Neutrino in the Universe

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Abstract. The evolution of oscillating neutrinos from the early stages of the Universe till now is described. The evolution of the neutrino energy spectrum distortion caused by matter or vacuum neutrino oscillations and the effect of the electron neutrino spectrum distortion on the early Universe is discussed.
Key words: cosmological neutrinos, neutrino oscillations

Неутриното във Вселената

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Описана е еволюцията на неутриното от ранните стадии на Вселената до днешния й етап в реалистичния случай на неутрино, участващо в неутринни осцилации. Дискутира се еволюцията на дисторсията на спектъра на неутриното, дължаща се на осцилации в среда или вакуумни осцилации, както и влиянието на тази дисторсия върху процеси в ранната Вселена.

Introduction

Today the cosmological neutrinos ν are expected to be the most numerous particles after the cosmic microwave background (CMB) photons γ_{CMB}.

According to the assumptions of the standard cosmological model (SCM) our Universe is filled with massless non-oscillating neutrinos. There exist three neutrino flavors, the lepton asymmetry is zero and neutrino number densities and spectrum are the equilibrium ones. For a review see Dolgov [2002].

In the last 10 years the observational, experimental and theoretical development in neutrino astrophysics revealed that most of these assumptions are wrong. There exists an experimental evidence, from CERN LEP experiment, that the number of neutrino species N_ν with ordinary weak interactions is $N_\nu = 3$. However, there still remains the possibility of inert SU(2)xU(1) singlet neutrino, the so called sterile neutrino ν_s, not participating in the electroweak interactions, i.e. *a fourth neutrino type is not forbidden.* In SCM those neutrinos should decouple much earlier from the thermodynamical equilibrium due to their extremely weak interactions and, hence, they are expected to have much lower energy, lower temperature and negligible number densities compared to the active flavor neutrinos.

Since 1998 *evidences for neutrino oscillations and, hence, for non-zero neutrino masses* have been found of at least two of the neutrino flavors.

And for the case of active-sterile neutrino oscillations it has been shown in Kirilova [1988], Barbieri & Dolgov [1990,1991], Chizhov & Kirilova [1996,1997], that both *the neutrino number densities and neutrino energy spectrum may considerably deviate from their equilibrium values and an asymmetry between the neutrino and antineutrino may exist*, generated during resonant neutrino transfers.

In what follows we will first discuss shortly the evolution of neutrinos in the Universe according to the SCM and then present the neutrino evolution in a model beyond the SCM accounting for neutrino oscillations. In more detail the case of electron-sterile neutrino oscillations is considered.

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Neutrinos in the standard cosmological model

In brief the neutrino evolution according to the SCM is as follows:

At temperatures above several MeV the tau ν_τ, muon ν_μ and electron neutrino ν_e are in thermodynamical equilibrium in the early Universe plasma thanks to their fast (in comparison with the Universe expansion rate) interactions with its other constituents. *SCM assumes three neutrino flavors, zero lepton asymmetry and equilibrium neutrino number densities and spectrum:*

$$n_\nu^{eq} = \exp(-E/T)/(1+\exp(-E/T)), \tag{1}$$

where E is the neutrino energy and T is its temperature.

However, in the process of Universe cooling when the energies become around several MeV, the neutrino interaction rates (predominantly with electrons and positrons at that epoch) become comparable with and lower than the expansion rate, i.e. neutrino starts freezing, and till 2 MeV all neutrino species are decoupled from the equilibrium plasma. The neutrinos keep their equilibrium Fermi-Dirac spectrum during the further expansion and cooling of the Universe due to the fact that their masses are negligible (if any). Their temperatures decrease, according to $T \sim 1/R(t)$ (for adiabatic expansion), where R(t) is the scale factor. As a result of the electron-positron annihilation, electron neutrino temperature becomes lower than the temperature of the photons, because the e^+e^- - annihilation heats only the particles in equilibrium, i.e. γ, not ν. Hence, today we expect that the Universe is filled with a cosmological neutrino phone with an equilibrium Fermi-Dirac spectrum and a temperature $T_\nu = (4/11)^{1/3} T_{cmb} \sim 1.9$ K, i.e. less than the temperature of the CMB $T_{cmb} \sim 2.7$ K. Neutrinos energy density is extremely low, since $\rho_\nu \sim T^4$ and its number densities are $n_\nu + n_{\tilde{\nu}} = 3/11 n_{cmb}$ per given neutrino species.

In contrast to CMB observations the detection of the cosmological neutrino is very difficult: first, because though neutrino is numerous, it is an extremely elusive particle due to its very weak interactions and second, because cosmological neutrinos are expected to have today extremely low energy. On the other hand, being a very weakly interacting particle, and hence, having a uniquely great penetrating capability, neutrinos carry precious information for the astrophysical processes in the dense star cores and from the early stages of the Universe evolution. Thus it is useful to study them.

Today our neutrino detectors are able to catch the neutrinos from the Sun and neutrinos from Supernovas bursts, i.e. only neutrinos with a considerable energy. In some near future, hopefully, apropriate detector facilities will be constructed and will be able to probe the early Universe by studying the cosmological neutrinos. The glimpse that we will have in neutrinos will correspond to the first seconds of the Universe existence (the epoch of neutrino decoupling).

Oscillating neutrinos in the early Universe

During the last 10 years it has been proved that neutrino oscillations between different neutrino species take place: solar, atmospheric and terrestrial neutrino experiments have provided evidences for neutrino oscillations.

The solar neutrino deficit, first observed in the Homestake solar neutrino experiment, was confirmed by Kamiokande, Gallex and SAGE. The atmospheric neutrino anomaly, observed in Kamiokande was confirmed by IMB, MACRO and SUDEN. The neutrino anomalies were successfully explained in terms of neutrino oscillations. In 1998 Super-Kamiokande obtained an evidence of atmospheric flavor neutrino oscillations and marked the discovery of a finite neutrino mass. The atmospheric neutrino oscillations were confirmed by terrestrial neutrino experiment K2K and MINOS. In 2002 SNO obtained the evidence of flavor-transformations of neutrinos being the dominant channels in explaining the solar neutrino anomaly and KamLAND detected the evidence of reactor antineutrino oscillations.

Thus, *the neutrino anomalies are explained in terms of neutrino oscillations, flavor neutrino oscillations being the dominant channels.* The role of sterile neutrinos and the active-sterile subdominant channels is being explored at present.

The hypothesis of neutrino oscillations was proposed more than 40 years ago by B. Pontecorvo [1958,1968] as a possible explanation to the solar neutrino deficit found at the Davis solar neutrino experiment. Oscillations are possible if mass eigenstates ν_i are distinct from the flavor eigenstates ν_f:

$$\nu_i = U_{if}\ \nu_f \qquad (f = e, \mu, \tau),$$

i.e. if neutrino has mass and non-zero mixing. Then in the two-neutrino oscillation case in vacuum, the probability to find a given neutrino type in an initially homogeneous neutrino beam of the same type is:

$$P_{ff}(t) = 1 - \sin^2 2\vartheta \sin^2(\delta m^2 t/4E),$$

where δm^2 - the neutrino squared mass difference and ϑ - the oscillations mixing angle are the oscillation parameters, E is the neutrino energy.

The neutrino oscillations parameters have been fixed from the data of solar and atmospheric neutrino anomalies and confirmed in recent years by the ground based accelerator and reactor experiments. The best fit point values of the masses and mixing angles determining the solar neutrino oscillations are $\delta m^2 \sim (7.2 - 9.2) \times 10^{-5}$ eV^2 and $\sin^2 2\theta \sim 0.25 - 0.39$. The best fit oscillation parameters for the available atmospheric neutrino data are $0.34 < \sin^2 2\theta < 0.68$ and $\delta m^2 \sim (2 - 3.2) \times 10^{-3}$ eV^2. For more detail see Maltoni et al. [2004], Goswami et al. [2005,2006], Choubey [2006], Fogli et al. [2007] and the references therein.

Thus, the neutrino experiments results confirmed the existence of non-zero neutrino mass and mixing. On the other hand, non-zero neutrino mass and neutrino oscillations influence the early Universe evolution by effecting the expansion rate and the neutrino energy spectrum, thus influencing the neutrino involved processes, as for example cosmological nucleosynthesis, structure formation, CMB, etc.

The flavor oscillations lead to a slight deviations in the equilibrium neutrino spectra, because of the almost equal temperatures of the different neutrino species, which equality is due to their very close decoupling times. Therefore, the influence of flavor oscillations on the Universe processes is negligible.

However, in case of oscillations between active neutrinos and initially non-equilibrium inert neutrino [1] the distortion of the energy spectrum of the active neutrinos caused by the oscillations may be considerable both for oscillations in vacuum, Kirilova [1988], and matter oscillations Chizhov & Kir-

[1] Non-equilibrium is the expected natural possibility, since the inert neutrino decouples much earlier in the Universe evolution and since its decoupling time the flavor neutrinos have been additionally heated by the annihilation processes that took place after inert decoupling

ilova [1996,1997,1998,2000], leading to a depletion of the active neutrino number densities and also to a generation of neutrino-antineutrino asymmetry.

Thus, active-sterile oscillations may considerably influence different Universe processes. The effect of neutrino oscillations on the early Universe evolution has been explored in numerous works, mainly discussing its BBN effect.

They were strongly restricted by cosmological BBN considerations (for 2 oscillation constraints see Kirilova & Chizhov [1998,2000], Kirilova, [2007], Kirilova & Panayotova [2006] and for 4 neutrino oscillation schemes see Villante & Dolgov [2003].

Here we consider the influence of neutrino oscillations on the energy spectrum and the evolution of the cosmic neutrinos from the early BBN stages till now. In the early dense epoch of the Universe the oscillations took place while neutrinos interacted with other particles from the Universe plasma, hence, first we will consider the matter oscillations case. While in the last section we will discuss the effects of vacuum oscillations which will correspond to later stages of the neutrino evolution, after the electron-positron annihilation epoch of the Universe.

Matter neutrino oscillations

It is known that the thermal background in the pre-nucleosynthesis epoch may strongly affect the propagation of neutrinos.

The medium distinguishes between different neutrino types due to different interactions with the particles from the plasma and different neutrino types aquire different average potentials, see Nötzold & Raffelt [1988]. The oscillations then depend on the characteristics of the medium as well. Although in general the medium suppresses oscillations by decreasing their amplitude, there also exists a possibility of enhanced oscillation transfer when the mixing in matter becomes maximal, independently of the value of the vacuum mixing angle, see Wolfenstein and Mikheyev & Smirnov [1978,1985].

Matter active-sterile neutrino oscillations taking place *before the active neutrinos decoupling* cause an increase of the relativistic degrees of freedom g_{eff}, thus speeding the Unverse expansion rate $H \sim \sqrt{g_{eff}}T^2$, see refs. Barbieri & Dolgov [1990,1991], Enqvist et al. [1990,1992] and as a result effecting process for which H is essential. The case of *oscillations effective after the flavor neutrino decoupling* was discussed as well. Such active-sterile oscillations may cause neutrino spectrum distortion and its effect on the early Universe has been also considered for the case of *matter neutrino oscillations* taking place before and during the cosmological nucleosynthesis epoch (BBN epoch). First estimates of an eventual depletion and spectrum distortion of electron neutrinos, due to $\nu_e \leftrightarrow \nu_s$ oscillations, and their corresponding influence on helium-4 were given, correspondingly in Kirilova, [1988] and Chizhov & Kirilova [1996,1997].

The accurate kinetic approach of Dolgov [1981] (vacuum flavor oscillations) and Kirilova, [1988] (vacuum active-sterile oscillations) to the description of the oscillating neutrinos in terms of *neutrino density matrix in momentum space*, was applied for extracting the kinetic equations, describing the evolution of neutrinos participating in matter oscillations.

The equations describing the kinetics of the neutrino ensembles ρ and antineutrino $\bar{\rho}$ read,Chizhov & Kirilova [1996,1997]:

$$\frac{\partial \rho(t)}{\partial t} = H p_\nu \frac{\partial \rho(t)}{\partial p_\nu} +$$
$$+ i \left[\mathcal{H}_o, \rho(t)\right] + i\sqrt{2} G_F \left(\pm\mathcal{L} - Q/M_W^2\right) N_\gamma \left[\alpha, \rho(t)\right] + \mathrm{O}\left(G_F^2\right), \qquad (2)$$

where $\alpha_{ij} = U^*_{ie} U_{je}$, p_ν is the momentum of electron neutrino, n stands for the number density of the interacting particles. The plus sign in front of $\mathcal{L}$ corresponds to the neutrino ensemble, the minus sign - to the anti-neutrino ensemble.

$\mathcal{H}_o$ is the free neutrino Hamiltonian. The 'non-local' term Q arises as a W/Z propagator effect, $Q \sim E_\nu\ T$. $\mathcal{L}$ is proportional to the fermion asymmetry of the plasma and is essentially expressed through the neutrino asymmetries $\mathcal{L} \sim 2L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau}$, where $L_{\nu_\mu,\nu_\tau} \sim (N_{\nu_\mu,\nu_\tau} - N_{\bar{\nu_\mu},\bar{\nu_\tau}})/N_\gamma$ and $L_{\nu_e} \sim \int \mathrm{d}^3p(\rho_{LL} - \bar{\rho}_{LL})/N_\gamma$.

Mixing just in the electron sector is assumed: $\nu_i = U_{il}\ \nu_l$ $(l = e, s)$. The initial condition for the neutrino ensembles in the interaction basis is assumed to be of the form:

$$\rho = n_\nu^{eq} \begin{pmatrix} 1 & 0 \\ 0 & S \end{pmatrix},$$

where $n_\nu^{eq} = \exp(-E_\nu/T)/(1+\exp(-E_\nu/T))$, while S measures the degree of population of the sterile state. The degree of population of ν_s may be different, depending on the concrete model of ν_s production.

These are exact kinetic equations for the neutrino density matrix in momentum space, which account *simultaneously* for expansion, neutrino oscillations and neutrino forward scattering.

These equations have been numerically solved for the full range of the oscillation parameters of the model. For a large range of oscillation parameters it has been found that oscillations cause considerable spectrum distortion of neutrinos. In case of electron-sterile oscillations the distortion of the electron neutrino spectrum may have strong effect on the pre-BBN kinetics, leading to an overproduction of primordially produced helium-4 Chizhov & Kirilova [1996,1997] , Kirilova & Chizhov, [1998,2000]. The dependence of the spectrum distortion on the initial population of the sterile neutrino state was studied by Kirilova [2004]. The spectrum distortion decreases with the increase of the initial population.

The figures from Kirilova [2004]) present the spectrum distortion of electron neutrino at different temperatures, namely 1 MeV and 0.7 MeV, correspondingly, and for different initial population of the sterile state.

In the resonant case, a growth of neutrino–antineutrino asymmetry generated by oscillations was registered, as well, see Foot, Thompson & Volkas [1996] and Chizhov & Kirilova [1996,1997], and its effect on helium-4 production was calculated Kirilova & Chizhov [1996,1997,1998,2000].

The oscillating neutrino species tend to reach statistical equilibrium, i.e. finally, when they reach the equilibrium, the active species will be depleted by a factor $p/(p+q)$, where p is the number of active neutrino species, participating into the oscillations with q sterile neutrino species. Besides, they will have lower energy than their equilibrium energy in the SCM and their spectrum will not be of the Fermi-Dirac form.

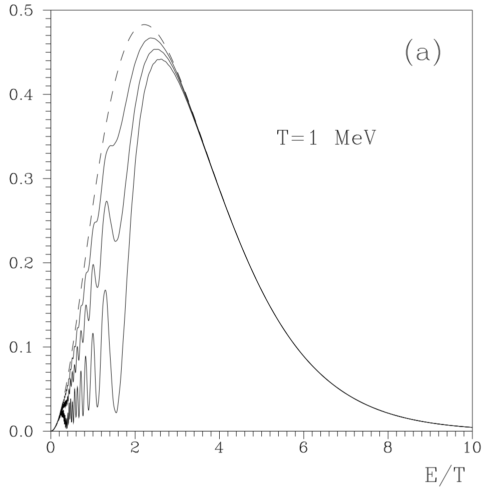

Fig. 1. A snapshot of the spectrum distortion caused by oscillations with mass difference $|\delta m^2| = 10^{-7}$ eV^2 and mixing $\sin^2 2\vartheta = 0.1$ at $T = 1$ MeV for different degrees of population of the steriles, namely $\delta N_s = 0$ (lower curve), $\delta N_s = 0.5$ and $\delta N_s = 0.8$ (upper curve). The dashed curve gives the equilibrium spectrum for comparison.

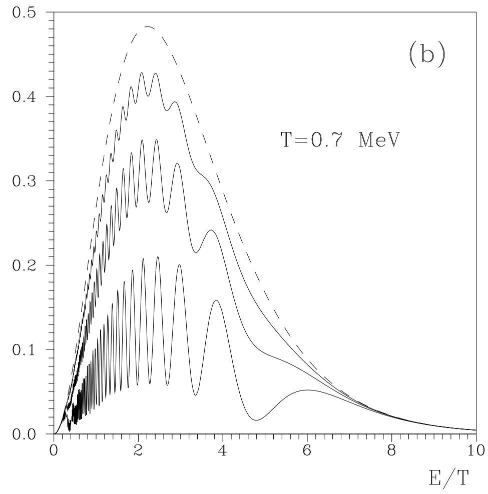

Fig. 2. A snapshot of the spectrum distortion caused by oscillations with mass difference $|\delta m^2| = 10^{-7}$ eV^2 and mixing $\sin^2 2\vartheta = 0.1$ at $T = 0.7$ MeV for different degrees of population of the steriles, namely $\delta N_s = 0$ (lower curve), $\delta N_s = 0.5$ and $\delta N_s = 0.8$ (upper curve). The dashed curve gives the equilibrium spectrum for comparison.

Oscillations effect on BBN

Precise numerical account of the distortion caused by oscillations reveals the possibility for 6 times higher helium-4 overproduction than the obtained in previous studies of BBN with neutrino oscillations not accounting for the spectrum distortion, see Kirilova [2003]. The distortion decreases with the increase of the initial population. Hence, the overproduction is maximal for the case of initially empty ν_s state $\delta N_s = 0$, Kirilova [2004].

Cosmological constraints on oscillation parameters, obtained on the basis of comparison of the calculated isohelium contours and the Y_p observational data have been obtained. The analytical fits to the exact constraints for the $\delta N_s = 0.0$ case are:

$\delta m^2 (\sin^2 2\vartheta)^4 \leq 1.5 \times 10^{-9}$ eV^2 for $\delta m^2 > 0$ and

$|\delta m^2| < 8.2 \times 10^{-10}$ eV^2 for $\delta m^2 < 0$ and large ϑ.

The constraints corresponding to other initial populations of the sterile neutrino were derived in Kirilova [2007] (corresponding to 3% helium overproduction) and Kirilova & Panayotova [2006] (corresponding to 5% helium overproduction).

Vacuum neutrino oscillations

An alternative possibility of active-inert oscillations exist, when oscillations occur after the electron-positron annihilation. Then the oscillations are of the vacuum type.

During this later epoch, even the residual interactions of neutrinos with the plasma can be safely neglected because just a negligible portion of electrons remains $n_e/n_{cmb} \sim 6.10^{-10}$ according to the charge neutrality of the Universe.

Hence, it is intriguing to consider the spectrum distortion of cosmological neutrino due to vacuum active-sterile oscillations. In Kirilova [1988] the nonequilibrium case of vacuum active–sterile oscillations, effective after the neutrino decoupling was discussed, i.e. the oscillation rate exceeds the expansion rate $\Gamma_{osc} \sim \delta m^2/(4E) > H$, while neutrino typical weak rates are less than the expansion rate $\Gamma_w < H$. At that time (i.e. 1988) the importance of the medium to oscillations was not yet known. Therefore the vacuum case was considered applicable since 2 MeV. As discussed in the previous section, we expect that the effect of the medium is not important after the electron-positron annihilation, i.e. after 0.5 MeV . However, the equations hold almost without change for the period after electron-positron annihilation.

The evolution of neutrinos, oscillating in vacuum, is described by integro–differential equations, similar to those in the case of matter oscillations, without the last two terms describing the interactions of neutrinos with the medium.

$$\frac{\partial \rho(t)}{\partial t} = H p_\nu \, \frac{\partial \rho(t)}{\partial p_\nu} + i \left[\mathcal{H}_o, \rho(t) \right], \tag{3}$$

We have solved the equations for the natural assumption of sterile neutrino number densities being initially negligible in comparison with the active neutrinos: as far as the number densities are proportional to their temperature $N \sim T^3$, and the sterile neutrinos have decoulped much earlier than the active ones, and hence $T_s < T_f$, and $N_s << N_f$. The evolution of the number density of electron neutrinos for the initially empty sterile state, $\delta N_s = 0$ reads:

$$n_{\nu_e} = \rho_{LL} = \left\{ 1 - 2c^2 s^2 + 2c^2 s^2 cos BT (T^{-3} - T_0^{-3})/E \right\} \times n_{\nu_e}^{eq} \tag{4}$$

where $B = 0.1 M_{Pl} g_{eff}^{-1/2} \delta m^2$ and $T_0 = 0.5$ MeV.

It was explicitly shown, that for a large range of oscillation parameters of the discussed model, oscillations may cause considerable spectrum distortion and/or depletion of the electron neutrino.

At later epoch, after the e^+e^--annihilation, $\cos[BT(T^{-3} - T_0^{-3})/E]$ is frequently oscillating and can be averaged. Then $\rho_{LL} = (1-2c^2s^2)n_{\nu_e}^{eq}$, and active neutrinos number densities are depleted by oscillations.

The depletion of active neutrino due to oscillations may influence different processes occurring in the epoch after electron-positron annihilation and should be studied. A probe of such possible effect of spectrum distortion of neutrinos at the CMB epoch might be the precise CMB measurements, for example. And in some more distant future the measurements of the cosmological neutrinos may also reveal the spectrum distortion caused by neutrino oscillations. Thus we will be able to reveal the specific neutrino oscillation parameters and have a glimpse in neutrino of the early Universe.

Conclusions

Exact kinetic equations describing the evolution of the oscillating neutrino during the Universe evolution are presented. Exact numerical solutions are found in the case of oscillations in the pre-BBN and BBN epoch. In the post BBN epoch, the kinetic equation describing the evolution of neutrino is solved analytically for the case of zero initial population of the sterile neutrino.

It is found that usually the energy spectrum of neutrinos is considerably distorted and the active neutrino number densities are depleted due to active-sterile neutrino oscillations effective after neutrino decoupling. In the resonant oscillations case a neutrino-antineutrino asymmetry may be generated by oscillations. The spectrum distortion, however, usually is the dominant effect of neutrino oscillations in the early Universe processes, like BBN, for example. The dependence of the spectrum distortion on the initial population of the sterile neutrino is studied. Distortion decreases with the increase of the initial population of the sterile neutrino.

The spectrum distortion effect on BBN is investigated. Precise numerical account of the distortion, caused by oscillations, reveals the possibility for 6 times higher helium-4 overproduction than the obtained in previous studies of BBN with neutrino oscillations not accounting for the spectrum distortion. BBN with nonequilibrium $\nu_e \leftrightarrow \nu_s$ oscillations allows to put stringent constraints on oscillation parameters. They were calculated for different isohelium contours and for different initial populations of the sterile neutrino.

In later epoch the oscillations of neutrino are expected to cause flavor neutrino depletion (for the sake of the sterile ones), reduced mean energy of the flavor neutrinos (for the sake of the increase in the sterile neutrino sector) and non Fermi-Dirac neutrino spectra. It is interesting to study the effect of these changed neutrino characteristics on the processes at late epochs in the Universe evolution, like the formation of CMB, large scale structure, etc.

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