_R(m)$, $\gamma_S(m)$, $\gamma_T(m)$ were calculated based on events of rising, setting and total set events respectively. In every set we considered events where the relation $|\Delta\theta| \leq 30^0$ was executed. To evaluate the consistency of each model correlation coefficient K, coefficient of determination R^2 and assessment of the significance of the regression equation $p - V$ were used.

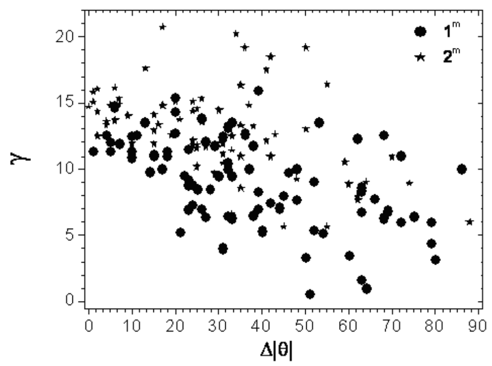

Fig. 2. The dependence of the arc of visibility γ on the difference of azimuths $|\Delta\theta|$ of the stellar rising (setting) θ^* and a solar rising (setting) θ_S for the events mentioned in the "Phases".

Appendix 4. The calculation of conditions for acronychal rising of Arcturus and Spica.

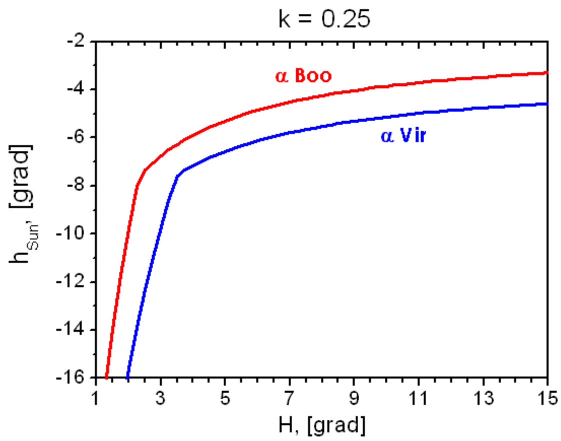

Fig. 3. The function $h_{Sun}(H)$ defines a set of parameters during acronychal rising (cosmic setting), when the extreme conditions of stellar visibility are realized. Above the function $h_{Sun}(H)$ stellar visibility is impossible; below this function an observer can register star. h_{Sun} - is a depression of the Sun below the horizon, H - is a stellar altitude. The calculation was made for the value of extinction coefficient $k = 0.25$.

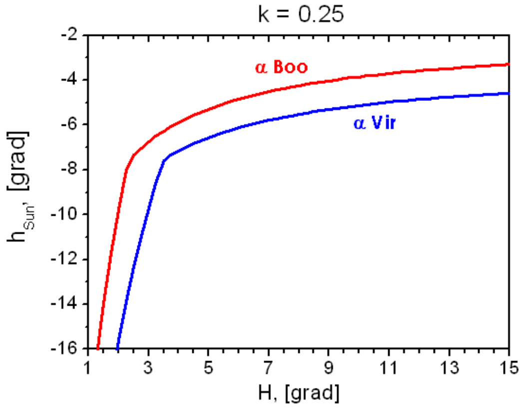

Fig. 4. Function $\gamma(h_{Sun}; H)$ defines a set of parameters during acronychal rising (cosmic setting), when the extreme conditions of stellar visibility are realized. Stellar visibility is possible on the area above each plot. The calculation was made for the value of extinction coefficient $k = 0.25$.

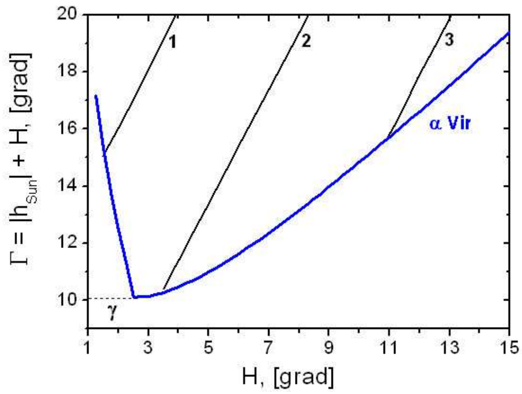

Fig. 5. In this figure the tracks of evening (acronychal) rising of Spica for the value of extinction coefficient $k = 0.25$ are presented. Each track corresponds to a certain date: (1) - 01.04.2013, (2) - 10.04.2013, (3) - 20.04.2013.

Appendix 5. A comparison of arcs of visibility of Sirius and Antares with other stars is shown.

Star	α CMa	β Ori	β Ori	β Ori
m_V	-1.46	0.12	0.12	0.12
$\Delta\theta$	52	52	48	46
γ	11.2	7.5	9.8	11.9

Table 4a. Heliacal rising of Sirius.

Star	α CMa	α Boo	α CMa	α Boo	α CMa	α Boo	β Ori	β Ori	β Ori	β Ori
m_V	-1.46	-0.04	-1.46	-0.04	-1.46	-0.04	0.12	0.12	0.12	0.12
$\Delta\theta$	53	50	48	45	40	39	39	39	33	33
γ	13.5	3.5*	10	8	5.5*	8.5	7	16	13.5	13

Table 4b. Heliacal setting of Sirius.

Star	α Sco	α Sco	α Lib	α Lib	α Sco	α Lib	α Sco	α Lib	β Lib
m_V	0.96	0.96	2.75	2.75	0.96	2.75	0.96	2.75	2.61
$\Delta\theta$	42	36	35	30	26	24	21	16	15
γ	18.5	19	16.5	14.5	8*	14.5	4*	13.5	14

Table 4c. Antares and bright stars in the Claws.

Marked values of the parameter γ correspond to the events when the star could not be observed. m_V - visual stellar magnitude; $|\Delta\theta|$ - difference between the solar and the stellar azimuth during the event of heliacal rising or setting; γ - heliacal arc of visibility.