Paralax effects on microlensing events caused by free-floating planets

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Abstract. Free-floating planets are recently drawing a special interest among the scientific community. Gravitational microlensing is up to now the exclusive method for the study of this Galactic population. In this work we find that the future Euclid space-based observatory can discover a substantial number of microlensing events in its field of view, caused by free-floating planets. Making use of a synthetic population, we investigate also the importance of using the parallax effect as an additional source of information. We conclude about the best positions of the Earth in its orbit for obtaining the greatest number of events with parallax traces.
Key words: Gravitational lensing; Galaxy:structure; Galaxy:center; Galaxy:disk

1. INTRODUCTION

A useful method for detecting extremely faint or completely dark objects in our Galaxy is the gravitational microlensing, which happens when the gravitational field of these bodies acts as a lens to magnify background source stars (Paczyński, 1986). A gravitational lens is characterized by its Einstein ring radius,

$$R_E(M,x) = \sqrt{\frac{4GMD_s}{c^2}x(1-x)}\ , \tag{1}$$

the radius of the ring image formed when the observer, the lens and the source are perfectly aligned. Here M is the mass of the lens; $x = D_l/D_s$ is the normalized lens distance; D_s, D_l are the source-observer and lens-observer distance.

In the case of Galactic lenses, the image separation is too small to be resolved and the observable feature is the variation in time of the light magnification, due to the lens-source relative motion. A microlensing event is thus obtained, whose key parameter is the Einstein radius crossing time, given by Paczyński (1986)

$$T_E = \frac{R_E}{v_T}\ , \tag{2}$$

with v_T-the relative transverse velocity between the lens and the source.

In the standard case, the lens and the source are considered as points with a relative motion linear and constant. The total magnification of the source luminosity in such microlensing events is found to be:

$$A_s = \frac{u^2(t)+2}{u(t)\sqrt{u^2(t)+4}}\ , \tag{3}$$

where

$$u(t) = \sqrt{u_0^2 + \left(\frac{t - t_0}{T_E}\right)^2} \ , \tag{4}$$

is the separation between the lens and the line of sight in units of R_E; u_0 is the minimum separation, named impact parameter, occurring at the moment of the peak magnification t_0. The light curve obtained by equation (3) is symmetric around t_0.

When the projected separation is $u = 1$, the amplification takes a specific value known as threshold amplification $A_{th} = 1.34$. However, the space-based telescopes can detect amplifications much smaller than 1.34 (Byalko, 1970). For $A_{th} = 1.001$, as expected in nowadays telescopes, the maximum value of u in equation (3) turns out to be $u_{max} = 6.54$.

A standard microlensing light curve (3) is described by three parameters, t_0, T_E and u_0, but only one of them, the Einstein radius crossing time T_E contains useful information about the lens: its mass M, the transverse velocity v_T and the distance D_l. For breaking this triple degeneracy, several additional methods are proposed, one of them based on the consideration of the parallax effects induced in the light curve due to the motion of the Earth around the Sun. Free-floating planets are a recently discovered population of planetary-mass objects either very distant from their host stars (over 100 AU) or entirely unbound. Their existence is reported by MOA-II mission (Sumi et al. 2011), based on evidences of light curves corresponding to small, planetary mass objects.

The aim of our work is the investigation of the best Earth positions in its orbit for observing parallax effects induced in microlensing events of free-floating planets. We are focused on the observations towards Galactic centre foreseen to be performed by Euclid, an European mission in preparation to be launched in 2017. In Section 2 we review the microlensing method and its parameters in the case of Euclid observations; in section 3 we show how the parallax motion have effects on microlensing curves; in section 4 we discuss about free-floating planets, their mass function and the spatial and velocity distribution models; in section 5 we discuss the results obtained; in section 6 we draw the main conclusions.

2. MICROLENSING EVENTS TOWARDS THE GALACTIC BULGE

Several microlensing surveys, as MOA (Microlensing Observations in Astrophysics) Collaboration (Bond et al. 2001) and OGLE (Optical Gravitational Lensing Experiment) Collaboration (Sumi et al. 2006), have been undertaken since about two decades towards the Galactic bulge with the aim of searching for MACHOs (Massive Astrophysical Compact Halo Objects) and exoplanets. These surveys have allowed the detection of several thousands of microlensing events, a fraction of them due to objects different from stars.

Microlensing observations are affected by the presence of the Earth atmosphere that is a source of Poisson's noise, which sets up an unavoidable lower limit to the mass of lens. A way to circumvent this limit is to go in space (Bennett & Rhie 2002). At present, there are two space-based missions which are planned for detecting microlensing events towards the Galactic bulge: the Wide-Field Infrared Survey Telescope (WFIRST)(Green et al. 2012) and Euclid.

Euclid is a Medium Class mission of the ESA (European Space Agency). During ten months, not necessarily consecutive, it will hunt for microlensing events towards Galactic Bulge. The Galactic coordinates of the Euclid line of sight are $b = -1.7°$, $l = 1.1°$, the distance of observation can be considered $D_s = (7-10)$ kpc, with mean value at $D_s = 8.5$ kpc and the observing image rate (cadence) is expected to be about 20 min (Laureijs et al. 2011).

In order to find the number of expected microlensing events to be detected by the Euclid mission we use the definition of the microlensing rate (Jetzer et al. 2002):

$$\Gamma = \int \frac{n(x) f(\boldsymbol{v}_l - \boldsymbol{v}_t) f(\boldsymbol{v}_s) dx d\boldsymbol{v}_l d\boldsymbol{v}_s}{dt} \ , \tag{5}$$

where $\boldsymbol{v}_l$, $\boldsymbol{v}_s$ and $\boldsymbol{v}_t$ are the lens, the source and the microlensing tube two-velocities in the plane transverse to the line of sight; $x = D_l/D_s$; $n(x)$ is the number density of the lens population. The microlensing rate is the number of events per unit time and per monitored star due to the lens population.

The velocity distribution functions $f(\boldsymbol{v}_l)$ and $f(\boldsymbol{v}_s)$ are assumed to have Maxwellian forms (Han & Gould 1995, Han & Gould 1996). The tube velocity is given by

$$v_t^2(x) = (1-x)^2 v_\odot^2 + x^2 v_s^2 + 2x(1-x) v_\odot v_s \cos\theta \ , \tag{6}$$

where $\boldsymbol{v}_\odot$ is the local velocity transverse to the line of sight and θ is the angle between $\boldsymbol{v}_\odot$ and $\boldsymbol{v}_s$.

The definition above corresponds to $A_{th} = 1.34$. For other values of A_{th}, we multiply the result by the corresponding value of u_{max} (Griest, 1991). In the case of observations towards Galactic center, the source stars are bulge stars which are distributed following a triaxial mass density model (Dwek et al. 1995):

$$\begin{aligned} \rho(x,y,z) &= \tfrac{M_b}{8\pi abc} e^{-s^2/2} \\ s^4 &= (x^2/a^2 + y^2/b^2)^2 + z^4/c^4 \ , \end{aligned} \tag{7}$$

where the parameters are $M_b \sim 2 \times 10^{10}\ M_\odot$, $a = 1.49$ kpc, $b = 0.58$ kpc and $c = 0.40$ kpc.

The limiting line flux of Euclid Telescope is $F_l = 3 \times 10^{-19} J s^{-1} m^{-2}$, whereas the flux of a Sun-like star situated at the Galactic center is $F_\odot = 4.44 \times 10^{-16} J s^{-1} m^{-2}$. Based on the mass-luminosity relation $\frac{L}{L_\odot} = (\frac{M}{M_\odot})^{2.4}$ for low-mass stars ($M < 0.8 M_\odot$), it can be directly shown that the telescope can observe all bulge stars.

The mean mass of bulge stars is $< M >= 0.28 M\odot$, found by using the mass function $\frac{dN}{dM} \sim M^{-2.35}$ (Salpeter, 1955). The Euclid's field of view is 0.54 square degree, hence the number of source stars in Euclid microlensing observations will be $N_{ED} = 2.23 \times 10^8$. This number has to be multiplied with the microlensing rate (5) and the time of observation for providing us the estimated number of microlensing events expected to be detected.

3. PARALLAX EFFECT

The motion of the Earth around the Sun may leave in microlensing events some traces which can be used to break the degeneracy of microlensing parameters in the standard curve. We are focused here on the investigation of parallax traces during the observation by the Euclid telescope of microlensing events caused by free-floating planets. For this reason, we make use of some geometrical relations (Dominik, 1998).

The Earth trajectory is an ellipse, given in polar coordinates (r, φ) by

$$r(\varphi) = \frac{a_{\oplus}(1-\epsilon^2)}{1+\epsilon\cos\varphi} \ , \tag{8}$$

where $a_{\oplus}$ is the semi-major axis, ϵ is the eccentricity of the earth orbit. The minimal value $r_{min} = a_{\oplus}(1-\epsilon)$ is obtained for $\varphi = 0$, and the maximal value $r_{max} = a_{\oplus}(1+\epsilon)$ for $\varphi = \pi$.

Therefore, one can express the curve with a parameter ξ as

$$r(\xi) = a_{\oplus}(1-\epsilon\cos\xi) \ . \tag{9}$$

The components along the semi-major axis (x-direction) and the semi-minor axis (y-direction) follow as

$$x(\xi) = a_{\oplus}(\cos\xi - \epsilon) \ , \tag{10}$$

$$y(\xi) = a_{\oplus}\sqrt{1-\epsilon^2}\sin\xi \ . \tag{11}$$

The time dependence $\xi(t)$ is implicitly given by

$$t = \sqrt{\frac{a_{\oplus}^3}{GM_{\odot}}(\xi - \epsilon\sin\xi)} \ , \tag{12}$$

so that $t = 0$ corresponds to the points $(r_{min}, 0)$. In general, this equation cannot be solved analytically, but only numerically.

Considering the position of the Earth on its trajectory, the amplification in a microlensing light curve, maintaining the point-like approximation for lenses and sources, will be calculated according to the following formulas (Dominik, 1998):

$$
\begin{aligned}
&A_p = \frac{u^2(t)+2}{u(t)\sqrt{u^2(t)+4}}, \\
&u^2(t) = p^2(t) + d^2(t), \\
&p(t) = p_0(t) + \cos\psi[x_1(t) - x_1(t_0)]+ \\
&+\sin\psi[x_2(t) - x_2(t_0)], \\
&d(t) = d_0 - \sin\psi[x_1(t) - x_1(t_0)]+ \\
&+\cos\psi[x_2(t) - x_2(t_0)], \\
&x_1(t) = \rho[-\sin\chi\cos\phi(\cos\xi(t) - \epsilon)- \\
&-\sin\chi\sin\phi\sqrt{1-\epsilon^2}\sin\xi(t)], \\
&x_2(t) = \rho[-\sin\phi(\cos\xi(t) - \epsilon)+ \\
&+\cos\phi\sqrt{1-\epsilon^2}\sin\xi(t)], \\
&\rho = \frac{a_\oplus(1-x)}{R_E}, \quad p_0(t) = \frac{(t-t_0)}{T_E}, \quad d_0 = u_0
\end{aligned}
, \tag{13}
$$

with ρ-the length of the semi-major axis projected to the lens plane measured in Einstein radii.

The parameters ϕ, χ and ψ in relations (13) give, respectively, the longitude measured in the ecliptic plane from perihelion towards the Earth motion, the latitude measured from the ecliptic plane towards the northern point of the ecliptic and the rotation angle in the lens plane which describes the relative orientation of velocity v_T to the sun-earth system. Using the relations between these two coordinate systems, we find the following values for the Euclid's line of sight: $\phi \simeq 167.8°$ and $\chi \simeq -5.4°$.

The parallax effect is estimated by calculating the residuals between the light curve $A_p(t)$ from eqs. (13) and the corresponding standard curve $A_s(t)$ from eq. (3), that is

$$
Res = |A_s(t) - A_p(t)| \ . \tag{14}
$$

This effect is sensibly dependent on the Earth position in its orbit around the Sun (ξ_0-the Earth position at the time of the maximum amplification). In Figs. 1, 2 we show the parallax effect in its two extreme cases, which correspond respectively to $\xi_0 = 75°$ and $\xi_0 = 165°$. The calculations consider the same object with mass $10^{-3}M_\odot$ situated at the distance $D_l = 4.5$ kpc.

Note that $\xi_0 = 165°$ would be the best position of the Earth for inducing the strongest influence of the parallax effect on microlensing light curves, since the maximum of the residuals is around ten times higher than for $\xi_0 = 75°$. In Section 5 we show and discuss our numerical calculations for the number of microlensing events with parallax effects resolvable by Euclid in different positions of the Earth around the Sun, based on the abilities of this satellite.

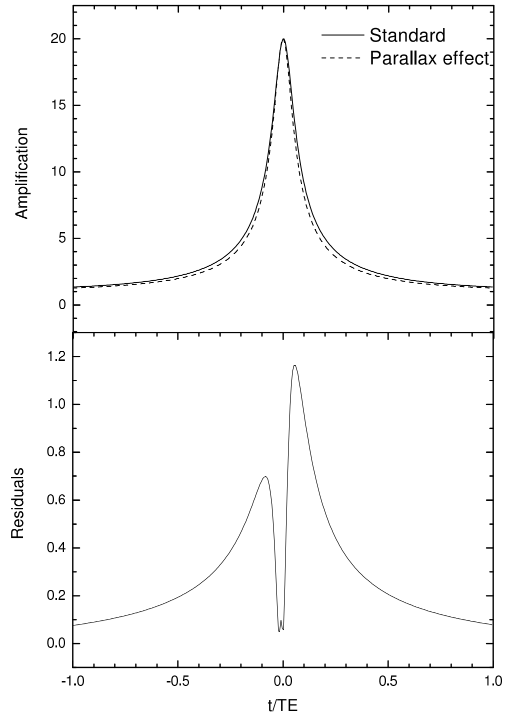

Fig. 1. Above: the standard curve (solid line) and the parallax curve (dashed line) for $\xi_0 = 165°$. Below: the residuals between them. The Einstein time is 2.24 days.

4. PLANETARY POPULATION

In a recent survey of the Galactic bulge, the MOA-II collaboration (Sumi et al. 2011) reported the discovery of free-floating planets, planetary-mass objects either very distant from their host stars (more than 100 AU away) or entirely unbound. By analyzing the timescale distribution of all the observed microlensing events, they found a statistically significant excess of events with timescale $t < 2$ days.

In order to account for the observed excess of these events, Sumi et al. (2011) assumed a free-floating planet's mass function with a power-law form as follows:

$$\begin{aligned} &\tfrac{dN}{dM} = k_{PL} M^{-\alpha_{PL}} \ , \\ &\alpha_{PL} = 1.3^{+0.3}_{-0.4} \ , \\ &10^{-5} M_\odot < M < 10^{-2} M_\odot \ . \end{aligned} \tag{15}$$

The derived number of planetary mass objects per star turned out to be very large, although rather poorly constrained: $N_{PL} = 5.5^{+18.1}_{-4.3}$, due to the low sensitivity of the instrument at the lower bound of lens masses (below $10^{-4} M_\odot$).

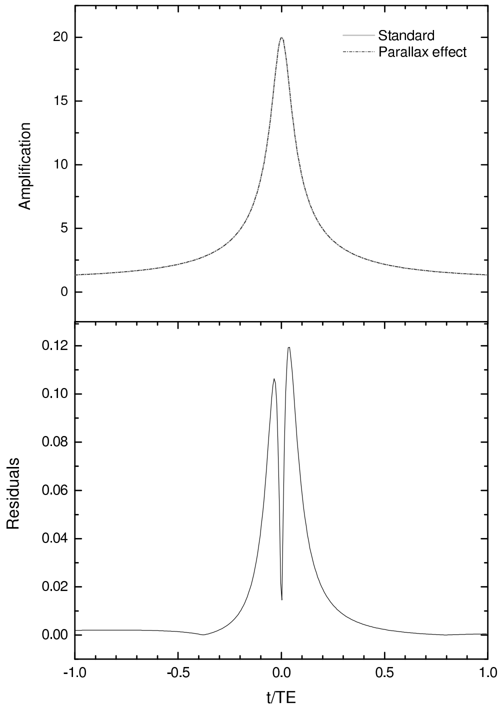

Fig. 2. Above: the standard curve and the parallax curve for $\xi_0 = 75°$. The distinction between them is too small to be noticed. Below: the residuals between them. The Einstein time is 2.24 days.

In regard to the free-floating planet distribution, we assume that these objects follow spatial and velocity star distribution. This assumption is based on the idea that they are most likely formed in proto-planetary disks and subsequently scattered into unbound or very distant orbits. Therefore, as to the free-floating planets spatial distribution, we use the same as for stars (Gilmore et al. 1989, De Paolis et al. 2001, Hafizi et al. 2004):

1. Double exponential disk,

$$\rho(R, z) = \rho_0(M)\ e^{-|z|/H}\ e^{-(R-R_0)/h}\ , \tag{16}$$

in cylindrical coordinates R (the galactocentric distance in the Galactic plane) and z (the distance from Galactic plane). The scale parameters are $H \sim 0.30$ kpc, $h \sim 3.5$ kpc for the thin component; $H \sim 1$ kpc, $h \sim 3.5$ kpc for thick component and $R_0 = 8.5$ kpc is the local galactocentric distance.

2. Triaxial bulge (Dwek et al. 1995, De Paolis et al. 2001, Hafizi et al.2004):

$$\begin{aligned} \rho(x, y, z) &= \rho_0(M) e^{-s^2/2} \\ s^4 &= (x^2/a^2 + y^2/b^2)^2 + z^4/c^4\ , \end{aligned} \tag{17}$$

where $a = 1.49$ kpc, $b = 0.58$ kpc, $c = 0.40$ kpc. For the free-floating planet velocity distribution we assume the Maxwellian distribution (Han & Gould 1995, Han & Gould 1996), where each component is given by

$$f(v_i) \sim \exp^{-\frac{(v_i-\overline{v}_i)^2}{2\sigma_i^2}} \quad , \quad i \in \{x, y, z\} \ , \tag{18}$$

where the coordinates (x, y, z) have their origin at the Galactic center and the x and z-axes point to the Sun and the north Galactic pole, respectively.

We are interested only in the perpendicular velocity with respect to the line of sight, namely on y and z components. For lenses in the Galactic bulge we use the mean velocity components $\overline{v}_y = \overline{v}_z = 0$, with dispersion $\sigma_y = \sigma_z = 100$ km/s; for lenses in the Galactic disk we use the mean velocity components $\overline{v}_y = 220$ km/s, $\overline{v}_z = 0$, with dispersion velocity $\sigma_y = \sigma_z = 30$ km/s for the thin disk and $\sigma_y = \sigma_z = 50$ km/s for the thick disk.

5. RESULTS

The first step was the calculation of the microlensing rate Γ (5) for all populations of free-floating planets: thin disk, thick disk and bulge, considering spatial distributions (16), (17), coupled with the mass functions (15), α_{PL} varying from 0.9 to 1.6. This microlensing rate, which corresponds to $A_{th} = 1.001$, needs to be multiplied by $u_{max} = 6.54$, as discussed before. The number of microlensing events is derived by the multiplication of Γ with the number of source stars in the Euclid field of view $N_{ED} = 2.23 \times 10^8$ and the time of observation, one month.

In Tab.1 we show the number of microlensing events per month towards our Galactic center, found theoretically based on considerations presented in previous sections. This number varies for different values of the index parameter α_{PL}.

Table 1. The estimated number of microlensing events per month caused by free-floating planets assumed with different values of the mass function index α_{PL}.

	$N_{PL} = 1.2$			$N_{PL} = 5.5$			$N_{PL} = 23.6$		
α_{PL}	$Bulge$	D_{thin}	D_{thick}	$Bulge$	D_{thin}	D_{thick}	$Bulge$	D_{thin}	D_{thick}
0.9	317	17	13	1451	76	61	6227	328	262
1.0	270	14	11	1238	65	52	5310	280	223
1.1	227	12	10	1038	55	44	4455	235	188
1.2	188	10	8	862	46	36	3701	195	156
1.3	156	8	7	713	38	30	3061	162	129
1.4	129	7	5	593	31	25	2544	134	107
1.5	109	6	5	498	26	21	2135	113	90
1.6	93	5	4	424	22	18	1820	96	77

Note that the bulge population of free-floating planets provides the most important contribution in the number of microlensing events observed. For estimating the ratio of events with resolvable parallax effect, we generate numerically a large number of synthetic events towards the Euclid field of view by Monte Carlo method. We draw:

a) lens distances D_l, based on the disk and bulge spatial distributions in eqs.(16), (17). The source stars are considered as fixed at the Galactic center, $D_s = 8.5$ kpc, for all events;

b) the relative transverse velocity based on the velocity distribution in eq. (18);

c) the impact parameter randomly distributed in a uniform interval [0,6.54] (Strigari, 2012);

d) the lens mass that follows the mass function distribution in eq. (15). Here, for the mass function index of free-floating planets in bulge, thin disk and thick disk we fix the value $\alpha_{PL} = 1.3$ as a mean value of the interval;

For each case we calculate the standard curve by eq. (3) and the curve containing the parallax effect by eq. (13), maintaining for all cases the point-like approximation for lenses and sources. The points in each curve are taken every 20 minutes, according to the observing image rate (cadence) of the instrument.
We vary the position of the Earth in its orbit taking one point for each month, beginning at $\xi_0 = 15°$ (January). This position corresponds to the time t_0 of the event's peak light curve, found by the implicit relation (13).

We assume that a microlensing event can be detected if in its light curve there are at least 8 points with amplification higher than the threshold amplification $A_{th} = 1.001$. The photometric error in this case is 0.1%:

$$A_{th}F - F = F(A_{th} - 1) = \Delta F \Rightarrow (A_{th} - 1) = \frac{\Delta F}{F} = 1.001 - 1 = 0.001 \ . \quad (19)$$

For estimating the parallax effect on the observed light curves, we consider only those containing at least 8 points with $Res > 0.001$ inside Einstein ring:

$$|A_s(t)F - A_p(t)F| > \Delta F \Rightarrow |A_s(t) - A_p(t)| > \frac{\Delta F}{F} \Rightarrow Res > 0.001 \ . \quad (20)$$

The efficiency of the parallax effect detection is defined as the ratio between the number of events fulfilling the condition (20) with the the total number of detectable events. In Tab. 2 we show the results of our calculations for the efficiency of the parallax effect detection during different months of the year. In fact, the perihelion corresponds to the beginning of January, so $\xi_0 = 15°$ is about the middle of January and the same for the other months.

Table 2. The efficiency of the parallax effect detection during different months for free-floating planets distributed in bulge, thin and thick disk.

Month	ξ_0	*Bulge*	D_{thin}	D_{thick}
January	15°	0.32	0.31	0.31
February	45°	0.27	0.27	0.27
March	75°	0.15	0.17	0.16
April	105°	0.26	0.26	0.26
May	135°	0.30	0.30	0.30
June	165°	0.32	0.31	0.31
July	195°	0.31	0.30	0.30
Augustt	225°	0.26	0.27	0.26
September	255°	0.15	0.17	0.16
October	285°	0.26	0.26	0.26
November	315°	0.31	0.30	0.30
December	345°	0.32	0.32	0.32

Let us fix our following discussion on the most plausible value of the number of free-floating planets per star: $N_{PL} = 5.5$. The theoretical number of microlensing events in such a case is shown in the fifth row and second column of the Table 1: 713 for planets in the bulge, 38 for planets in the thin disk and 30 for planets in the thick disk. We multiply the efficiency with these numbers to find the theoretical number of microlensing events with parallax effect, as in Tab. 3.

Table 3. The theoretical number of microlensing events with parallax effect predicted to be detected by Euclid during different months of one year.

Month	ξ_0	*Bulge*	D_{thin}	D_{thick}	*Total*
January	15°	225	12	9	246
February	45°	192	10	8	210
March	75°	105	7	5	117
April	105°	184	10	8	202
May	135°	217	11	9	237
June	165°	227	12	9	248
July	195°	220	12	9	240
August	225°	188	10	8	206
September	255°	104	7	5	116
October	285°	184	10	8	202
November	315°	220	11	9	241
December	345°	229	12	10	251

Note that the best period of the year for performing microlensing surveys by Euclid are December and June, if we look for having the highest number of light curves with resolvable traces of the parallax effect. Being aware that the threshold amplification $A_{th} = 1.001$ is only an instrument limit, we furthermore tested our results for a higher threshold amplification $A_{th} = 1.01$, taking into account observational conditions like the variability of stars (Ciardi 1998).

With this new limit and based on similar calculations we find an efficiency about 10% lower than before and a number of microlensing events with parallax effect about 2.5 times lower, too. Anyway, this change did not affect our main result about December and June as the best periods for such kind of observations.

6. CONCLUSIONS

Obviously, the aim of our study is the investigation of the parallax effect, induced due to the motion of the Earth around the Sun, as an additional condition for a best observation of free-floating planets towards the Galactic bulge. These observations are planned to be carried out by the future Euclid space-based observatory, via detection of microlensing light curves of the source stars. We look for those curves which could provide us with supplementary information on lenses, aside from the standard parameter, Einstein radius crossing time.

We settle on recent knowledge about free-floating planets, their mass function and their velocity and space distributions. We are limited here on point-like source and lens approximation, without considering the finite dimensions of sources and lenses. Making use of numerical methods for generating synthetic microlensing events observable from the Euclid mission in different positions of the Earth around its orbit, we find that the best periods for this observation are December and June.

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