Peculiarity of the motion of the asteroid 108 Hecuba

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Особености при движението на астероид 108 Хекуба

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The determination of the Hecuba's orbital elements is a special case of the three-body problem. The mean motion of Hecuba is approximately two times bigger then the Jupiter's one. To achieve more precision ends in solving the restricted three-body problem Sun, Jupiter and asteroid 108 Hecuba we improve the Kiril Popoff doctor's dissertation "Sur la mouvement de 108 Hecube" with supervisor Henry Poincare [Popoff, 1912]. The results obtaining by Kiril Popoff were very sufficient with existing observation data. Our main goal is to popularize and to develop his dynamical theory in order to its implementation in determination the orbital elements of the new discovered objects that are in resonance 2:1.

The peculiarities of the variations of the Hecuba orbital elements are observed after 1980. In order to explain this fact we include the terms up to the fourth order about the Hecuba's eccentricity in the perturbation function [Borisov, Shkodrov, 2006]. The application for the system for the manipulation of symbolic and numerical expressions "Maxima"[http://maxima.sourceforge.net/] to develop the perturbation function up to the arbitrary order is done [Borisov, Shkodrov, 2007 a].The presence of observational data enables us to take the date 18.08.2005 for the epoch [Batrakov Y. at al., 2005]. This is important, because then the angle that is the difference between the mean anomaly of Hecuba and the double difference between the mean longitude of Jupiter and the mean longitude of Hecuba is approximately equal to 180°. At this way the series in the solution become alternative and they converge faster. The zero point time responds to minimum value of the Hecuba eccentricity for a period more than 30000 years.

The observation data are taken from "Ephemerides of minor planets". We show the expansions of some functions of the radius vector and the eccentric anomaly, which are used in the development of the perturbation function in series of the trigonometric functions of mean anomaly. We present the derivations of perturbation function in a different way using the new formula for partial derivatives of homogeneous function [Borisov, Shkodrov, 2005]. This result enables to calculate more terms in Taylor series expansion of perturbation function. For this purpose we define the S-numbers and show its obtaining and properties. We introduce binomial transforms of n-tuples in the perturbation theory [Borisov, Shkodrov, 2006]. At this way the perturbation function is developed in series by the generalized binomial transform.

To resolve the problems of summation of the series in the results we prove the new method for the summation of the alternating divergent series with general term the k-binomial transform of n-tuple [Borisov, Shkodrov, 2007 b]. We defined another generalization the p-binomial transform. Using it we find a relation between Cauchy numbers and binomial coefficients.

The generalization of Euler transform is derived [Borisov, Shkodrov, 2007 b]. We introduce new canonical variables in the solution of the restricted three body problem using the exact formulation of the third Kepler's low. Differential equations are solved approximately using the Maclaurin series expansion up to the second order about the Jupiter's mass expressed in solar masses. In the integration we include shortperiodical terms that consist up to eight times the difference between mean longitudes of Hecuba and Jupiter. The Sun gravitational attraction is counted more precisely and it is expanded up to the fourth order too. All terms that are not trigonometric are included in the zero approximation and all longperiodical terms in the second approximation in the equations for all variables. The constants of integration are derived by nine iterations. The variations of orbital elements are presented graphically for a period about 300 years and they are compared with these obtained by observation data [Borisov, Shkodrov, 2008]. We compute periods of these variations. Because of the fact that some of them are long we show another graphs for a period about 3000 and 30000 years [Borisov, Shkodrov, 2009].

Kiril Popoff dynamical theory predicts Hecuba's orbital elements satisfactorily for a period about 20-30 years [Popoff, 1912]. We include all terms up to the fourth order of Hecuba's eccentricity in the perturbation function in our computations. So the accuracy of determination of Hecuba's orbital elements especially of inclination and longitude of ascending node increases and our model determines them with better accuracy for a period about 150 years. In addition our computations predict that the Hecuba's eccentricity will raise this year and the Hecuba's inclination after 2032 [Borisov, Shkodrov, 2008]. Future observations may confirm this.

Key words: asteroid, Hecuba, Jupiter, restricted three-body problem, perturbation function, binomial transform, divergent series

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