Variations of orbital elements of the asteroid 108 Hecuba

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(Conference talk)

Abstract. This paper is a continuation of our previous works "On the movement of the asteroid 108 Hecuba" and "The main part of the perturbation function in the restricted three-body problem". The first one presents an application for the computing the perturbation function in the restricted three-body problem Sun, Jupiter and Hecuba. And the second one shows the expression of the main part of the perturbation function. The determination of Hecuba's orbital elements is a special case of the three-body problem. Hecuba's mean motion is approximately two times bigger then Jupiter's one. The variations of orbital elements, presented graphically here, are calculated by the theoretical model developed by Kiril Popov in his doctor's dissertation. We improve this method including terms up to the fourth order of Hecuba's eccentricity in the perturbation function. It is possible because of the development of computer technology. All calculations are made by the system for the manipulation of symbolic and numerical expressions "Maxima". The presence of observational data enables us to take the date 18. 08. 2005 for epoch. The differential equations are solved approximately using the Maclaurin series expansion up to the second order about Jupiter's mass expressed in solar masses. The constants of integration are derived by iterations.
Key words: Jupiter, Hecuba, mean motion, orbital elements, semimajor axis, eccentricity, inclination, argument of pericenter, longitude of ascending node, mean anomaly.

Изменение на орбитните елементи на астероид 108 Хекуба

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Тази статия е продължение на предишните наши статии "Върху движението на астероид 108 Хекуба" и "Главната част на пертурбационната функция в ограничената задача за три тела". Първата дава едно приложение за пресмятане на пертурбационната функция в ограничената задача за три тела Слънце, Юпитер и Хекуба. А във втората се показва изразът за главната част на пертурбационната функция. Определянето на орбитните елементи на Хекуба е един специален случай на задачата за три тела. Средното движение на Юпитер е приблизително два пъти по-голямо от това на Хекуба. Измененията на орбитните елементи на астероид 108 Хекуба, представени графично тук, са получени въз основа на един теоретичен модел на ограничената задача за три тела, разработен от Кирил Попов в докторската му дисертация. Ние подобряваме този метод като в израза за пертурбационната функция включваме членове до четвърти порядък относно ексцентрицитета на орбитата на астероида. Това е възможно поради развитието на компютърните технологии. Всички изчисления са направени с помощта на системата за аналитично пресмятане "Maxima". Наличието на наблюдателни данни ни позволява да изберем датата 18. 08. 2005 год. за начален момент. Диференциалните уравнения се решават приблизително като се развиват в ред на Маклорен с точност до втори прядък относно масата на Юпитер, изразена в Слънчеви маси. Интеграционните константи са получени чрез последователни приближения.

Introduction

This paper is a continuation of our previous works "On the movement of the asteroid 108 Hecuba" [1] and "The main part of the perturbation function in the restricted three-body problem" [2]. The first one presents an application for the computing the perturbation function in the restricted three-body problem Sun, Jupiter and Hecuba. And the second one shows the expression of the main part of the perturbation function using the generalized binomial transform [3].

The variations of 108 Hecuba's orbital elements are calculated by the theoretical model developed by Kiril Popov in his doctor's dissertation [4]. We improve this method including the terms up to the fourth order of Hecuba's eccentricity in the perturbation function [1], [2]. It is possible because of the development of computer technology. We make the calculations by the system for the manipulation of symbolic and numerical expressions "Maxima" [5]. The obtained orbital elements (see Fig. 1-6) are shown only graphically as the analytical results are very enormous.

The applications for the calculations will be presented in next works. If we succeed with reducing the analytical solution using generalized binomial transform [3], we shall publish it too.

Why do we choose 18.08.2005 *for epoch*?

- As a first Andoayer proved [4] that the angle ψ that is the difference between Hecuba's mean anomaly (M) and the double difference between Jupiter's mean longitude (ζ_1) and Hecuba's mean longitude (ζ) has to be approximately equal to 180° at epoch 18.08.2005. Then we have:

$$\psi = M - 2\,(\zeta_1 - \zeta) \approx 180^\circ \tag{1}$$

- As a second the angle ψ changes with a period about 300 years and now we have the incredible possibility to include observational data from 2005 in our computations. The orbital elements calculated by observations are taken from "Ephemerides of minor planets" [6].

1 Equations of motion

We assume that Jupiter's orbital elements are constant.

Initially Hecuba's orbital elements are transformed on the plane defined by the Jupiter orbital elements [6] at epoch 18.08.2005 [J2000]:

$$a_1 = 5,2018733[AU], \quad e_1 = 0,048957, \quad i_1 = 1^\circ,30376,$$
$$M_1 = 190^\circ,4223, \quad \omega_1 = 274^\circ,21437, \quad \Omega_1 = 100^\circ,50891.$$

Hecuba's orbital elements [6] at epoch 18.08.2005 [J2000] are:

$$a_0 = 3,2409744[AU], \quad e_0 = 0,0524662, \quad i_0 = 4^\circ,24713,$$
$$M_0 = 224^\circ,63705, \quad \omega_0 = 191^\circ,05215, \quad \Omega_0 = 300^\circ,37926.$$

a_0, a_1 – the samimajor axis of Hecuba and Jupiter respectively,
e_0, e_1 – the eccentricity of Hecuba and Jupiter respectively,
i_0, i_1 – the inclination of Hecuba and Jupiter respectively,
M_0, M_1 – the mean anomaly of Hecuba and Jupiter respectively,
ω_0, ω_1 – the argument of pericenter of Hecuba and Jupiter respectively,
Ω_0, Ω_1 – the longitude of ascending node of Hecuba and Jupiter respectively.

The equations of motion are expressed using the Delone variables [7] as follows:

$$\begin{aligned} \frac{dL}{dt} &= \frac{\partial F}{\partial M}, & \frac{dG}{dt} &= \frac{\partial F}{\partial \omega}, & \frac{d\Theta}{dt} &= \frac{\partial F}{\partial \upsilon}, \\ \frac{dM}{dt} &= -\frac{\partial F}{\partial L}, & \frac{d\omega}{dt} &= -\frac{\partial F}{\partial G}, & \frac{d\upsilon}{dt} &= -\frac{\partial F}{\partial \Theta}, \end{aligned} \tag{2}$$

where:

$$\upsilon = \Omega - \zeta_1, \tag{3}$$

$$F = n_1 \left(\frac{1}{2L^2} + \Theta + \hat{\mu} R \right), \tag{4}$$

$$\hat{\mu} = \mu_1 \left(1 + \mu_1\right)^{-1/3}, \tag{5}$$

$$R = a_1 \hat{R}. \tag{6}$$

ζ_1 – the mean longitude of Jupiter,
n_1 – the mean motion of Jupiter,
μ_1 – the mass of Jupiter in solar masses,
$\hat{R}$ – the perturbation function.

The relations between the Delone variables and the orbital elements [7], [8] are:

$$\begin{aligned} L &= (1 + \mu_1)^{1/6} \sqrt{\frac{a}{a_1}}, \\ G &= L\sqrt{1 - e^2}, \\ \Theta &= G \cos i. \end{aligned} \tag{7}$$

The Delone variables are replaced with new variables following [4]:

$$\begin{aligned} U &= L + S + T, & S &= L - G, & T &= G - \Theta, \\ \varphi &= M + \omega + \upsilon, & s &= -M - 2\omega - 2\upsilon, & \tau &= -M - \omega - 2\upsilon, \end{aligned} \tag{8}$$

$$\begin{aligned} \tilde{x} &= \sqrt{2S} \cos s + \tilde{e}, & y &= \sqrt{2S} \sin s, \\ \xi &= \sqrt{2T} \cos \tau, & \eta &= \sqrt{2T} \sin \tau, \end{aligned} \tag{9}$$

$$\tilde{x} = \varepsilon \cos \alpha, \qquad y = -\varepsilon \sin \alpha,$$
$$\xi = \iota \cos \beta, \qquad \eta = -\iota \sin \beta, \tag{10}$$

where $\tilde{e}$ is a constant that is determined by the following condition:

The coefficient of Hecuba's eccentricity in the perturbation function must be equal to zero.

Then the equations of motion become:

$$\frac{dU}{dt} = \frac{\partial F}{\partial \varphi}, \qquad \frac{d\varepsilon}{dt} = -\frac{1}{\varepsilon}\frac{\partial F}{\partial \alpha}, \qquad \frac{d\iota}{dt} = -\frac{1}{\iota}\frac{\partial F}{\partial \beta},$$
$$\frac{d\varphi}{dt} = -\frac{\partial F}{\partial U}, \qquad \frac{d\alpha}{dt} = \frac{1}{\varepsilon}\frac{\partial F}{\partial \varepsilon}, \qquad \frac{d\beta}{dt} = \frac{1}{\iota}\frac{\partial F}{\partial \iota}. \tag{11}$$

Finally the differential equations are solved approximately using the Maclaurin series expansion about $\hat{\mu}$ including terms up to the second order [8]. The quantity $\hat{\mu}$ is approximately equal to Jupiter's mass expressed in solar masses μ_1. The constants of integration are derived by iterations.

All calculations are made by the system for the manipulation of symbolic and numerical expressions "Maxima" [5].

2 Differences between our model and Kiril Popov's model

Initially we exclude only the terms depending on φ in the perturbation function. Kiril Popov excluded all terms depending on φ, e_1, and $\tilde{e}$.

All longperiodical terms are included in the second approximation of our solution. Kiril Popov included only some of them [4].

The angle ψ at zero time point is approximately equal to $0°$ in Kiril Popov's model. It is approximately equal to $180°$ in our model. In this way the series become alternative and they converge faster.

To determine the integration constants we take only the initial orbital elements of Hecuba at epoch 18.08.2005. Kiril Popov determined the integration constants influenced by available observational data.

We include all terms up to the fourth order of Hecuba's eccentricity in the perturbation function in our computations. So the accuracy of determination of Hecuba's orbital elements especially of inclination and longitude of ascending node (see Fig. 4, 5) increases. Kiril Popov used the expression of the perturbation function including the terms up to the second order [7]. He took the terms up to the fourth order only in the Taylor series in the equations for α and β.

3 Results

Figures (1-6) show the comparisons between the calculations of K. Popov [4] (dotted curves), calculations based on the results of the presented work (solid curves) and available observational data [6] (dots).

The variations of Hecuba's semimajor axis are shown in Fig. 1. The amplitude calculated by Kiril Popov is smaller then the real amplitude as he

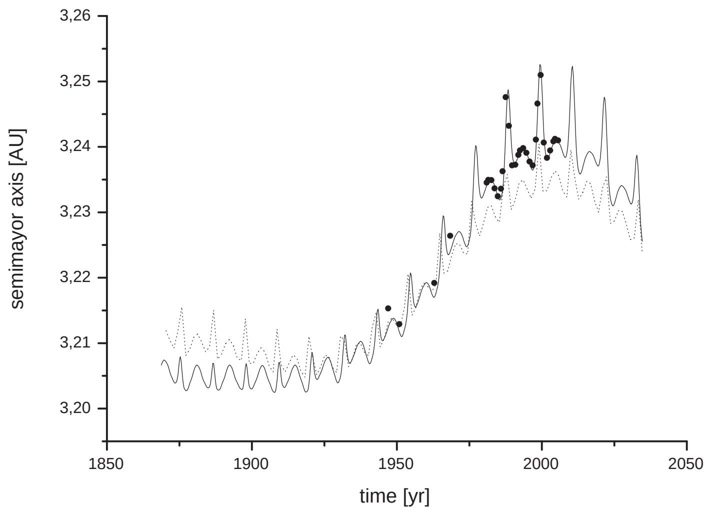

Fig. 1. Variations of semimajor axis, calculations of K. Popov (dotted curve), calculations based on the results of the presented work (solid curve), observational data (dots)

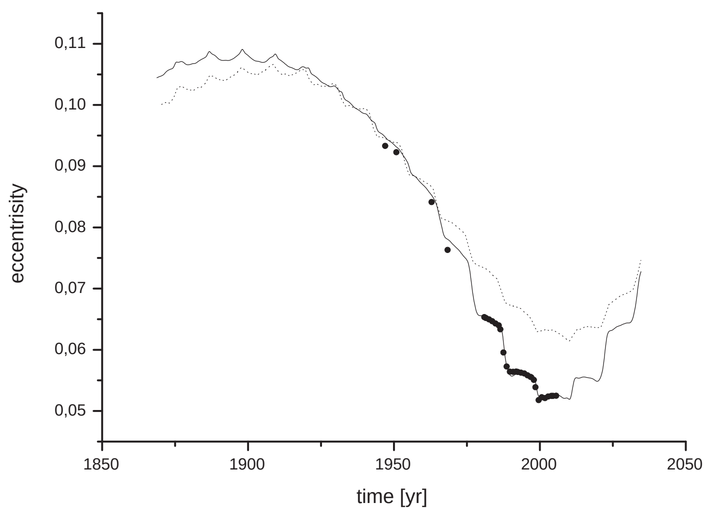

Fig. 2. Variations of eccentricity (see also Fig. 1)

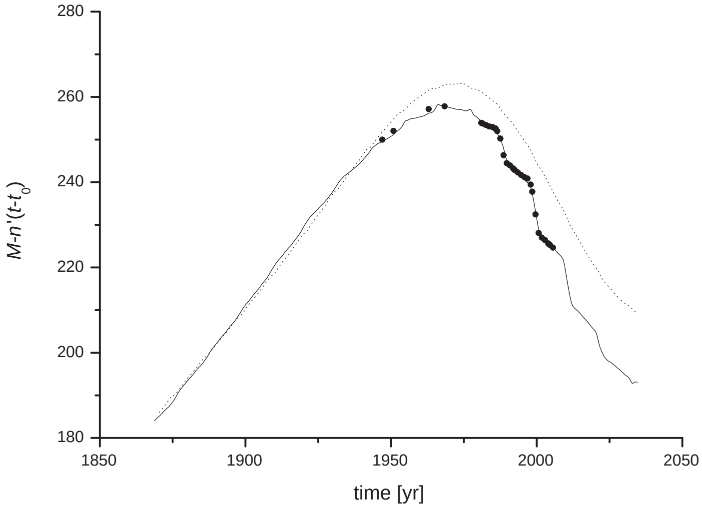

Fig. 3. Variations of mean anomaly (see also Fig. 1)

included a part of the longperiodical terms. The same thing is observed in the diagram describing the variation of eccentricity (see Fig. 2).

Because the variations of mean anomaly are very fast ($M \sim t$), the variations of the quantity $M - n'(t - t_0)$ are shown in Fig. 3 where $n' = 62°/yr$.

It is seen that the variations of inclination and longitude of ascending node determined by Kiril Popov (see Figs. 4, 5, dotted curve) are disparated with the observational data (dots). That is because he included only the terms up to the second order in the equations of inclination.

Including the terms up to the fourth order in the Taylor series in the equations for α and the introduction of the quantity $\hat{e}$ explain more accurate determination of Hecuba's argument of pericenter by Kiril Popov (see Fig. 6, dotted curve). But it is obviously that our calculations (solid curve) are more precise.

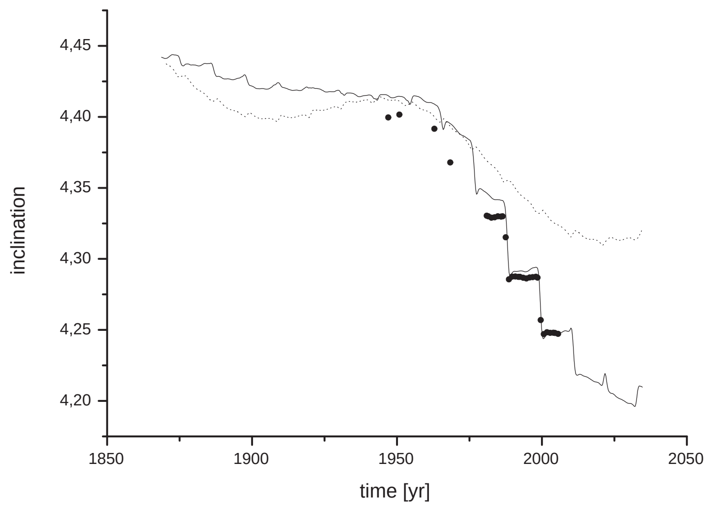

Fig. 4. Variations of inclination (see also Fig. 1)

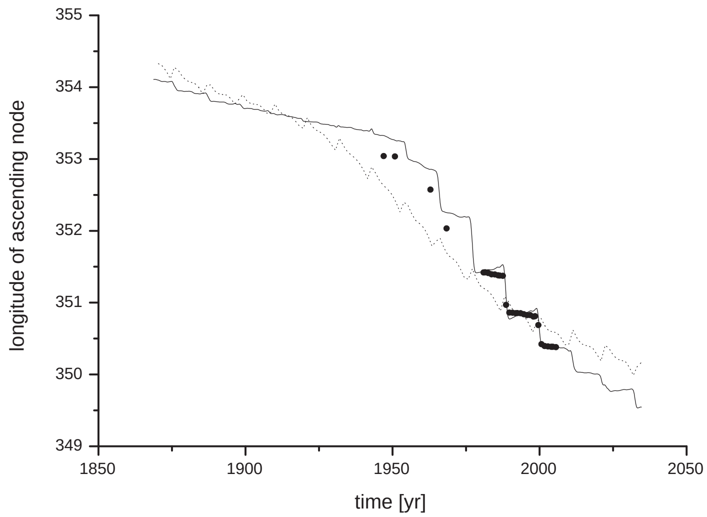

Fig. 5. Variations of longitude of ascending node (see also Fig. 1)

Conclusions

The accuracy of determining Hecuba's orbital elements is increased with including the terms up to the fourth order. The assumption that Jupiter's orbital

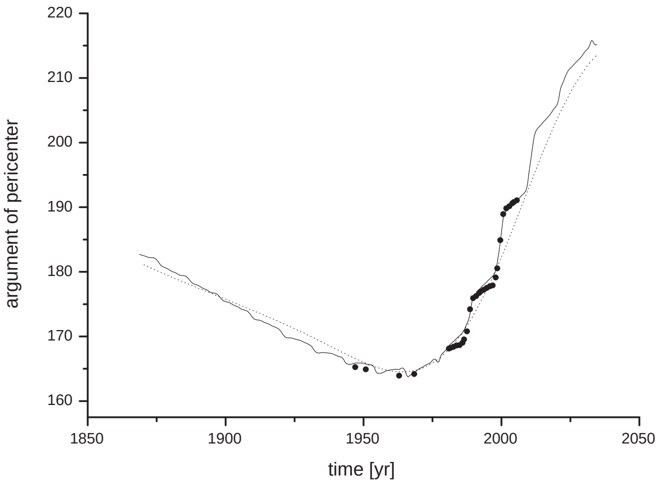

Fig. 6. Variations of argument of pericenter (see also Fig. 1)

elements are constant and excluding the shortperiodical terms in the second approximation do not allow to obtain better accuracy.

Kiril Popov predicted Hecuba's orbital elements satisfactorily for a period about 20-30 years. Our model determines them with better accuracy for a period about 150 years. In addition our computations predict that Hecuba's inclination will raise after 2031. Future observations may confirm this.

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ОРБИТНИ ЕЛЕМЕНТИ НА
АСТЕРОИД 108 ХЕКУБА
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