Modified Fisher's criterion for detection of faint stellar variability

Tsvetan B. Georgiev
New Bulgarian University, BG-1618, Sofia;
Institute of Astronomy and NAO, BG-1784, Sofia
tsgeorg@astro.bas.bg

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Abstract. We present a modified Fisher's criterion for characterization of low amplitude stellar variability and, in principle, time series. It is based on the ratio S_0/S_M, where S_0 is the standard deviation of the data in respect to their average and S_M is the standard deviation of the data in respect to the M-degree polynomial fit. The last one is that, which produces the smallest standard deviation. (The polynomial with degree $M+1$ produces larger standard deviation because the data changes are not normally distributed and uncorrelated, the number of data is not large and the calculation errors accumulate). In our cases M lies between 6 and 19. We apply the criterion on 10 light curves of close binary stars with mass transfer and find that in respect to the photometry error one of them shows negligible variability. We give also a graphical presentation of the criterion for fast establishment of suspected variability, or for fast planning of future monitoring.

Key words: stars: variability – techniques: statistic

Introduction

Usually the stellar variability on time scales from minutes to hours is ascertained by light curves (LCs), obtained by monitoring. The single exposure times and the reading times between two exposures last from seconds to decades of seconds. In fact, the LC is a time series which contains form decades to hundreds data which approximately equal time separation. Sometimes small lapses in the time series exist.

The stellar flickering is a special kind of irregular variability, characterized by fast changes of the brightness, with amplitude up to a few tenths of magnitude (some tens of percent of the luminosity). The flickering gives information about the mass transfer in close binary stars - symbiotic and cataclysmic variables (cf. Di Clemente A., et al., 1996, Sokoloski et al. 2001). Applications of the fractal analysis on LCs of flickering is presented by Bachev et al. (2011) and Georgiev et al (2012). However, when the variability is faint in respect to the standard error of the measurement, the reality of the variability is not certain.

In this paper we present a method for characterization of the low amplitude stellar variability, based on the Fisher's criterion. We show the application of this method on ten stellar LCs, taken by monitoring of light flickering of close binary stars. We give also a graph for fast establishment of suspected variability, or for fast planning of future monitoring.

1 The problem and the observational material

The problem which provoked this investigation is illustrated in Fig.1. The LCs of 2 stars with obvious flickering, extracted from the data base of Zamanov (2012), are shown in the left panel of Fig.1. The LCs of 3 stars with preliminary suspected variability, investigated later by Stoyanov (2012), are shown in the right panel of Fig.1. The large scale trends of the LCs, fitted by 1st and 9th degree polynomials, are shown everywhere. In Fig.1 each 9th degree polynomial elucidates some system of large scale details in the LC. In the right LCs these details seem to be 2-3 times lower than in the left LCs, but the right magnitude scale is 5 times shorter. Therefore, the large scale details in the right, with amplitudes about 0.01 mag, are at least 10 times lower than the details in the left. The reasons of the large scale faint details in the right LCs may be (i) faint stellar variability, (ii) variations of the atmosphere transparency or (iii) both.

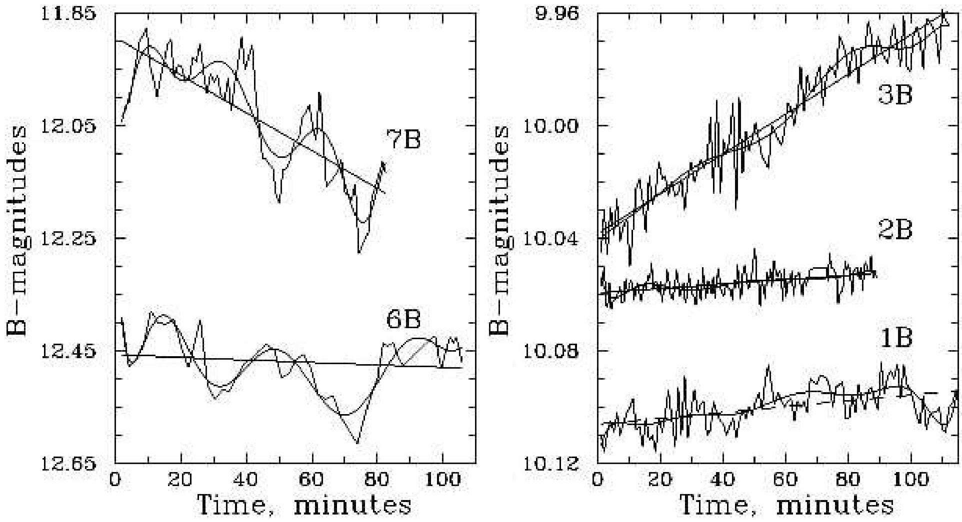

Fig. 1. Stellar LCs (jagged curves) and their fits by 1st degree polynomials (straight lines) and 9th degree polynomials (smooth curves). Left panel: Two LCs with undoubted flickering (Zamanov 2011). Right panel: Three LCs with preliminary suspected flickering (Stoyanov, 2012). Note that the magnitude axis in the right graph has five times smaller scale in respect to this on the left. The stars are presented in Table 1.

All LCs in Fig.1 show well pronounced error noise. The photometric error here is suspected to be about 0.02 mag in the left panel and about 0.005 mag in the right panel (2 % and 0.5 %, respectively). Consequently, the important problem is whether the observed variability in the right LCs may be considered as real. The result of Stoyanov (2012), based on the Fishher's criterion, is that faint, but significant flickering is present in the top and bottom LCs and negligible flickering is observed in the middle one.

Further we present a general solution of this problem and apply it on ten LCs and two numerical generations of noise. The stars are observed by the telescopes of the Rozhen NAO and the LCs are extracted from the data base of Zamanov (2012). The data about the stars and their LCs are presented in Table 1. Figure 2 shows the distributions of the main observing parameters: data number N (20-120), total monitoring time T_{TOT} (20-120 min), single exposure times T_{EXP} (10-300 s) and standard deviations of the data in respect to their averge values S_0 (0.004-0.12 mag). It is seen that the used LCs inhere very different observing parameters, without mutual correlations between them (Fig.2).

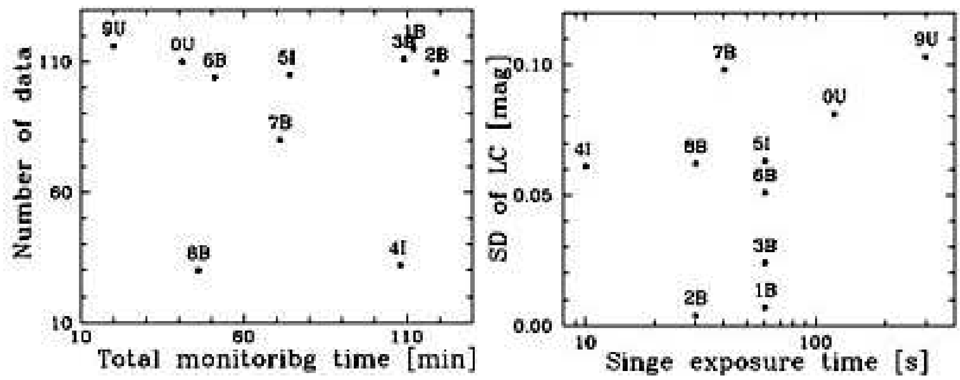

Fig. 2. Left panel: Distribution of the observing parameters of the stars

2 The method and the results

We adapt the statistical criterion of Fisher. It is strictly correct when the data changes in the time series are normally distributed and uncorrelated. We suppose that these requirements are implemented enough well.

Each observed time series may be characterized simply by average value and standard deviation S (in respect to the average value). In rare cases, when the standard error of the measurements (which are supposed normally distributed and uncorrelated) has accurately known value σ. Then the significance of the changes in the time series may be established by the ratio S/σ (with $S > \sigma$). The context of this method is comparing between the observed time series with standard deviation S and a time series of normal noise with standard error σ. The ratio S/σ is a random value with Pearson's distribution (chi-square distribution). When the ratio S/σ is larger than the theoretic value, the variability is considered as significant.

However, in practice σ is not well known and only an estimation s of σ is available. Then the significance of the changes in the time series may be established by the ratio S/s (with $S > s$). This ratio is a random value with Fisher's distribution and when it is larger than the respective theoretic value, the variability is consider as significant. (Cramer 1946, Tucker 1962, Neter et al. 1992). The theoretic values of the distributions of Pearson and Fisher depend on the number of data and on degree of freedom. They may be found everywhere in the literature.

We consider that the standard error σ is unknown and use its estimation S_M. That is why we fit the LC consequently by polynomials with degrees 0, 1, 2, ..., M and calculate the respective standard deviations S_0, S_1, S_2, ..., S_M. We stop the process when the condition $S_{M+1} > S_M$ is reached and use the value of S_M as estimation of σ. We note, that the polynomial with degree

$M + 1$ produces larger standard deviation because the data changes are not normally distributed and uncorrelated, the number of data is not large and the calculation errors accumulate. In our cases M lies between 6 and 19. In the end we detect variability by the modified Fisher's criterion $S_0/S_M > F_N$. Here F_N is the theoretical value of the Fisher's distribution for N data, with account of the degree of freedom. We apply this criterion for significances of 95 % and 99 %.

The results for ten LCs, described in Table 1, are shown in Fig.3. Nine LCs take places above the curves, corresponding to the criterion 95 % and 99 % significance. Seven of them have well pronounced variability, one LCs shows low, but statistically significant variability (star B1) and one LC does not show statistically significant variability (star B2). Thus, under the given observing conditions, the last star can not be considered as variable, that confirms the conclusion of Stoyanov (2012).

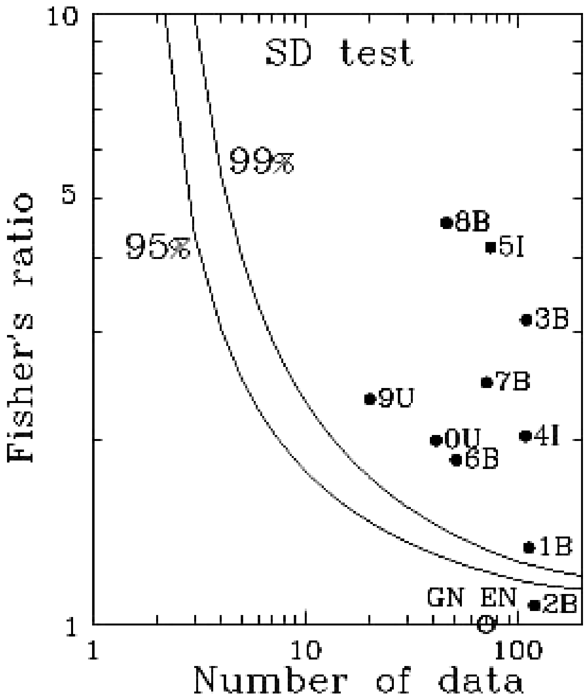

Fig. 3. Distributions of the stars on the diagram of the Fisher's criterion. Simulated "light curves" with 77 points as normal noise and equivalent noise are shown by the circle in the bottom coordinate of the diagram, noted by GN and EN (see also Table 1)

For comparison, simulated "light curves" with 77 points, produced by generators of normal noise and equivalent noise, are processed by the described method. As it is expected, they do not show variability above the noise and are situated on the bottom coordinate axis of Fig.3.

3 Detection diagram of the Fisher's method with use of relative data

The graph in Fig.4 presents the Fisher's criterion in relative units, here in percentages. It may be directly used in astronomy, where the stellar magnitudes are relative values and hundredths parts of the magnitude corresponds approximately to the percentages of the luminosity.

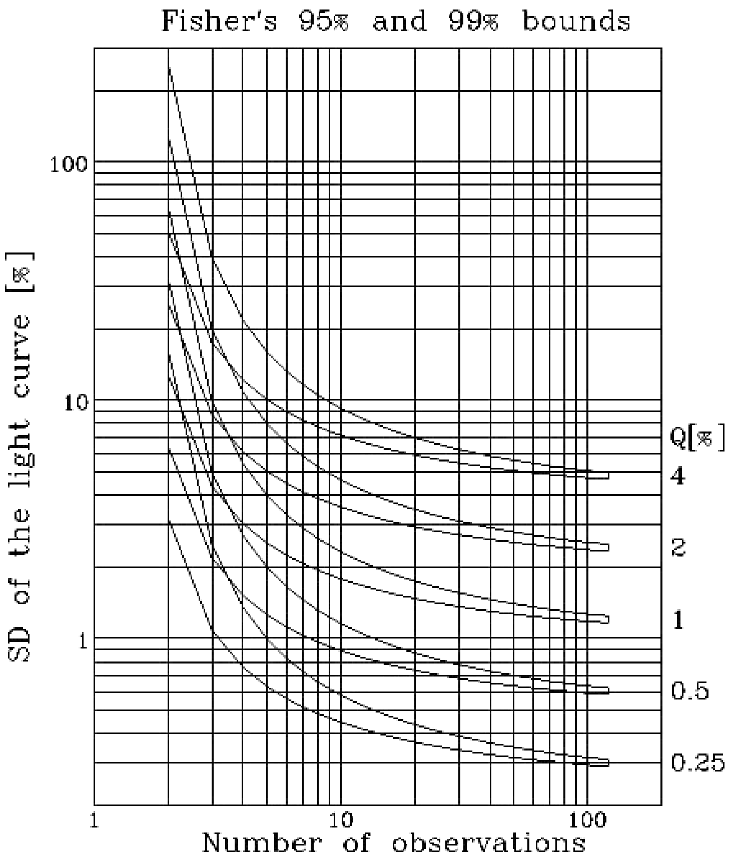

Fig. 4. Diagram of the Fisher's criterion for detection of faint variability in time series

Five pairs of curves, each for 95 % and 99 % significance, corresponding to five values of the relative standard error Q, expressed in percentages, are shown in Fig.4. For example, let us imagine a monitoring of stellar variability, carried out with photometry standard error Q (f.e. 0.01 mag or 1 %), number of observations N (f.e. 20), and standard deviation in respect to the average value SD (f.e. 0.011 mag or 1.1 %). According to the diagram in Fig.4, the numerical values, given here, correspond to an object whose variability has more than 99 % significance.

Generally, the diagram, shown in Fig.4, solves two practical problems. The straight problem is the determination of the detectable variability SD (%) under known Q (%) and given N. The reverse problem is the determination of the necessary N under known Q (%) and desired SD (%).

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Table 1. Basic data about the observations: 1 – Number; 2 – Star; 3 – Date; 4 – Telescope (Schm – Schmidt in Rozhen, Cass – Cassegrain in Rozhen, 2mRC – 2m in Rozhen; 5 – Photometry band; 6- Number of exposues; 7 – Single exposure time, in seconds; 8 - Total observing time, in minutes; 8 – Source (S – Stoyanov, Z – Zamanov)

# 1	Star 2	Date 3	Telescope 4	Band 5	N_E 6	T_{EXP} 7	T_{TOT}	Source
1B	ZZ CMi	2011 Jan 25	Cass	B	113	60	115	S
2B	NQ Gem	2011 Jan 25	Cass	B	119	30	105	S
3B	BF Cyg	2011 Mar 27	Cass	B	109	60	111	S
4I	V425 Cas	2006 Aug 25	2mRC	I	108	10	32	Z
5I	RS Oph	2009 July 21	Schm	I	74	60	105	Z
6B	RS Oph	2009 July 06	Schm	B	50	60	105	Z
7B	RS Oph	2009 July 21	Schm	B	70	40	80	Z
8B	RS Oph	2009 July 07	Schm	B	46	30	30	Z
9U	RS Oph	2008 July 06	2mRC	U	20	300	115	Z
0U	RS Oph	2009 July 07	Schm	U	41	120	110	Z