On the nature of the short-term variability of the cataclysmic binary star KR Aurigae

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Abstract. We study the character of the optical variability of the dwarf nova KR Aur on intra-night time scales. Based on high-accuracy light curves in different colors we performed a number of tests, aiming to reveal the nature of the variability and the processes that drive it. The typical auto-correlation times of the light curves are 30–100 min. The slopes of the structure functions give a hint that the so called self-organized criticality within the accretion disk may drive the continuum variability. We find no clear indications for inter-band time delays, time asymmetry, and the presence of low-dimensional chaos in the light curves. The obtained results from the tests are promising, but so far mostly inconclusive.
Key words: stars: binaries: close – binaries: cataclysmic – stars: individual (KR Aur)

Върху природата на бързата променливост на катаклизмичната двойна звезда KR Aurigae

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Изследван е характерът на оптичната променливост на новата-джудже KR Aur в рамките на по една нощ. Приложени са няколко теста върху високо-точни криви на блясъка, целящи да изявят природата на променливостта и процесите, които я управляват. Типичните авто-корелационни времена на кривите на блясъка са 30–100 мин. Наклоните на стуктурните функции намекват, че може би променливостта на континуума се предизвикна от т.н. само-организирана критичност на диска. Не са намерени ясни индикатори за изоставане на промените в блясъка в един цвят спрямо друг, на асиметрия на промените в блясъка и на присъствие на хаос с ниска размерност в кривите на блясъка. Получените от тестовете резултати са обещаващи, но са все още незаключителни.

1 Observations and reduction

We used light curve (LC) data of KR Aur, obtained during several nights, quasi-simultaneously in different colors and/or telescopes. The telescopes in use were the 2m RCC telescope, the 50/70cm Schmidt camera and 60cm Cassegrain reflector of the Rozhen NAO, Bulgaria, as well as the 60cm Cassegrain reflector of Belogradchik observatory, Bulgaria.

All telescopes were equipped with CCD cameras and standard UBVRI filter sets. Repeating exposures of 30–120 s were taken with one or more instruments in one or more filters to achieve a multicolor quasi-simultaneous coverage of the object's variations. After flat field and dark current corrections, the aperture magnitudes of the variable objects were extracted and calibrated through standard stars in the filed, and the LCs of KR Aur were built. The time durations of the LCs were 2–9 hours.

Only high-quality data was used for this analysis, i.e. when KR Aur was in a high state and the respective photometric errors were below 0.01–0.02 mag. Provided the typical intra-night variations reached 0.5–1 mag, the precision was high enough to successfully perform the variability tests, mentioned below. A LC example is shown in Fig.1.

The short-term variability (flickering) of KR Aur is investigated also by Georgiev et al. (2012) with help of the fractal dimensions based on the local amplitude and local RMS of the light fluctuations. The main result is that the flickering seems to be at least bi-fractal.

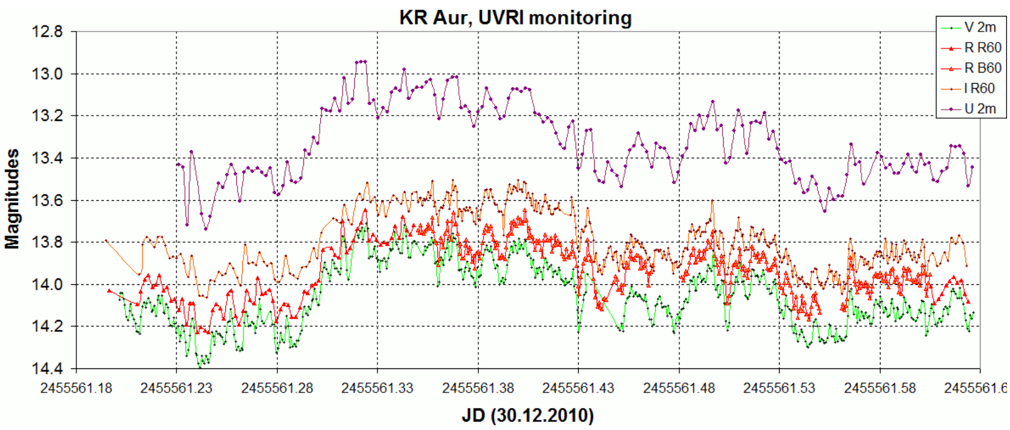

Fig. 1. Rapid variations of KR Aur during the night of 30.12.2010, observed in 4 colors, quasi-simultaneously with 3 telescopes.

2 Tests on the light curves

Although repeating exposures were taken to build the light curves, for a number of reasons the time interval between these exposures was not precisely a constant. Furthermore, the exposure times often differed with the telescopes/wavebands, making the spacing between the points of the light curves not equal and not constant. Some of the tests we performed require equal spacing and simultaneity. For that purpose simultaneous, equal-spaced light curves were built from the real data by linear interpolation over the missing parts.

2.1 Auto-correlation function (ACF) – searching for correlation times

Building the auto-correlation function is useful for finding self-similarities in the light curve and to find the correlation time, i.e. the time after which the light curve will have ”no memory” about the current conditions. Oscillating pattern of the ACF is often indicative of (quasi)periodicity.

Figure 2 shows 4 ACFs built from observations with the 2m RCC telescope. The observations from 30.12.2010 produce 2 light curves due to gaps in some of the data sets. ACF shows sharp decline and crosses zero after 30–100 min, determining various correlation times for different LCs.

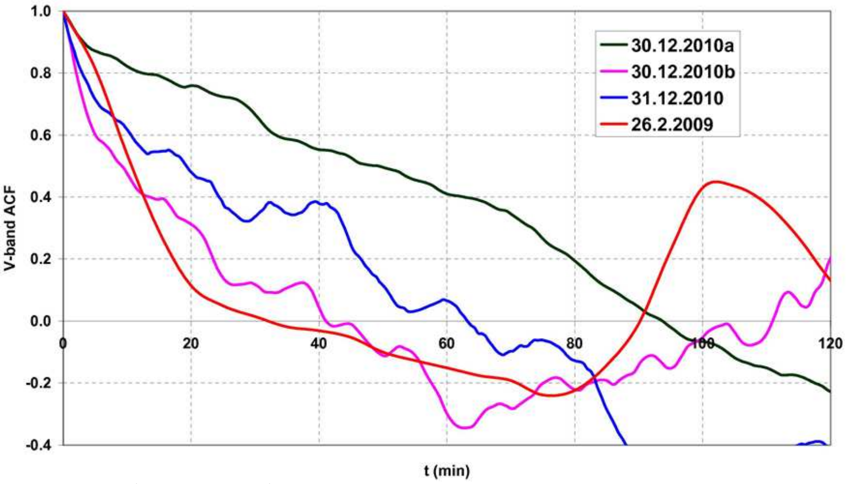

Fig. 2. The ACF of KR Aur from 4 observations with their evening dates shown.

2.2 Inter-band cross-correlation function (CCF) – searching for time delays

Finding possible time lags between LC of various wavebands could give a strong clue about the nature of the processes driving the variability. For instance, if red color variations lead the blue ones, it might be an indication for evolving disturbance, traveling down the accretion disk, taking into account that the disk temperature drops with the radial distance. The opposite behavior may indicate explosive processes with slow temperature evolution (decrease), reprocessing of hard radiation from the center, etc. In any case finding any "wavelength – time lag relation" would serve as a boost for the theoretical modeling.

To find inter-band time lags, a similar to ACF approach was used, this time correlating 2 LCs of different colors and obtaining a time dependant CCF. Unlike the ACF, always showing a maximum (equal to one) at zero lag, the CCF will have a maximum at non-zero lag, provided the two LCs show repeating patterns with a time offset, one in respect to the other. If the two LC's are completely independent, the CCF will always be close to zero and if they are exactly the same, the CCF will be identical to ACF.

Figure 3 shows the CCF between V-band and I-band, during the night of 30.12.2010. We used the interpolated CCF method of Gaskell & Sparke (1986) to build the CCF. The maximum is at about -1 unit, which in this case corresponds to -30 sec, indicating a possible lag of V-band behind I-band, i.e. longer wavelengths leading. Taking into account that the typical exposures were 30 sec (V-band) and 60 sec (I-band), as well as the CCF uncertainties (not shown here), the lag should be considered as close to zero. Furthermore, on several other occasions with other wavebands used, the CCFs were found to peak at zero. In other words we find no conclusive evidence of inter-band time delays exceeding ≈ 30 sec.

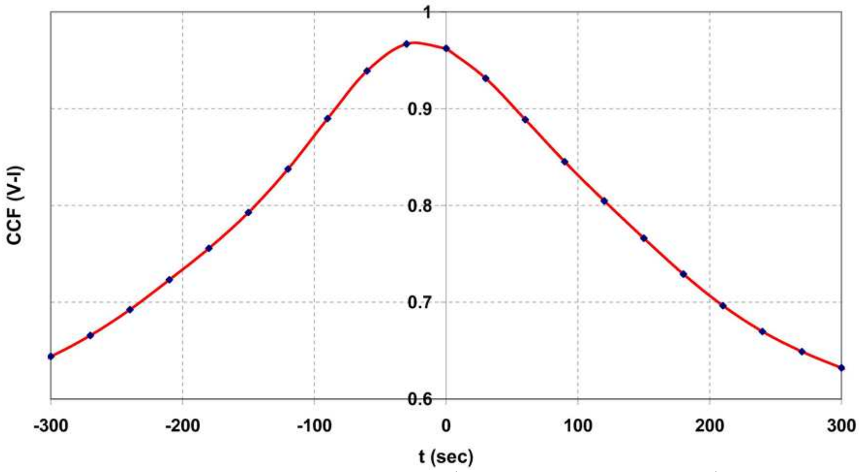

Fig. 3. Time-dependent CCF between I-band (60 cm Rozhen telescope) and V-band (2 m Rozhen telescope) during the night of 30.12.2010

2.3 Structure function (SF)

The first order structure function (e.g. Di Clemente et al., 1996) is often used to analyze unevenly spaced time series. It is defined as:

$$SF(\tau) = \sqrt{\frac{\pi}{2}}(< |m(t+\tau) - m(t)| >^2 - \sigma^2),$$

where $m(t)$ is the LC, σ is the standard error of the photometry (here we adopt σ=0.01 mag) and the averaging is over all magnitude differences, time separated by τ within some $\Delta\tau$.

The SF will generally be a rising function with 2 plateaus, one at τ approaching 0, where the photometric errors will dominate the variability and the other at some τ_{max}, where a saturation of the variability amplitude will occur.

Of importance is the slope of the linearly rising part of the SF,

$$\beta = d \log SF(\tau)/d \log \tau.$$

This slope is of significance for understanding the processes, driving the variability (Kawaguchi & Mineshige, 1999). A larger slope, e.g. $\beta > 0.5$, might be an indication of a shot-noise driven variability, i.e. the variations are due to many independent, explosive events ("shots"). The opposite case, $\beta < 0.5$, gives preference to accretion disk instability models (Kawaguchi et al., 1998), e.g. those based on so called *self-organized criticality* (Bak et al., 1988).

Figure 4 shows the V-band SFs for 4 epochs of observation. One notices the absence of a plateau for $\tau \to 0$, indicating variability structure exceeding photometric errors on time scales less than 30 sec. This means that for the high states of KR Aur, exposures of 10-15 sec are preferable (at least with a 2m class telescope) in spite of the slightly larger photometric errors. On the other hand there are hints of a saturation around 5-10 ksec, however, as this is the typical monitoring interval, one should be careful with the conclusions concerning τ_{max}.

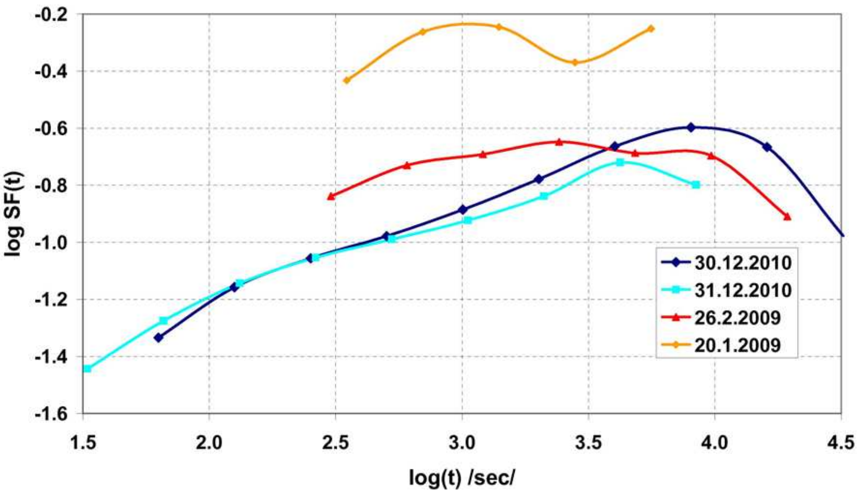

Fig. 4. V-band SFs for KR Aur for 4 epochs of observations. 2m RCC telescope data.

The slopes of the SFs for all nights of monitoring are consistent and are below 0.35, being consistent with the disk instabilities model, but seems to rule out the shot noise model.

2.4 Time asymmetry of the light curves

Additional test to discriminate between different models is based on the time asymmetry of the light curve.

Explosive processes will naturally produce sharp inclines with slow declines in the light curve. On the other hand, slowly evolving disturbances (e.g. in an accretion disk) will produce slow inclines with a sharp decline when the disturbance reaches the inner radius of the accretion disk, where it will eventually abruptly dissipate. On the other hand, stochastic processes like random walk, or dynamical systems with a *Hamiltonian*, not directly depending on time, would mostly produce time symmetric light curves.

The time symmetry of a light curve, even the non-uniformly sampled one, can successfully be tested by building the SF separately – for the rising and decaying trends of the light curve, $SF^{+}(\tau)$ and $SF^{-}(\tau)$, respectively (see

Kawaguchi et al., 1998, for details). The method works in a case where the overall brightness is due mostly to a superposition of different, light generating events (but not to shadowing, for instance). The difference gives the time asymmetry of the light curve as a function of time.

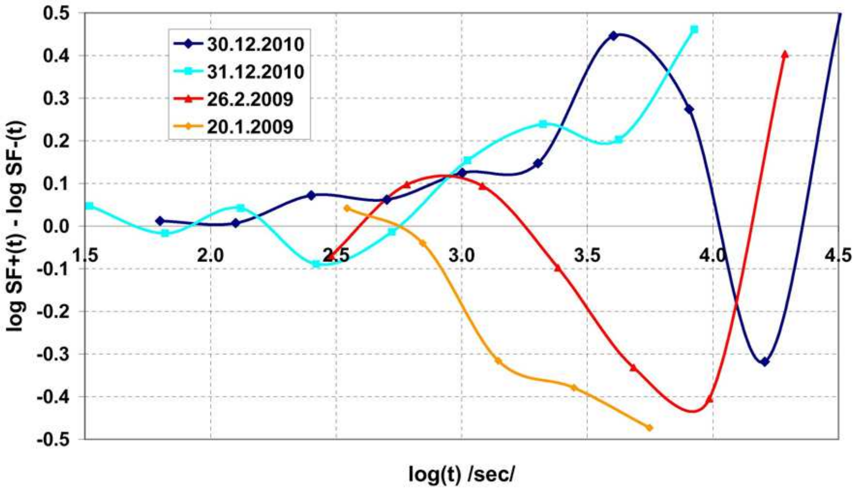

Fig. 5. V-band time asymmetry search of KR Aur light curve.

Figure 5 shows the time-asymmetry test applied to the light curves used in Fig. 4. A positive $SF^{+}(\tau) - SF^{-}(\tau)$ difference in this case means predominantly slow inclines and fast declines in the light curve, i.e. as in the disk-instability models. The results from different nights, however, are not consistent. For intervals shorter than about 1 ksec, there is no apparent time asymmetry, meaning perhaps that some time symmetric random processes govern the LC at these scales.

2.5 Search for chaos

Time series of many astrophysical objects show variability patterns that can often be described as "random", since the power spectra do not reveal any significant periodicity. Except for the "true" random behavior, e.g. the *Brownian* motion, where the dynamical system has an immensely large number of degrees of freedom (in order of the number of particles in the liquid), there are cases of dissipative systems, where the dynamics is governed by only a few $(3 - 5)$ degrees of freedom (variables), but still the behavior appears to be "random" or "chaotic".

In the famous *Lorenz attractor* case (Lorenz, 1963), a system of N=3 nonlinear differential equations has a solution, that once entering some volume of the phase space, stays bound and never leaves this sub-space of (non-integer) dimension $d < N$.

These solutions are so called "strange attractors" and such a behavior is often described as "deterministic chaos". Trajectories of a strange attractor evolve into a finite volume in the phase space, never returning to the same points. The divergence between two close trajectories increase exponentially in time and the long-term predictions are impossible. Consequently, any variable of a strange attractor system can appear as "random" as a function of time, even if it is deterministic in nature.

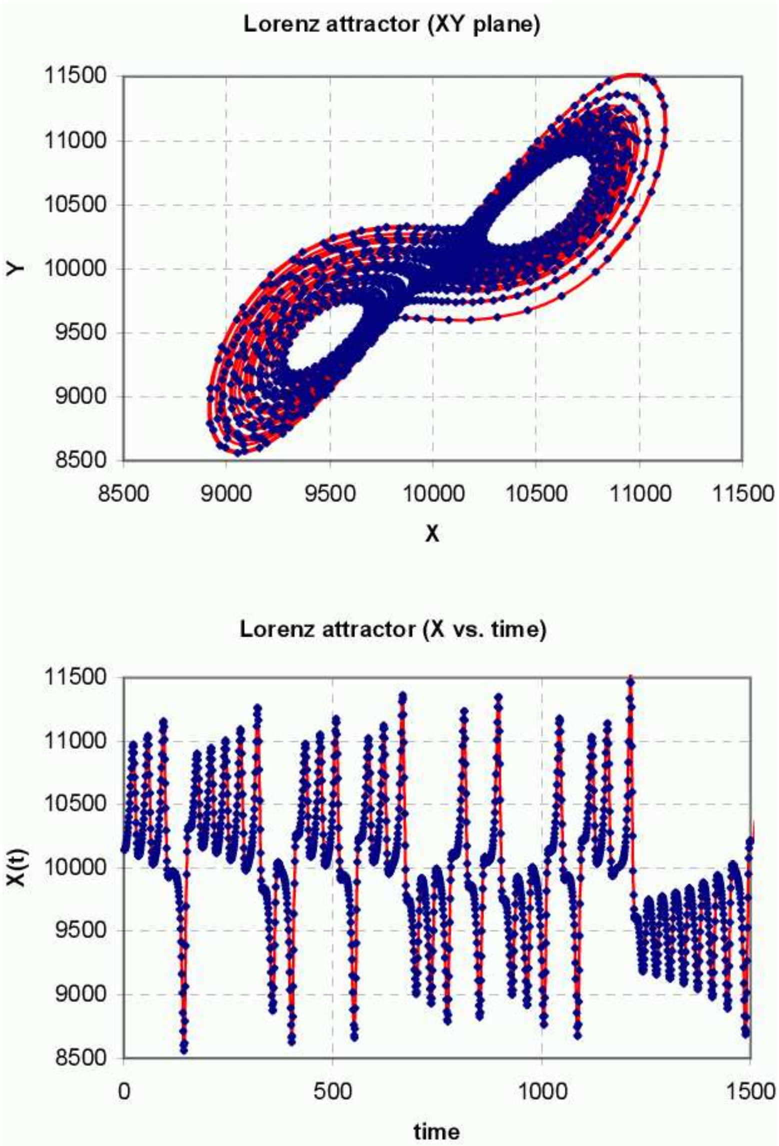

Fig. 6. Lorenz attractor.Upper panel - X-Y plane, lower panel - X(t).

To check if the short-term variations of KR Aur could possibly be governed by a low-dimensional dynamical system, we applied the *correlation integral* (CI) method (see Vio et al., 1992; Lehto et al., 1993; Provenzale et al., 1994; for application to other accreting sources) to the V-band light curve of the object. This method is based on the construction of a new (empirical) phase space from the available N discrete data points. The data set (e.g. the light curve) is separated into strings of length d observational points. Thus, each string can be considered as a d-dimensional vector (X_i), embedded in the d-dimensional empirical phase space. The number of vector pairs with a distance smaller than r, as a function of r, is computed for different d and related to the total number of pairs (n_p) for that d.

Thus, the correlation integral can be written as:

$$C_d(r) = \frac{1}{n_p} \sum \Theta(r - |X_i - X_j|);\ i, j = 1, ..., N; i \neq j$$

where Θ is the Heaviside's function. So, if the dimension of the attractor is D, then:

$$C_d(r) \propto r^d, \text{ if } d < D \text{ and } C_d(r) \propto r^D, \text{ if } d > D.$$

Therefore, increasing the embedding dimension d leads to saturation when $d > D$, thus allowing estimation of the attractor dimension. Generally

$$D \rightarrow (d \ln C(r) \ / \ d \ln r), \text{ when } r \rightarrow 0$$

where D is the correlation dimension of the attractor and can be a non-integer value.

Knowledge of D allows determination of the number of differential equations, describing the dynamical system, N, i.e. the first integer value, larger than D and therefore makes possible drawing conclusions about the physical process driving the variability.

As an example, Figure 6 shows the 3D Lorenz attractor. The upper panel shows the solution of the Lorenz system in X-Y plane, and the lower one – only one axis (say X) as a function of time, which might represent the only variable we observe in a real situation (e.g. the continuum flux).

We analyzed 3 segments of the V-band light curve of KR Aur taken during 2 nights with the 2m RCC telescope in order to search for signatures of low-dimensional chaos.

Figure 7 shows the results from the application of CI method to different LCs – the lower panels, from left to right, are Lorenz attractor, white noise and a combination of attractor with white and shot noise (a number of shots with sharp rises and slower decays, randomly distributed). The upper panel is the KR Aur data for the 3 segments (30.12.2010a, b and 31.12.2010 respectively). Each panel shows the correlation integral vs. its slope for different embedding dimensions (from 1 to 15, in different colors). Saturation for some slope value is considered an indication for the presence of low-dimensional chaos. Such is visible (and expected) for the Lorenz attractor toward $D \approx 2$ (2.06 is the real Lorenz attractor dimension), correctly identifying the number of the variables (equations) that govern this dynamical system – 3 (the first integer larger than 2.06).

Naturally, the white noise shows no saturation (infinite degrees of freedom) and for the combination of both the situation becomes more complicated. Clearly however, the KR Aur results resemble mostly this case, but in general shows no clear indications of low-dimensional chaos. Of course, one is to take into account the so called Ruelle criterion, stating that the maximal embedding dimension that can be determined from a series of N points is $D_{min} \leq 2\log N$, which in our case means about 5. Since the accretion process is typically governed by at least 5 equations, one clearly needs more data points in the light curve to search successfully for a low-dimensional chaos.

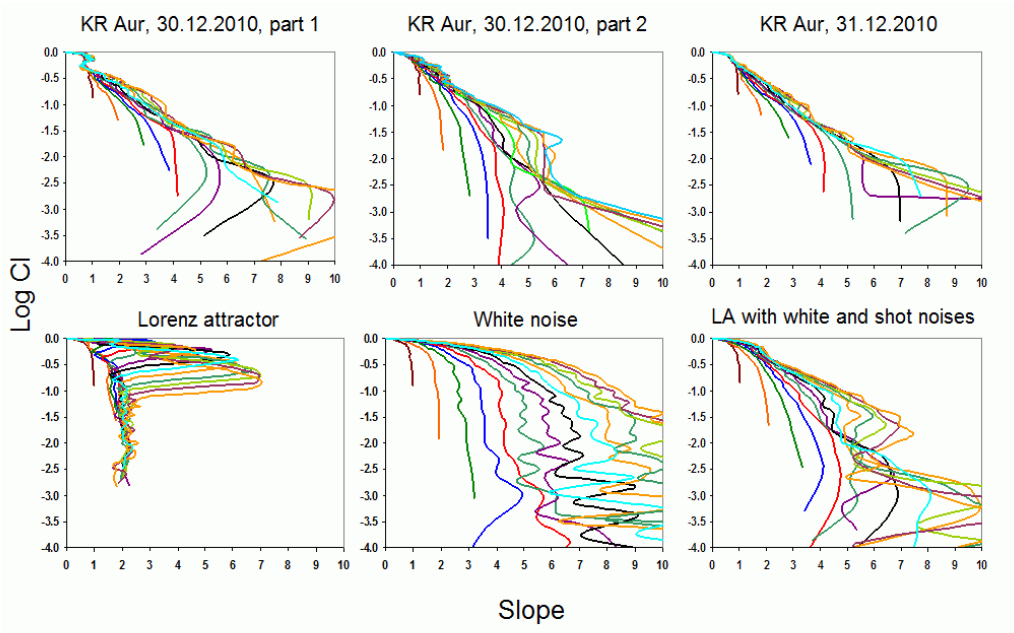

Fig. 7. Application of the CI method. Upper panel — KR Aur data. Lower panel — artificial data, from left to right: Lorenz attractor, white noise and a combination of white and shot noises. Different embedding dimensions (from 1 to 15) are given in different colors.

3 Summary

We performed various tests on the intra-night light curves of KR Aur. Some of the tests, like the tilt of the structure function, the inter-band time delays (if real) suggest that the variability on time scales of 10–100 min might be driven by evolving disturbances, traveling from the periphery to the center of an accretion disk (disk instability model). These tests, however, are not enough to be considered as a proof and different interpretations are also possible. Furthermore, the time asymmetry test gives inconsistent results. Shorter variations (on scales less that 5-10 minutes), appear to be predominantly stochastic in nature. We were not able to establish the shortest time scale of KR Aur variability but the object certainly appears to show variations much exceeding the photometric errors (e.g. 0.01 mag) even on a 30 sec time scale. We have not been able also to confirm that a dynamical system, governed by less than 4-5 parameters (equations) drives the variability.

Acknowledgements

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