Unsuccessful attempt for dating the Ulugbeige's catalogue by the method of Dambis and Efremov

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Abstract. In the paper "Dating Ptolemy's star catalogue on the basis of proper motions: a thousand-year problem is solved" [1] A.K. Dambis' and Yu.N. Efremov applied so called "bulk method" in an attempt for dating the "Almagest" star catalogue. They obtained T=89±112 B.C. and concluded that this catalogue was created in Hipparchan epoch (about 130 B.C.), rejecting Ptolemy's (about 130 A.D.) authorship. In order to check the reliability of "the bulk method", we applied this method for dating the Ulugbeig's catalogue, selecting from it different samples of fast stars. The main variant of the check points out a time interval of the catalogue's completion between 1149÷1275 A.D. Correspondence to the traditionally accepted epoch of this catalogue, about 1437 A.D., is only achieved upon exclusion the seven fastest stars. We explain the obtained significant discrepancy as a result of numerous assumptions in the method of Dambis and Efremov which lead to underestimating of the errors and to displacement the epoch of dating.
Key words: history of astronomy; problems of dating of the Almagest star catalogue

Неуспешен опит за датировка на каталога на Улугбек по метода на Дамбис и Ефремов

Михаел Г. Никифоров

В статията "Датировка на звездния каталог на Птоломей по собствените движения на звздите: хилядолетният проблем е решен" [1] А.К.Дамбис и Ю.Н.Ефремов приложиха така наречения "колективен метод", опитвайки се да датират каталога "Алмагест". Те получиха T=89±112 пр.н.е. и заключиха, че този каталог е създаден в епохата на Хипарх (около 130 г пр.н.е.), отхвърляйки авторството на Птоломей (около 130 г сл.н.е.). За да проверим надеждността на "колективния метод" ние го приложихме за датировка на каталога на Улукбек, селектирайки от него различни набори бързи звезди. Според главния вариант на проверката каталогът е съставен в периода 1149÷1275 сл.н.е. Съответствие с традиционно приетата епоха на каталога, около 1437 г сл.н.е., се получава само след отстраняването на седемте най-бързи звезди. Ние обясняваме полученото значително несъответствие с множеството допускания в метода на Дамбис и Ефремов, водещи до подценяване на грешките и отместване на центъра на датировката.

Introduction

In A.K. Dambis' and Yu.N. Efremov's paper "Dating Ptolemy's star catalogue based on proper motions: a thousand-year problem has been solved" [1] an attempt of dating the "Almagest" star catalogue with the help of a "bulk method" worked out by authors was made. As result of calculations the authors obtained the date of drawing up the "Almagest" catalogue $T = -89$ with an error of 122 years. This result allowed A.K. Dambis and Yu.N. Efremov to conclude that "Almagest" star catalogue was made in Hipparchan epoch and to reject Ptolemy's authorship at a 94% confidence level. In this thesis was the "solution of a thousand-year problem" declaring in the heading of article.

The statement about possibility of determination of the authorship of the catalogue turned out to be so interesting that A.K. Dambis' and Yu.N. Efremov's work has been rechecked by D. Duke [2]. Dennis Duke rechecked the "Almagest" star catalogue by a "bulk method" and came to the conclusion that because of error of the method makes it impossible to attribute the authorship of the catalogue to Hipparchus or Ptolemy with certainly. Therefore, A.K. Dambis' and Yu.N. Efremov's statement about the Hipparchus authorship is groundless.

However, the question on the results of the dating the "Almagest" catalogue by "bulk method" executed by A.K. Dambis and Yu.N. Efremov can't be considered completely by two reasons. First, the "bulk method" of dating star catalogues by the authors is new and hasn't been approved on other star catalogue which authorship and dates are beyond of doubt. Secondly, constructing a regress the authors believe for some reason that errors in vicinities of all stars are identical that is obviously erroneous. In this connection there is a doubt that the "bulk method" suggested by A.K. Dambis and Yu.N. Efremov can basically provide an adequate result of dating.

To solve these fundamental questions we have applied "a bulk method" on Ulugbeige's star catalogue [3], dated 841 the year of Khidzhra, that corresponds to 1437 A.D. Ulugbeige's star catalogue was chosen among other medieval catalogues because it has the identical structure of fast stars and similar value of the random measurement error to catalogue "Almagest".

Technique of dating by a bulk method

Let's describe in brief a technique suggested by A.K. Dambis and Yu.N. Efremov in their work [1]. Assume the star catalogue was made in a certain year T. Using a modern highly precise star catalogue, formulas of precession and considering proper motions of the stars we shall calculate true ecliptic coordinates λ and β of all stars of investigated catalogue for a certain year T.

We shall determine discrepancies of coordinates $\Delta\lambda$ and $\Delta\beta$ for each star of catalogue as a difference of value of quantity taken from the dated catalogue and calculate quantity for date T:

$$\Delta\lambda = \lambda_{cat}\cos\beta_{cat} - \lambda_{calc};\ \Delta\beta = \beta_{cat} - \beta_{calc}$$

Around each star with a large proper motion (designed with the symbol *) we shall choose a group of several nearest stars of comparison N_{nei} and determine a systematic error in this vicinity in longitude and latitude from the values of discrepancies of the stars of comparison. The authors determine the systematic error by a median that allows eliminate emissions.

$$\Delta\lambda^*\cos\beta^* - \langle\Delta\lambda^{nei}\cos\beta^{nei}\rangle = \frac{1}{60}\left(\mu_\lambda^* - \langle\mu_\lambda^{nei}\rangle\right)T_{cat} + \Delta\lambda_a'^*\cos\beta \quad (1)$$

$$\Delta\beta^* - \langle\Delta\beta^{nei}\rangle = \frac{1}{60}\left(\mu_\beta - \left\langle\mu_\beta^{nei}\right\rangle\right)T_{cat} + \Delta\beta_a'^* \quad (2)$$

where μ_λ and μ_β are projections of the proper motion in longitude and latitude in seconds of an arch per year; the factor 1/60 transforms the proper motion from seconds per year to minutes per year; T_{cat} is the age of catalogue counted from the assumed date of drawing up T; $\Delta\lambda_a'^*$ and $\Delta\beta_a'^*$ are random measurement errors which are unknown. The general idea of method is shown by authors in fig. 1.

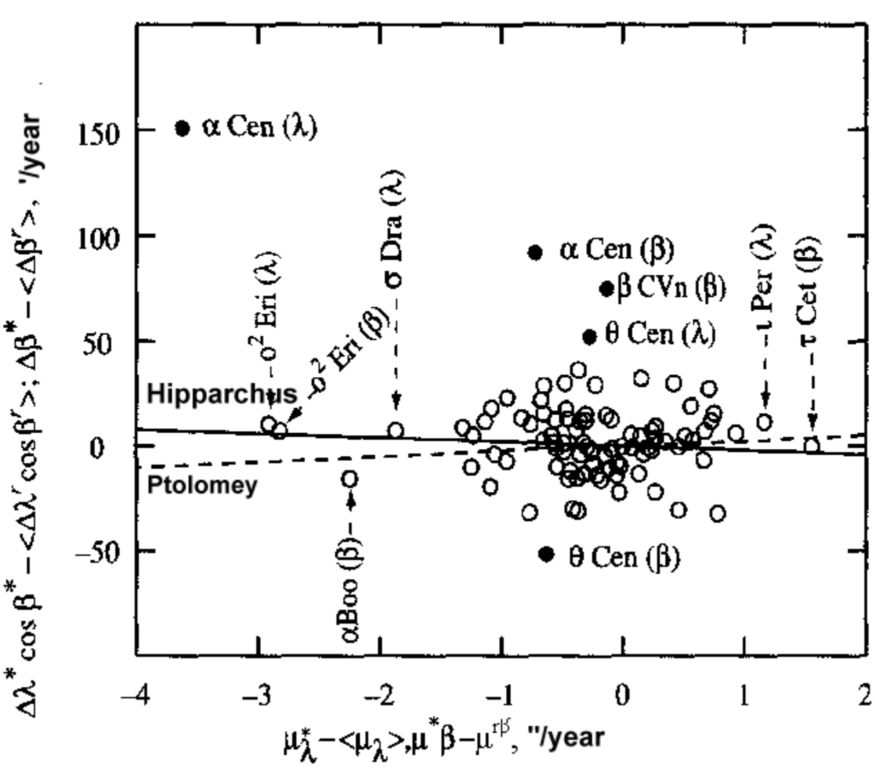

Fig. 1. *The difference of coordinates adduced in "Almagest" and computed for epoch of 1 B.C. minus such an average difference for the six nearest slow stars, depending on corresponding component of proper motion for the 50 fastest stars of the "Almagest".*

On the X-axis is takes the component of proper motion of a fast star $\mu_{\lambda;\beta}$ with deduced of an average velocity of vicinity $\left\langle \mu_{\lambda;\beta}^{nei} \right\rangle$. On the axis of ordinates is takes the difference of discrepancies of a fast star and an average discrepancy of vicinity $\Delta\lambda$ and $\Delta\beta$ for the given component of velocity. The points with the value $\Delta\lambda$ or $\Delta\beta$ exceeding the trebled root-mean-square error are eliminated from future investigation. Then using the method of least squares authors draw a regression line of the type across the rest set of points and assign the tangent of inclination k. Hence, the date of drawing up of catalogue will be assigned as and the error of dating will be determined through an error of tangent of an angle as $T_{cat} = T + 60 \cdot k$. Solving separately the system of equations for longitudes (1) and latitudes (2) the authors obtain two dates T_λ and T_β with the errors σT_λ, σT_β and also average random errors $\sigma\lambda_a^{\prime nei}$ and $\sigma\beta_a^{\prime nei}$. Solving in common the system of the equations (1) and (2) with weights inversely proportional and the authors obtain more accurate dating $T_{\lambda\beta}$.

The technique of dating catalogue by Dambis–Efremov's method the choice of a number of the stars of comparison of neighbourhood is rather essential. The authors solve this problem in the following way:

"We have applied the described in the previous chapter method to the whole "Almagest" catalogue to be more precise to it's 1020 stars having eliminated four repeated records and four non-star objects. Futher we have obtained the datings of the catalogue using all possible combinations of the parameters $N_{nei} = 2 \ldots 21$ and $N_{fast} = 11 \ldots 100$ (i.e. using from 11 up to 100 fasted stars from 2 up to 21 nearest basic stars for each fast star). Thus the basic stars were selected by their proximity to a fast star according to the "Almagest" coordinates so that this selection wouldn't depend on initial epoch (1 B.C. in our case). For the beginning it is necessary to select the optimum number of basic stars N_{nei} for each fast star. It is apparently such a number of basic stars which allow to predict systematic error of a coordinate difference with the highest accuracy. The efficiency of such a prediction is measured by a root-mean-square error of the corresponding coordinate, determined the solution of the systems of equations (9) and (10) [in our numbering (1) and (2)] by the method of the least squares: ".

In fig. 2 and fig. 3 the algorithm of the selection of optimum number stars of vicinity (neighbourhood) N_{nei} and quantities of fast stars N_{fast} is shown.

Thus, determining the dating of the catalogue the authors used 40 fast stars and the neighbourhood of comparison concerning which the estimation of the position of a fast star was made consistent of 6 stars. As a result of the following datings were obtained $T_\lambda = -110\ \pm 230$, $T_\beta = -80\ \pm 150$, $T_{\lambda\beta} = -90\ \pm 120$ and root-mean-square errors of ecliptic coordinates $\sigma\lambda = 18'$ and $\sigma\beta = 13'$.

Remarks to Dambis-Efremov technique

Despite of seeming reliability of the "bulk method" the authors make a number of essential simplifications and unstipulated assumptions which make the work methodologically unsolved. We shall formulate some basic remarks.

<u>Remark 1</u>. A serious drawback of the technique is elimination of information on the random measurement errors in vicinities of a fast star of calculation of the datings and . Really in the equations for longitude (1) and latitude (2) the last terms making sense of the random measurement are simply equated with zero. Thus if the height of a point on an X-axis is completely equated with the first right number of the equation which describes a proper motion of a star. Defining an inclination of regression the authors use the information only on the difference of a proper motion of a projection of velocity of a fast star and neighbourhood (axis X) and the difference of velocity of a fast star and an average velocity of neighbourhood (axis Y). These

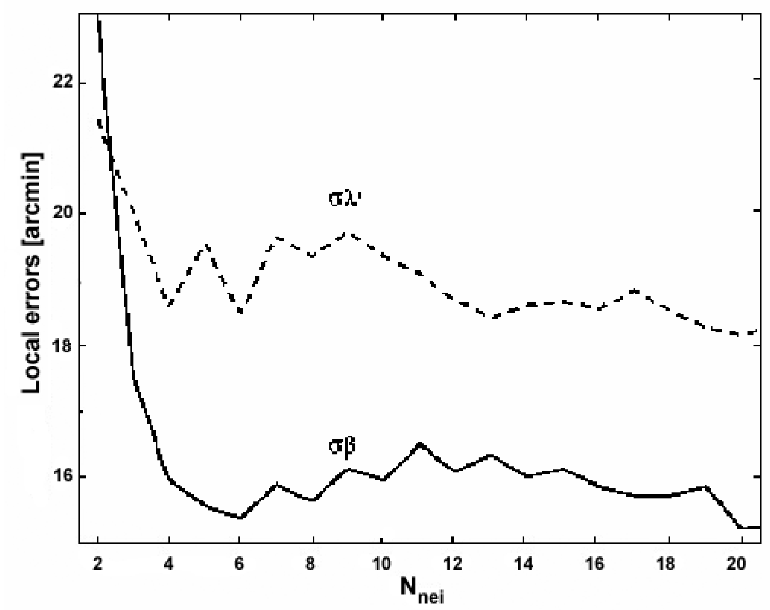

Fig. 2. *Local root-mean-square random errors and of ecliptic coordinates $N_{fast} = 100$ fastest stars of "Almagest" depending on the number of the used neighbor basic stars N_{nei}. It is visible that error of both coordinates cease decreasing at $N_{nei} > 6$.*

quantities don't contain the random measurement errors. Here A.K. Dambis and Yu.N. Efremov make two assumptions at once. They believe that the measurement error in all vicinities is the same and equal zero. However, the value of quantity of the random measurement error considerably differs in different neighbourhood that is very easily determined by the errors of neighbor stars (we'll return to this question below). Therefore the quantities T_λ and T_β themselves are determined incorrectly and with the underestimated values of an error. The authors as if remember about the members of equations $\Delta\lambda_a'^* \cos\beta$ and $\Delta\beta_a'^*$ only after the determination of independent dating on projections T_λ, T_β and use them when calculating a mixed dating $T_{\lambda\beta}$.

The efficiency of performance of amendments suggested by the authors can be checked up very easily. Having led the regression at once all points (in longitude and latitude) we have calculating the dating $T_{\lambda\beta}$ without determination of intermediate dating on projections T_λ, T_β with-out using any weights, fig. 4 Besides we didn't subtract the average projection nof the velocity of neighbourhood from the projection of the own velocity of a star, since for the overwhelming majority of fast stars such an amendment is insignificant.As a result of calculation the factor of regression $k = -1.41 \pm 2.07$ has been obtained in recalculation it gives the calendar date $T = -85 \pm 124$ years, that almost doesn't differ from A.K. Dambis and Yu.N. Efremov's result $T = -89 \pm 122$ years, fig. 4.

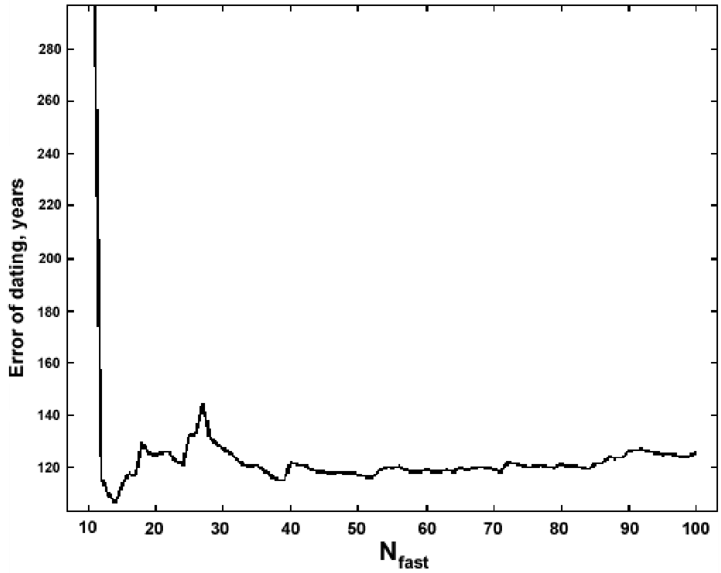

Fig. 3. *A root-mean-square error of the joint dating in latitude and longitudes on the number N_{fast} of the used fastest stars of "Almagest" for $N_{nei} = 6$. It is visible that error of the dating remain practically constant after the number of fast stars exceed 40.*

It means that the authors' method of calculating the mixed dating $T_{\lambda\beta}$ suggests considering an individual error in coordinates of a star, but gives the same result which is obtained when a mixed dating in latitudes and longitudes is calculated directly without any amendments. Hence the given technique is ineffective.

Remark 2. The technique of selecting the number of neighbour stars suggested by the authors is inefficient and is unable to adjust the optimum number of stars of comparison. In fact, in fig.2 at $N_{ref} = 6$ the minimum in a local error of a longitude and latitude is observed, however, both of these minimum are statistically indistinguishable, fig. 5.

From the figure it is visible that the value of the local error in the longitude $\sigma\lambda$ $N_{nei} = 6$ makes about $18,5'$. But at the same time, the local error does not essentially change at N_{ref} from 4 up to 21 stars, when $\sigma\lambda$ is about $19'$ and deviates from this quantity less than $1'$ at different values N_{ref}. Since the value $\sigma\lambda$ itself is determined with accuracy not more than $5 \pm 10\%$ from the value of quantity local minimum at $N_{nei} = 6$ can't be defined. Besides it is possible to allocate two more local minima at $N_{nei} = 4$ and 13 where the last minimum is the deepest minimum out of these three ones. On the other hand, with a loss of accuracy in $2'$ it is possible to use even three stars of comparison.

The same situation is observed with the dependence of a local error in latitude $\sigma\beta$ on N_{ref}. At the same values N_{ref} the quantity of a local error $\sigma\lambda$

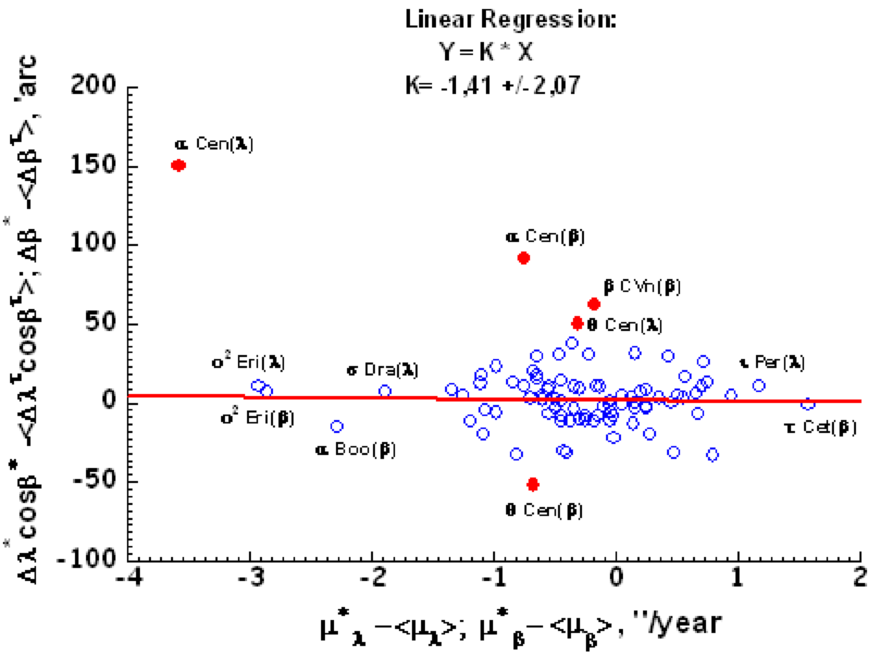

Fig. 4. *Calculation of dating mixed $T_{\lambda\beta}$ without estimation of particular dating T_λ and T_β.*

will make about 16′ and change in the whole interval less than 1′. However, the value $\sigma\beta$ is also determined with the error about 1′, therefore it is impossible to calculate authentically an optimum number N_{ref} on the basis of presented dependence.

Thus, on the basis of presented technique of selection N_{ref} it is impossible to determine the optimum number of stars of comparison statistically. The possible reason of it is the following: the neighbour stars of comparison are very different and it should not be searched a universal value N_{ref} which applied absolutely to all vicinities of fast stars, but it would be better to suggest a transparent working algorithm which would define N_{ref} for each vicinity automatically.

Remark 3. Let's return to a question on errors in vicinities of stars. For this purpose we shall consider how an error in a neighbourhood of a fast star changes depending on number N_{ref} fig. 6.

For example, discrepancies defined on a median are steady in longitude for Keid (o^2 Eri), Arcturus (α Boo), τ Cet and in latitude for Sirius (α CMa). Therefore, for such vicinities it is quite possible to use a common value N_{ref} from 3 up to 10 stars. And vice versa latitude discrepancies of slow stars of vicinities of Keid, τ Cet and longitudinal discrepancy of neighbourhood of Sirius are unstable and considerably change depending on N_{ref}. For example,

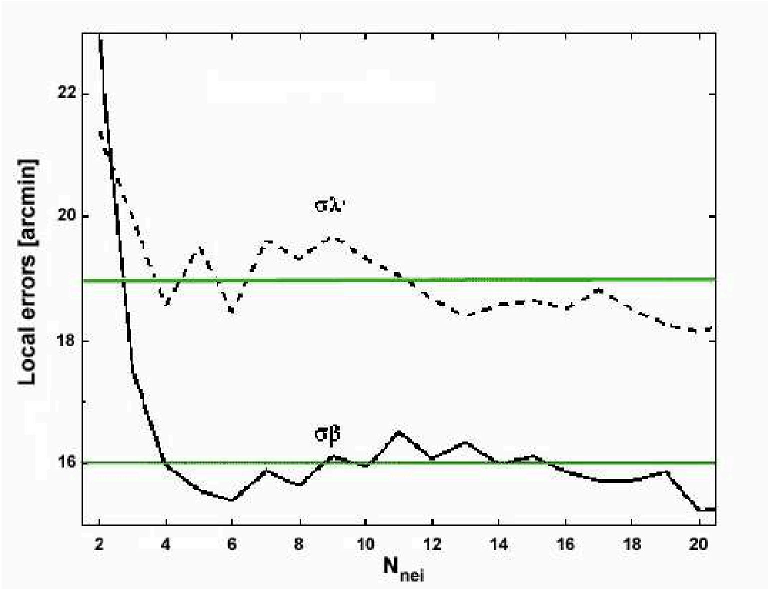

Fig. 5. *Local root-mean-square errors $\sigma\lambda$ and $\sigma\beta$ ecliptic coordinates $N_{fast} = 100$ fastest star of the "Almagest" depending on the number of the used neighbour basic stars N_{nei}.*

the error of Keid neighbourhood at $N_{ref} = 4$ and $N_{ref} = 8$ changes for $16'$, the error of the neighbourhood of τ Cet when transiting from $N_{ref} = 6$ stars to $N_{ref} = 10$ stars will change for $16'$, and the error of neighbourhood of Sirius changes for $12'$ when transiting from $N_{ref} = 4$ to $N_{ref} = 6$ stars. Note, that instability in the considered vicinities occurs at different values N_{ref}, therefore it is impossible to assign the common number of stars of comparison for such vicinities correctly.

Let's return to the question of selection of the optimum number of the fast stars N_{fast} and stars of comparison N_{ref} fig. 2, fig. 3. On the grounds of the last figure the authors make a conclusion, that the error of dating ceases changing essentially at $N_{fast} = 40$, therefore they use this number in the further calculations. Now, after analysis of discrepancies of stars of comparison Keid, Arcturus, τ Cet and Sirius it becomes absolutely clear why the functions of a local error from N_{ref} practically do not depend on the number of stars of comparison on significant interval N_{ref}. A part of vicinities is steady from N_{ref} (slightly changes when changing the number of stars of comparison) therefore it is possible to take it them various N_{ref}. These vicinities are mixed with unstable vicinities in which a local error can change by dozens of minutes with a little change N_{ref}.

But if there are a lot in unstable vicinities, their contributions to the general local error is averaged, therefore the functions of local errors in fig.2 have no strongly pronounced minima. If the authors had calculated the functions of local errors from N_{ref} for a greater number of the fast stars, for example,

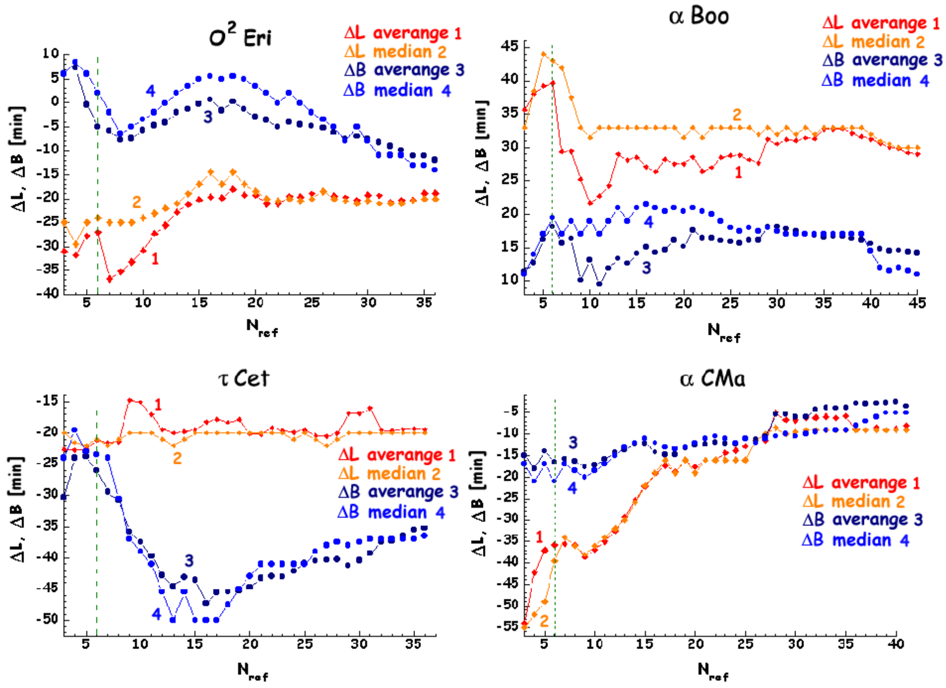

Fig. 6. *The functions of discrepancies of coordinates in the neighborhood o^2 Eri, α Boo, τ Cet and α CMa from the presented figures it is visible, that the average discrepancies of stars of comparison behave differently in the neighborhood $N_{ref} = 6$.*

$N_{fast} = 60$ or 80, the functions of local errors would depend on N_{ref} even less. And vice versa with a smaller number of fast N_{fast} starts the dependence of local errors on N_{ref} would be expressed move strongly though the value N_{ref} can appear different and not the only one.

In conclusion, we shall state to fig. 3 one more small, but essential remark. With the change of value N_{fast} from 10 up to 11 in occurs a sharp (in several times) reduction of the error of dating. This result is incomprehensible as at the same time the values of functions of local errors change poorly and the contribution of one fast star (in this case η Cas) is insignificant.

Remark 4. A particular remark. The application of common value N_{ref} in rarefied vicinities or small constellations in some cases obviously deduces out of the limits of the area of identical regular errors.

For example, the star if the δ Tri has high velocity in longitude $(+0,93''$ per year) in latitude (-0,63" per year) and is presented at once by two significant points. However, the constellation Triangulum in the "Almagest" catalogue contains only four stars, one of which is a fast star, therefore with $N_{ref} = 6$ it is necessary to take the rest three stars from neighbour constellations which have quite different systematization. Let's calculate the discrepancies of coordinates of slow stars of the vicinity of the δ Tri for the year 1 B.C., Table 1.

No.	Baily	Bayer/Flamsteed	$\Delta\lambda \cdot \cos\beta$	$\Delta\beta$
1	359	β Tri	-36'	-13'
2	361	γ Tri	-14'	-12'
3	358	α Tri	-58'	+16'
4	219	16 Per	+9'	+9'
5	349	γ And	+26'	-21'
6	377	39 Ari	+2'	-18'
7	378	35 Ari	+19'	-2'

In this case three stars from three different constellations with different group errors are taken to the vicinity of the stars of comparison. In the errors of stars B219, B349 and B377 in latitude approximately correspond to the errors of stars of contour of constellation of the Triangulum then longitudinal error of the added stars have quite different systematization that inexitably affects dating. In fact, if we define a vicinity by the three nearest stars the error of the vicinity in longitude will make up –3.6' on a medial and –33' on an average. In the vicinity made up of 6 nearest stars longitudinal errors will be equal to –6' on a median and –12'on an average, that is they will change concerning the previous result by $20' \div 30'$. It will change the dating by one and a half thousand years.

Let's make one more experiment. We shall replace the last star of vicinity B377 (=39 Ari) with the following next star B378 (=35 Ari) which is removed from the Triangulum approximately by 0,5 $^\circ$ further then B377.

The star 35 of Aries has discrepancies in a longitude and latitude correspondingly (+19', -2') that corresponds to the errors of stars of the Triangulum even worse and will lead to an even greater error. Thus when a fast star is in a rarefied vicinity and the vicinity is completed with the nearest stars of neighbour constellations, the error of a vicinity can change in the most unpredictable way.

The nearest vicinity of the star of the ξ UMa (B32 according to Baily) with a latitude velocity –0,71" per year consists only of one star ν Uma (B31). The next star removed from B32 least of all is the star B490 which belongs to the "informate" of the Leo constellation and all the vicinity of comparison with $N_{ref} = 6$ will consist of stars ξ (UMa), B490 (54 Leo), B481 (Leo), B489 (41 LMi), B30 (UMa) and B495 (15 Com) and have the following errors in latitude:

No.	Baily	Bayer/Flamsteed	$\Delta\beta$
1	31	ν UMa	+13'
2	490	54 Leo	+55'
3	481	δ Leo	+37'
4	489	41 LMi	+32'
5	30	ψ UMa	+11'
6	495	15 Com	-15'
7	380	61 Leo	+33'

Table 2.

It follows from the adduced table that the stars with absolutely different errors are selected to the vicinity of comparison of the ξ UMa. Stars B481, B489 and B490 have the regular error which is typical for the constellation of the Leo. In the "Almagest" star B495 also belongs to the Leo, but has its own different from the error of all the rest stars of the vicinity. The sub situation of this star to a bit more removed star B480 (60 Leo) makes the situation even worse in this case the movement of the Ursa Major is estimated by stars B480 and B489 of the constellation of the Leo. It would be reasonable to assume that the coordinates of ξ UMa were measured together with the stars of the Ursa Major, but not the Leo, therefore the systematic error of the vicinity made up by such a method bears no relation to this star.

Remark 5. On the distribution of errors in vicinities. The authors have calculated, that the optimum value N_{ref} from the Almagest' star catalogue is equal to 6, whence on a median group errors of a vicinity in a longitude $< \Delta L_{nei} >$ and in latitude $< \Delta B_{nei} >$ have been calculated. The advantage of using the method of calculating an error on a median lies in the fact, that in contrast to the calculation of an error on average this method allows to estimate emissions effectively.

However, using a median we obtain a correct result only in the case when errors in a vicinity are distributed under Gauss law. The authors do not investigate the question of distribution of error in vicinities.

In the case where the vicinity includes drop-outs (which can be seen as a step of distribution), the N_{ref} value will decrease and the distribution form will diverge from the normal distribution pattern even further. In the case of only a few takes done, the distribution pattern is more accurately described by Student's methods, however in this case a less accurate assessment of accidental measure error is achieved).

Thus, by replacing the real error distribution patterns with normal ones, the authors introduce an error very hard to assess, which tells on the dating centures and the dating inaccuracy alike.

Remark 6. All the previous speculations had a goal of defining the vicinity of a fast star which shares a group error close to that of the fast star. It was assumed that the distribution of individual discrepancies in a vicinity of star is entirely defined by the accidental measurement error. However, this theory proves wrong in a number of cases, since in the longitude vicinities systematical errors are applied to the accidental errors. For example, the vicinity of Sirius is rather compact and the six stars required for the comparison are found rather easily in a 5 $^\circ$ radius around the star. In the current example, even if the comparison stars share diametrically opposite positions, the systematical error will be insignificantly small. On the other hand, the vicinity of τ Cet embraces the six comparison stars in the vicinity radius of 11 $^\circ$, the angular distance between the farthest stars of the vicinity σ Cet and η Cet is about 21 $^\circ$, and the distance between them and the τ Cet exceeds 10 $^\circ$. First, in this case, there are no reasons to assume that all these stars were measured basing on the same anchor star (otherwise the group errors can differ due to the fact that different anchor stars can have different proper coordinate error, which are inherited by all the associated vicinities), and the coordinates of these three stars were measured at once by the same anchor star. Thus, these stars can happen to have totally different errors and comprising them into one vicinity is invalid.

Secondly, the additional error affecting the long vicinity is introduced through the systematical error. For example, the authors [4]÷[6] assess the difference of the plane declination of the ecliptic of the "Almagest" from the calculated one as large as $\gamma = 18'$ to $21'$. This means that the extreme stars of a vicinity with a radius of 10 $^\circ$ will display different systematic to $\gamma \sim 20\,^\circ{=}7'$. In principle, this isn't that big of a value as compared to the accidental measurement error of $\sim 20'$, moreover when the group error of the vicinity os calculated by the latitude. Nevertheless, this error isn't considered and, as seen by the authors, doesn't affect neither the dating itself nor the dating's proper error.

If the fast stars is moving along the longitude and the group error is likely to be deduced from the longitude as well, then ignoring the systematic can lead to invalidating inaccuracies. As a sample we can quote the stars B57 (σ Dra) and B61 (χ Dra), which reside in the higher latitudes. The vicinity of σ Dra at $N_{ref} = 6$ is comprised from B55 (ε Dra), B58 (υ Dra), B59 (τ Dra), B54 (δ Dra), B56 (ρ Dra) and B53 (π Dra), and the vicinity radius is about 5 $^\circ$. However, a number of stars from this vicinity are quite distant from each other and the longitude difference sometimes reaches tens of degrees.

Baily	Bayer/Flamsteed	$\varDelta\beta$
31	ν UMa	+13'
490	54 Leo	+55'
481	δ Leo	+37'
489	41 LMi	+32'
30	ψ UMa	+11'
495	15 Com	-15'
380	61 Leo	+33'

Table 3.

Taking into consideration the fact that the longitudes of the "Almagest" are afflicted by a systematical error similar to the one which is applied to the latitudes in [5], the presence of systematic leads to an error applicable to different stars up to $15' \div 20'$. So the guess that in this case the stars of the vicinity taken have the same group error is apparently wrong. The same situation is applicable to the vicinity of the relatively fast star of χ Dra, which is similarly positioned on a high latitude but moves on longitude. Nevertheless, the authors do not introduce a systematic modifier for none of the vicinities.

Conclusion

The remarks found throughout the article vary in character and touch upon matters ranging from the very general methodical problems considering the finding out of an optimal number of stars for the comparison and the consideration of the systematical errors, to some particular facts which cast doubt over the validity of calculating dates by using isolated stars. Doing so in some cases led to erroneously later dates, in other the effect was opposite, the method for picking the right number of comparison stars failed to work, the systematical errors in the vicinities weren't considered at all, and the only attempt at considering an accidental error in the calculation of the combined dating $T_{\lambda\beta}$ was unsuccessful. The summarized flaws of the method suggested by A.K. Dumbis and Yu.N. Efremov give us grounds to state that the real inaccuracy of the method is much greater than claimed by the authors, virtually being unable to distinguish between the epochs of Ptolemy and Hipparchus. This conclusion is in full accordance with the conclusion of D. Duke's paper [2] who applied the aforesaid method to date the "Almagest" star catalogue.

Our preliminary conclusion is as follows: the bulk method suggested by A.K. Dumbis and Yu.N. Efremov has a dating inaccuracy a few times greater than claimed by the authors, at that the dating centuries is often deduced with serious deviations from the real composition date of the catalog. To prove that hypothesis we are going to apply the said a bulk method to Ulugbeig's star catalog, which displays a similar coordinate accuracy value and a similar list of fast stars.

Finding out errors in Ulugbeig's catalogue

To calculate the earlier positions of the stars we use the contemporary data about the coordinates and projections of the stars [7] and an algorithm used for calculated the star coordinates [8], [9] which considers the precession and the proper motion of the stars.

Let's define the modified ecliptic coordinates of the stars which are found in Ulugbeig's catalog for the year of 1437 and compare it to the star coordinates of the contemporary catalogue, define the longitudinal and latitudinal discrepancies for each star. All stars with absolute discrepancy values above $120'$ are excluded from further consideration. Moreover, the 27 southern stars are automatically left out since Ulugbeig himself asserted that the calculations for these were borrowed by him from As-Sufi's work.

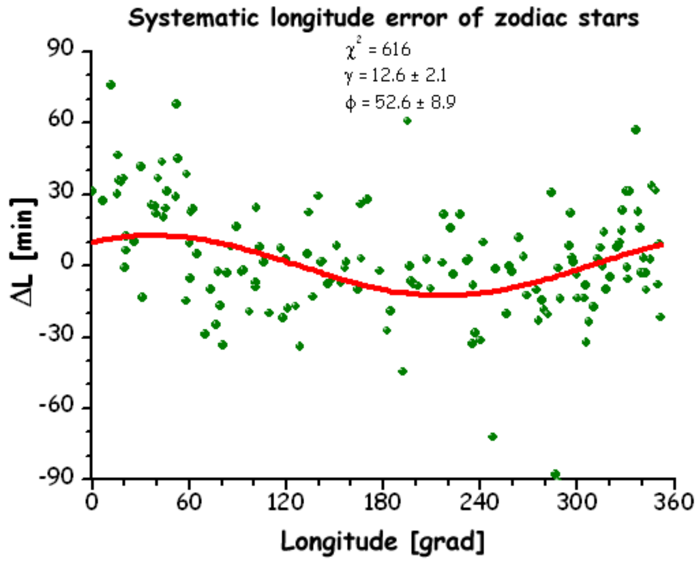

Fig. 7. *Systematical longitudinal error calculated from the multitude of zodiacal stars.*

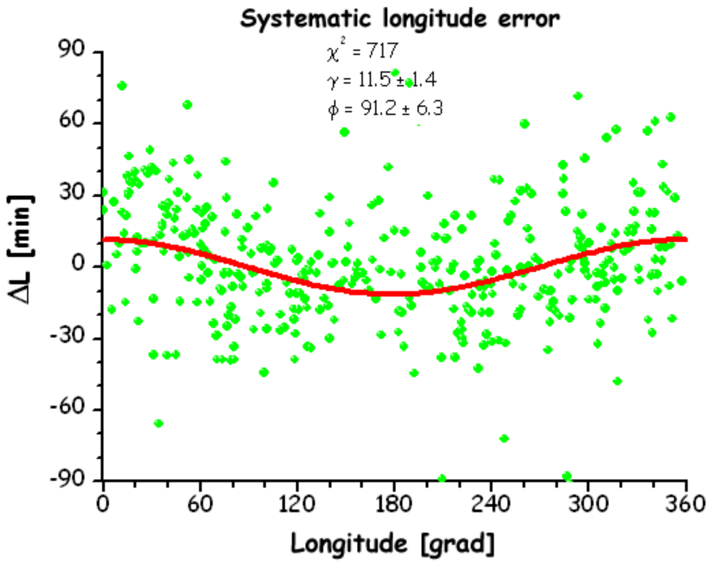

Fig. 8. *Systematical longitudinal error calculated from the multitude of zodiacal stars.*

To define the values of systematical errors let's build up a number of distribution dependencies of the longitudinal and latitudinal discrepancies and approximate the found error distribution value with a function like $\Delta S_{\lambda;\beta} = \gamma_{\lambda;\beta} \sin(\lambda - \varphi_{\lambda;\beta})$ which is estimated to produce the values for $\gamma_{\lambda;\beta}$, $\varphi_{\lambda;\beta}$ and the values for the corresponding inaccuracies. In the figures 7 $\div$10 are shown the approximations done for the longitudinal and latitudinal discrepancies calculated from the multitude of zodiacal stars and all the stars from the catalogue.

To compensate for the systematical error by the longitude, we get a value of $\gamma = 11.5 \pm 1.4$ by the zodiacal stars and $\gamma = 12.6 \pm 2.1$ by the whole multitude of stars. The values of the sinusoidal phases are slightly different from each other and amount to $\varphi = 53^o \quad \pm 9^o$ and $\varphi = 91^o \quad \pm 6^o$, correspondingly. However with the real phase measurement inaccuracy considered as $10^o \div 20^o$, the credible ranges of these values intersect.

By compensating the systematical latitudinal error we get values of $\gamma = -9.8 \pm 1.7$ by the zodiacal stars and $\gamma = -8.8 \pm 1.2$ by the whole multitude of stars. The values of the sinusoidal phases are slightly different from each other and amount to $\varphi = 33^o \quad \pm 9^o$ and $\varphi = 61^o \quad \pm 8^o$. correspondingly.

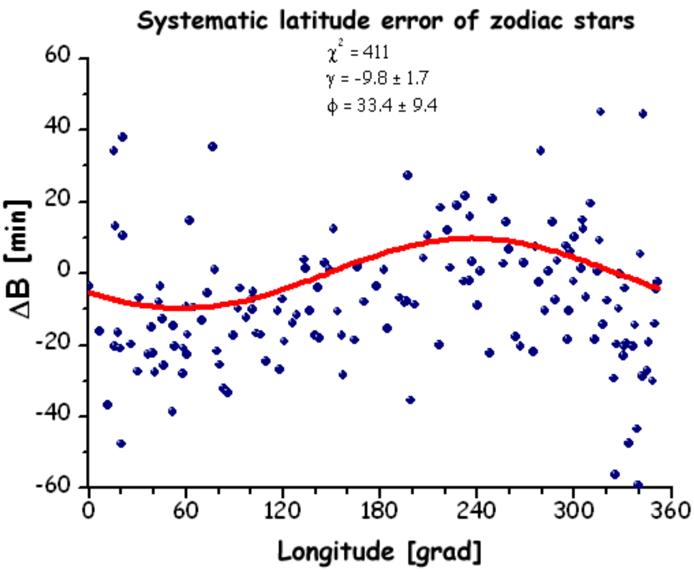

Fig. 9. *Systematical latitudinal error calculated from the multitude of zodiacal stars.*

Let's leave out the systematical longitudinal and latitudinal errors with the help of the compensation sinusoid with the parameters γ and φ which correspond to the whole multitude of stars. After that, we define the residual errors in star coordinates and calculate the root-mean-square error at the first approximation, amounting to $29'$ for the longitude and $27'$ for the lat-

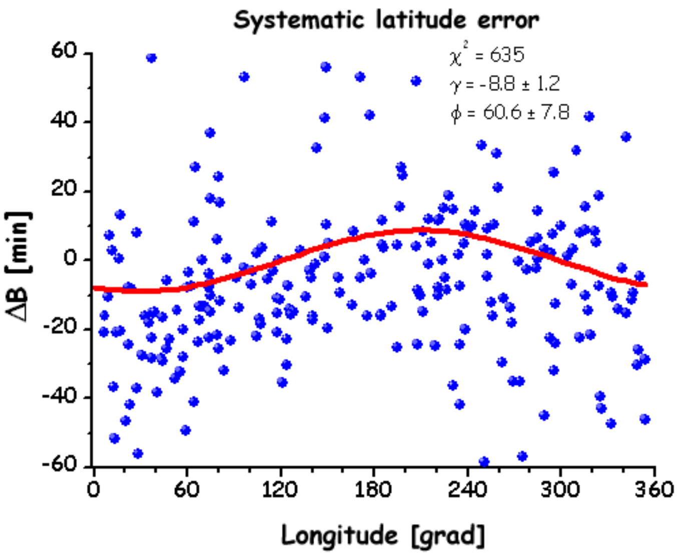

Fig. 10. *Systematical latitudinal error calculated from the multitude of all stars.*

itude. The real accuracy of the coordinate measurements is slightly higher, to calculate it correctly it's necessary to leave out the drop-outs. A drop-out is considered to be a kind of calculation which produces a discrepancy value that doesn't fall into the double range of the found root-mean-square, which amounts to 58′ for the longitude and 54' for the latitude.

Performing the whole procedure all over again, we deduce the values of the root-mean-square errors which amount to 23' for the longitude and 21′ for the latitude. The found values of the accidental measurement errors roughly correspond to the accuracy of the "Almagest". Since the star coordinate measurement accuracy in the catalogues of Ptolemy and Ulugbeig's is the same and so are the lists of fast stars, the dating inaccuracy for the both catalogues should be likewise the same.

Dating Ulugbeig's star catalogue using Dumbis and Efremov's method

For the first approximation we will use group of comparison stars of $N_{ref} = 6$. This choice is motivated by the following factors. As it was shown in the figure, the large value of N_{ref} reveals no clear minimum for the residual discrepancy neither by longitude nor by latitude. Since dating of the both catalogues is done using the same stars (save for a few apparently badly measured), the catalogues display roughly the same measurement errors, so we have reasons to expect that the rest of summary discrepancies of the stars in Ulugbeig's catalogue will be hardly dependent from the N_{ref} value, similar to the "Almagest". That's why we take the number of comparison stars of $N_{ref} = 6$ for the first calculation, the same numbers used by A.K. Dumbis and Yu.N. Efremov. Further it will be possible to quote with a different value of $N_{ref}$and assess the stability of the result.

Variant $N_{ref} = 6$

To produce a correct dating, we'll engage all the stars the projections of which by the latitude and longitude exceed 0.45"/year. If a speed projection along one of the axes is less than this value, then the projection won't be included into the dating calculation. It's noteworthy that in their variant of the calculation, A.K. Dumbis and Y.N. Efremov use both components of the star speed even in the case where the speed by the slow component is close to zero (e.g. the longitudinal speed projection of Arctur). Though the slow speed component leaves practically no impact on the dating centrum, to out opinion including it to the dating is a methodical mistake since in this case the authors would have to include all the stars with the same speed likewise. This could have been done, however in this case the residual discrepancy of the star will be defined not by its proper motion, but rather by the accidental measurement error, which is senseless. Let's exclude the stars with discrepancies out of correspondence with the coordinate discrepancies of the stars from the closest vicinity from the common multitude, figure 11.

The latitudinal discrepancies of the stars o^2 Eri (Keid), δ Eri and τ^6 Eri as well as the long-lasting discrepancies of β and γ Vir, ε Sco don't fall into the error interval of 2σ which is $46'$ for the longitude and $42'$ or the latitude. The quoted coordinates are invalidated by the measurement inaccuracy, so they can't be used for credible dating with N_{ref} from 4 to 10. The calculation reveals that the discrepancies of all the enumerated stars except Keid don't fall into the doubled measurement inaccuracy interval. The latitudinal discrepancy of Keid starts shrinking as the vicinity radius grows, when the radius reaches $R = 10^\circ$ it equals to about $+1.5^\circ$, with $R = 14^\circ$ it decreases to $40'$ and doesn't fall into the doubled measurement inaccuracy interval. However it's clear that the distant stars involved into the comparison bear no relation to the group error of the Keid vicinity, so performing calculations for such vast vicinities lacks reason.

Let's note that the longitudinal discrepancy of Keid falls into the range of an accidental measurement error, thus it will be used in the following calculations. The doubled measurement inaccuracy interval also misses the discrepancy of the longitudinal projection of γ Ser (the orange spot), which is equal to $48'$, so this star is excluded from the main calculation. However, considering the proper values of accidental measurement errors are defined with a certain degree of inaccuracy, we include the longitudinal projection of γ Ser in the complementary calculation merely for the sake of future stability examination.

Let's note that, diverging from A.K. Dumbis and Yu.N. Efremov, we defined the dependency along the X axis only using the proper speed of the μ star and not the difference between the proper speed of the aforementioned star and the average speed of $< \mu_{nei} >$. By engaging the faster stars into the calculation, the speed value difference provided by the vicinity is quite insignificant and affects the calculation in the same way (in the majority

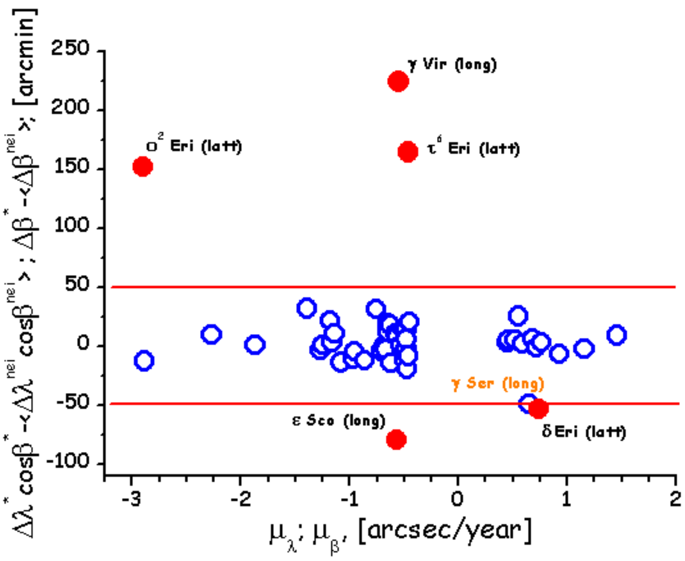

Fig. 11. *Fast star distribution in Ulugbeig's catalog by the "speed – vicinity discrepancy" coordinates.*

of cases, the star speed just degrades). Moreover, this kind of difference is likely to be rather insignificant, if the assumed calculated date is close to the real date of the catalogue's composition. Finally, the question of proper consideration of the vicinity speed is somewhat awkwardly posed: since the vicinity error is calculated by the median, it's defined by the discrepancies of one or two stars which might have totally different speed not corresponding to the speed of the rest of the stars belonging to the vicinity. Thus, calculating an average speed for the vicinity by examining only a part of its stars and calculating by the median may lead to unpredictable results.

After forming a multitude of fast stars which will further be used as a basis for dating and defining the number of stars used for the comparison in the vicinity of the fast star, let's calculate the intermediate datings by the projections of longitude and latitude. The values of the regression coefficients will be equal to $k_\lambda = 1.42 \pm 2.18$ and $k_\beta = -6.72 \pm 3.12$ which can correspond to an approximate date of $\grave{T}$ = *1437 +60k*. Thus we deduce $T_\lambda = 1522 \pm 131$ years and $T_\beta = 1034 \pm 187$ year. Thus the dating centrums diverge by the latitude and longitude by 488 years, and at the same time the inaccuracy values of the two datings don't intersect.

Because the dating result can depend to a significant extent on the few fastest stars, let's quote an additional investigation of the stability of the regression declination coefficient. For that, we'll be consequently excluding the projection of the faster stars' speed from the dating calculation. The results are represented in the fig. 12.

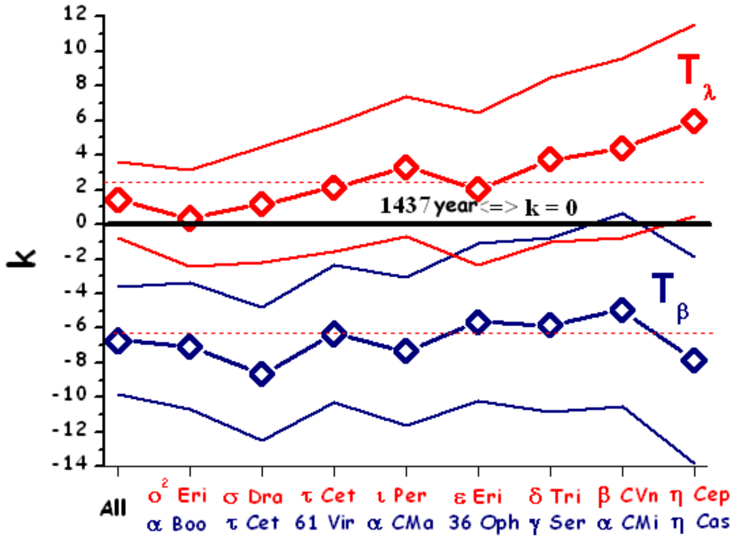

Fig. 12. *The stability of the regression declination coefficients for* T_λ, T_β *with the fast stars excluded.*

In the figure, the solid bold line indicates the catalogue completion date by Ulugbeig in 1437 which corresponds to the regression declination coefficient $k = 0$. The red bold line indicates the defined dependency of the regression declination coefficient k_λ, the thin line indicates the possible dating interval with a credibility level of 2σ. The dark blue color indicates the corresponding clause of the k_β coefficient.

From the represented figure it is seen, the dating centrums are insignificantly changed after the five stars with the fastest longitudinal and latitudinal projections are excluded from the vicinity. The average value of the coefficient for the longitudinal regression is equal to $k_\lambda = 2.74$ for the longitudinal regression and for the latitudinal regression $k_\beta = -6.7$. It is important to note that the credible interval of the regression coefficients (=datings) begin intersecting only when the ten fastest speed projections (including ε Eri and 36 Oph) where the intersection occurs at the width of the credible interval of 2σ. To make the coefficients correspond to each other at the width of the credible interval of σ, it's necessary to remove a few more fast stars from the selection. There's little practical sense to it, since the position of the dating centrums will vary only slightly when the stars are excluded, and the intersection of the credible intervals will occur inlay because of the partial data exclusion, which effectively increases the inaccuracy.

Thus, calculating the datings T_λ and T_β by the speed projections revealed their significant discrepancy, which is either due to the overly low inaccuracy of each of the particular datings or the incorrect definition of the dating centrums, but the most likely variant is the combination of the both factors. Let's calculate the combined dating $T_{\lambda\beta}$ by assuming 1437 AD as the estimated completion date, as seen in figure 13.

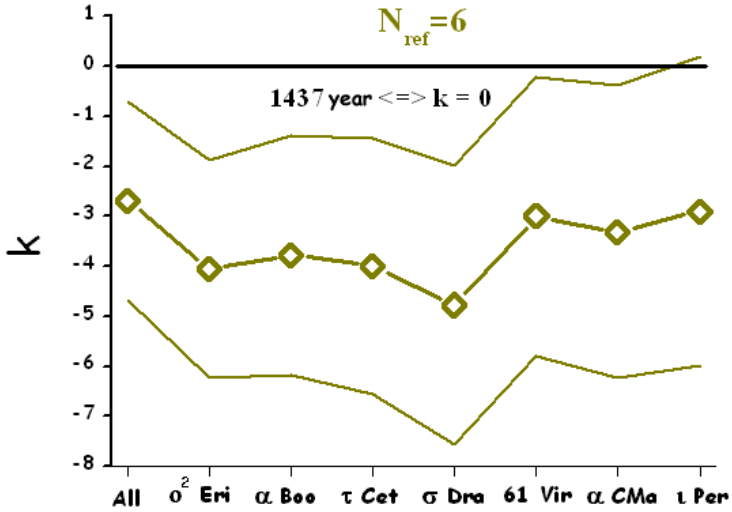

Fig. 13. *Dating Ulugbeig's catalog at $N_{ref} = 6$ with the fast stars excluded.*

In the main variant, the value of the regression declination coefficient is limited to the range of $k_{\lambda\beta} = -4.8 \div -2.7$, which corresponds to the calendar dates of the catalogue's completion between $1149 \div 1275$ AD. However, the correspondence to the traditional date is only achieved upon exclusion from the selection of the seven fastest stars including ι Per.

Variant $N_{ref} = 6 + \gamma$ Serpens

Let's quote another calculation variant at $N_{ref} = 6$, but taking into account the longitudinal projection of the γ Serpens. From the formal point of view, this star shouldn't be included into the calculation due to its discrepancy differing from the discrepancy of the vicinity by $52'$ against the maximum possible limit of $48'$. Nevertheless, the dating calculation with the longitudinal projection of γ Serpens included is of significant interest at least because of two factors. The discrepancy of this star is on the verge of penetrating into the double credible interval at $N_{ref} = 4$ and $N_{ref} = 6$ and clearly falls into it at $N_{ref} = 8$. It is quite interesting to find out how stable the dating centrum is depending on the inclusion of such a star, and the star's influence on the dating inaccuracy. Let's quote another variant of the calculation with $N_{ref} = 6$, but considering the longitudinal projection of γ Ser.

According to the calculation performed, the inclusion of the longitudinal projection of γ considerably affects the results of dating because the average value of the regression declination coefficient is changed by $\Delta k = 1.11$, considering the relatively slow longitudinal speed of that star ($\sim$ 0.65"/year). The dating inaccuracy factor is increased by 15%, and the dating spectrum is shifted in such a fashion that the historically accepted date doesn't correspond to the calculated date even if the increased inaccuracy is considered.

Variant $N_{ref} = 4$

Let's examine a dating variant providing for only four stars compared in the vicinity, that is $N_{ref} = 4$. To perform that, similar to the previous calculation, let's define the particular datings by the T_λ, T_β, projections, and then define the complete dating by $T_{\lambda\beta}$.

The mixed dating $T_{\lambda\beta}$ with o^2 Eri and α Boo corresponds to the traditionally assumed composition date by the credibility level 2σ. Leaving out the five fastest stars effectively moves the dating centrum to the beginning of the XV century.

Variant $N_{ref} = 8$

Let's consider a dating variant that assumes a vicinity composed of eight stars used in the comparison, $N_{ref} = 8$. Let's deduce the particular datings, then calculate T_λ, T_β using the projections and the complete dating T_λ, T_β and the complete dating $T_{\lambda\beta}$. The regression declination coefficients are enclosed in ranges between $k_\lambda = +0.4 \div -2.0$ and $k_\beta = -4.8 \div -7.8$, which produces the average values of $k_\lambda = -1.1$ and $k_\beta = -6.1$. In this case, the varying datings meet each other to a maximum extent, so the credible intervals of the datings coincide in all variants once the Keid and Arctur projections are left out.

Neither variant corresponds to the historically accepted composition date. The $k_{\lambda\beta}$ value hardly affects the scene when enclosed in the $k_{\lambda\beta} - 2.8 \div -5.8$ range with some stars left out and the average value of $k_{\lambda\beta} = -4.1$. The dating inaccuracy slowly increases as more stars are left out, this being the only reason why the traditional date eventually falls in the credible interval revealed through this method of dating.

The variant corresponding to the larger values of N_{ref}

Provided $N_{ref} = 10$ the vicinity radius becomes $R = 11^\circ \div 13^\circ$ degrees for about half the vicinities, and each sixth vicinity exceeds it. We may limit the vicinity radius to 14° degrees and consider this variant as the extreme. However in this case the $k_{\lambda\beta}$ will vary between -4 and -8 with the fastest stars excluded. However this sort of result can't be deemed credible, since the vicinity of the fast stars includes stars with totally different systematics. For instance, the dating becomes afflicted by the latitudinal component o^2 Eri, which moves the estimated date towards an earlier period. The latitudinal projection of this star missed from the previous calculation variants due to the reason that the latitudinal discrepancy of the star doesn't correspond to the latitudinal discrepancy of the vicinity. However, if the vicinity radius is equal to $R = 14^\circ$ the vicinity includes a number of stars with totally different group error, which leads to a difference between the Keid's discrepancy and the vicinity up to $+40'$. This effect is equally observed for the majority of vicinities, which in some cases leads to incorrect datings suggesting a much earlier or later epoch.

Analyzing the dating results

Let's sum up the data we collected over the course of the analysis of Ulugbeg's star catalog using the bulk method. Presuming the values of $N_{ref} = 6$, $N_{ref} = 8$ and $N_{ref} = N_{ref}$ $(R = 14^\circ)$ we came to an estimated catalogue completion date belonging to an earlier period of time than suggested. Only removing a few of the faster stars and taking the value of $N_{ref} = 4$ keeps the bulk method relatively valid, producing a more or less credible dating. Thus, we have to find out which approach is the most accurate before we do any judgements on the bulk method's overall validity.

Let's check out at once that the calculation variant presuming $N_{ref} = 4$ has to peculiarities. First, the discrepancy factor afflicting the vicinity is defined by the median value which is applicable from the gaussian error distribution, provided that a significant number of takes were performed. However if the stars under comparison are as few as $N_{ref} = 4$ the discrepancy factor distribution of the stars belonging to a vicinity can differ to great extent from the gaussian distribution, and the final result will be affected by drop-outs. This factor leads to the resulting discrepancy factor of the stars within the vicinity being assessed incorrectly. Secondly, it is doubtful that only a single calculation with some stars left out was enough for an accurate dating while the rest of calculations produced erroneous data. These two considerations are just enough to cross out the calculation done with $N_{ref} = 4$.

However, the selection of the most optimal variant can be based on a number of mathematical criteria, if the χ^2 value is correctly defined for each calculation. In fact, the χ^2 parameter is an value indicating how dense the vestigial discrepancies (the difference between a fast star's discrepancy and its vicinity) are grouped around the regression line. The lower values of χ^2 stand for lower average values of the vestigial discrepancy, which means a more accurate correspondence between a star's discrepancy and the discrepancy of its vicinity. This peculiarity may be used as a criterion for selecting an optimal value for N_{ref}, as seen in fig. 14.

Calculation taken with $N_{ref} = 6$ with proper consideration of the 2σ γ Ser missing from the interval leads to a serious increase in the χ^2 parameter; it is worth noting that even if that star fell into the necessary interval, the value χ^2 will vary but slightly. By adding to the formula $N_{ref} = 4$ and $N_{ref} = 6$ of γ Serpens, which takes part in the calculation with $N_{ref} = 8$, the optimal value of χ^2 is achieved at $N_{ref} = 8$.

In connection to this, it is important to revise D. Duke's remark pointing out that A.K. Dumbis and Yu.N. Efremov excluded the longitudinal projection of the θ Centauri from the calculation, the discrepancy of which belongs to the 2.5σ interval [2], however initially the credible interval was defined as

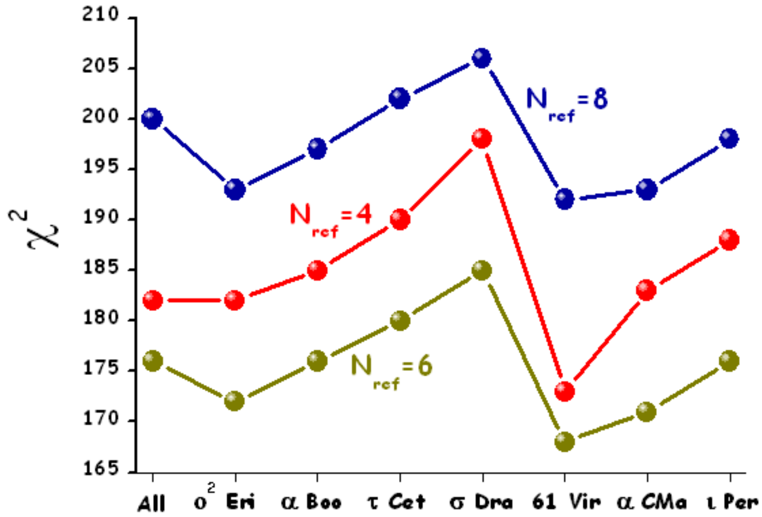

Fig. 14. *The behaviour of the χ^2 with different number of stars of vicinity.*

3σ. Let's point out that the latitudinal speed component of θ Cen has as much significance as the longitudinal component of γ Serpens. Thus, adding the θ Cen star into the calculation of A.K. Dumbis and Yu.N. Efremov would effectively shift the dating to a later epoch and increase the inaccuracy factor of the dating by $15 \div 30\%$ (a few possible dates depending on the fast stars excluded from the calculation one by one) which eliminates the possibility of dating the catalogue to have been composed between the births of Hipparchus and Ptolemy, even while staying in the framework of the method suggested by A.K. Dumbis and Yu.N. Efremov.

The minimum value of the criterion χ^2 stands for a calculation involving a vicinity comprised of six stars compared, that is N_{ref}. However the dating result with $N_{ref} = 6$ produced by the bulk method by A.K. Dumbis and Yu.N. Efremov diverges from the historically established date of the catalogue's completion, which only further supports the idea that this method lacks accuracy and validity.

Conclusion

1. Ulugbeig's star catalogue performed using the bulk method suggested by A.K. Dumbis and Yu.N. Efremov fails to produce a credible date, taking into account a different number of stars engaged in the comparison and the inaccuracies that apply. Either Ulugbeig's catalogue was composed some two or three centuries before the date presumed by the historians, or the method used by Dumbis and Efremov fails to provide enough accuracy to be able to produce credible datings. Potential reasons which could lead to this are reviewed in the Remarks $1 \div 6$. The combination of these factors leads to the inaccuracy being underestimated and an incorrect choice of the anchor date used for dating, which is proved by the numerical calculation.

2. Once the bulk method is applied to the "Almagest", even greater deviations of the anchor date used in the assessment from the real completion date of the "Almagest" arise, which are explained by the larger scale of systematic errors and a number of different systematics used in the "Almagest" [6]. There are about twice as few systematical errors in Ulugbeig's catalogue than in the "Almagest", and what is more important, all of Ulugbeig's errors stem from the same systematical error. Taking this into consideration, it's obvious that the "Almagest" suffers even worse inaccuracies in dating that Ulugbeig's catalogue, however no precise values or quantitative assessments can be drawn up.

3. The "Almagest" star catalogue produces an invalid dating with the bulk method of A.K. Dumbis and Yu.N. Efremov applied to it. This method brings no credible evidence of the catalogue having been composed between the days of Hipparchus (130 BC) and Ptolemy (130 AD), but also leaves high chances for alternative possible dates ranging vastly outside the suggested time bracket.

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