Galaxies, Cosmology and Dark Matter

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Summer semester 2000

Chapter 7

Elliptical Galaxies

7.6 Mass Determination for Spherical Galaxies using Velocity Dispersion Profiles

For spherical galaxies the Jeans equation reads:

$$\frac{d\rho\overline{v_r^2}}{dr} + \frac{\rho}{r}\left[2\overline{v_r^2} - \left(\overline{v_\theta^2} + \overline{v_\varphi^2}\right)\right] = -\rho\frac{d\Phi}{dr} \tag{7.1}$$

with $\rho =$ number density (not necessarily mass density!) and $\overline{v_\theta^2} = \overline{v_\varphi^2}$

Using:

$$\boxed{\beta \equiv 1 - \frac{\overline{v_\theta^2}}{\overline{v_r^2}}}$$

$\beta = 0$: **isotropic** velocity dispersion
$\beta < 0$: **tangential** anisotropy
$0 < \beta < 1$: **radial** anisotropy

equation (7.1) can be transformed into:

$$\boxed{-\overline{v_r^2}\left(\frac{d\ln\rho}{d\ln r} + \frac{d\ln\overline{v_r^2}}{d\ln r} + 2\beta\right) = r\frac{d\Phi}{dr} = v_c^2 = \frac{GM(<r)}{r}} \tag{7.2}$$

Let the stellar density of the galaxy fall off with a power law (like in the case of a Jaffe or an $r^{1/4}$ profile):

$$\rho(r) \propto \rho_0 r^{-k}$$

$$\rightsquigarrow \; d\ln\rho/d\ln r = -k$$

Then equation (7.2) transforms to:

$$\boxed{M(r) = \frac{r}{G}\left[\frac{k-2\beta}{1-\beta}\sigma_\theta^2 - \frac{r}{1-\beta}\frac{d\sigma_\theta^2}{dr} - r\sigma_\theta^2 \frac{d(1-\beta)^{-1}}{dr}\right]}$$

$\beta(r)$ can principally be chosen arbitrarily, thus an exact determination of the the mass is impossible. But if $\sigma_\theta \simeq$ const., $\beta \simeq$ const., then:

$$M(r) = \frac{\sigma_\theta^2 r}{G} \cdot \frac{k-2\beta}{1-\beta}$$

$k \simeq 3$ applies at r_e (Jaffe model) and so in the case of:

isotropic orbits: $\beta = 0 \quad M(r) = 3\frac{\sigma_\theta^2 r}{G}$

tangential orbits: $\beta \to -\infty$: $M(r) = 2\frac{\sigma_\theta^2 r}{G}$

radial orbits: $\beta \to 1$: $M(r) \to \infty$

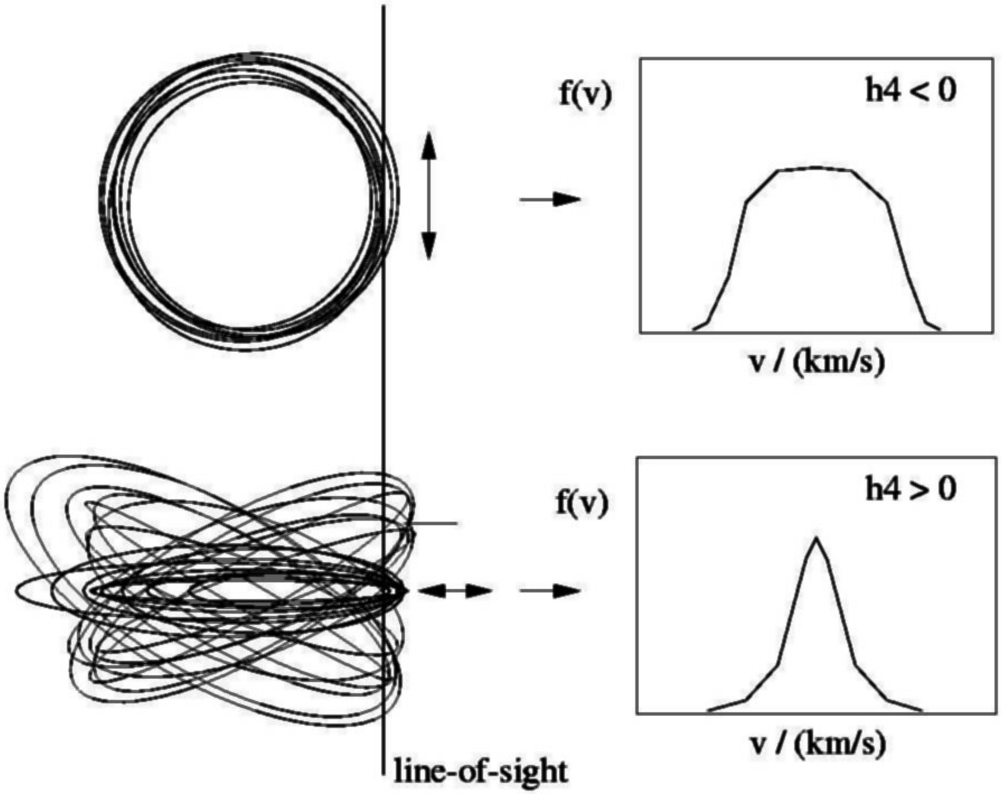
I.e. a lower limit for mass can be determined, but the result is generally not robust enough to conclude about the presence of dark matter.

The β**-ambiguity** can be circumvented using higher moments of the velocity distribution:

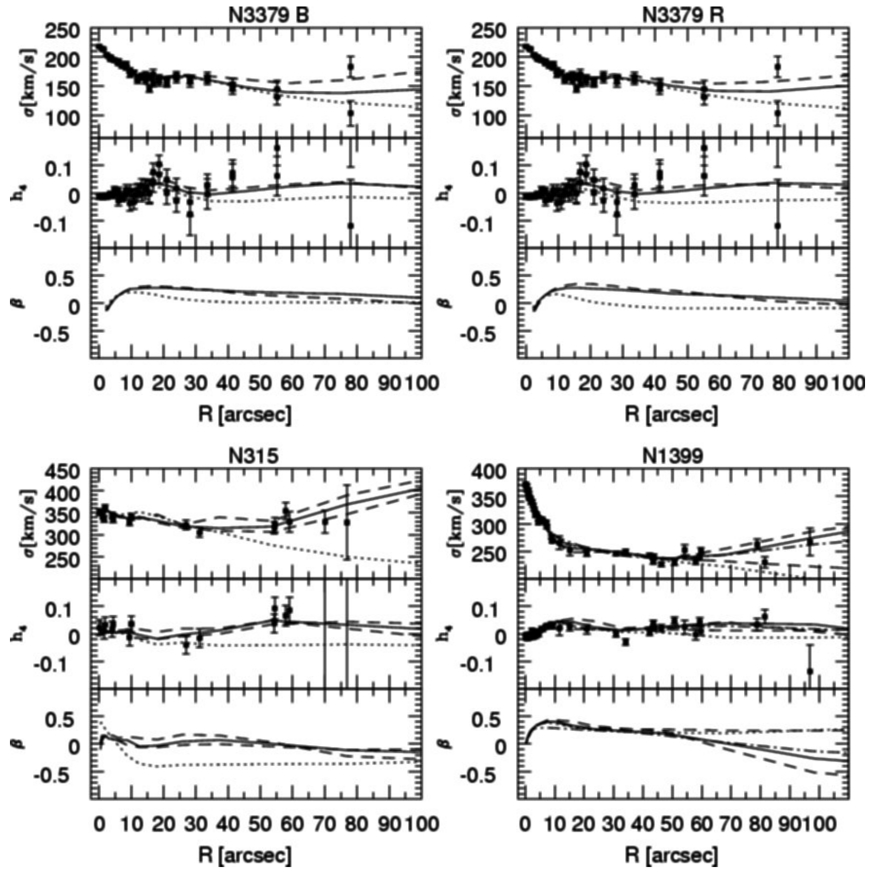
$\rightarrow$ tangential orbits cause a box-shaped velocity profile.

$\rightarrow$ radial orbits cause triangular-shaped velocity profiles.

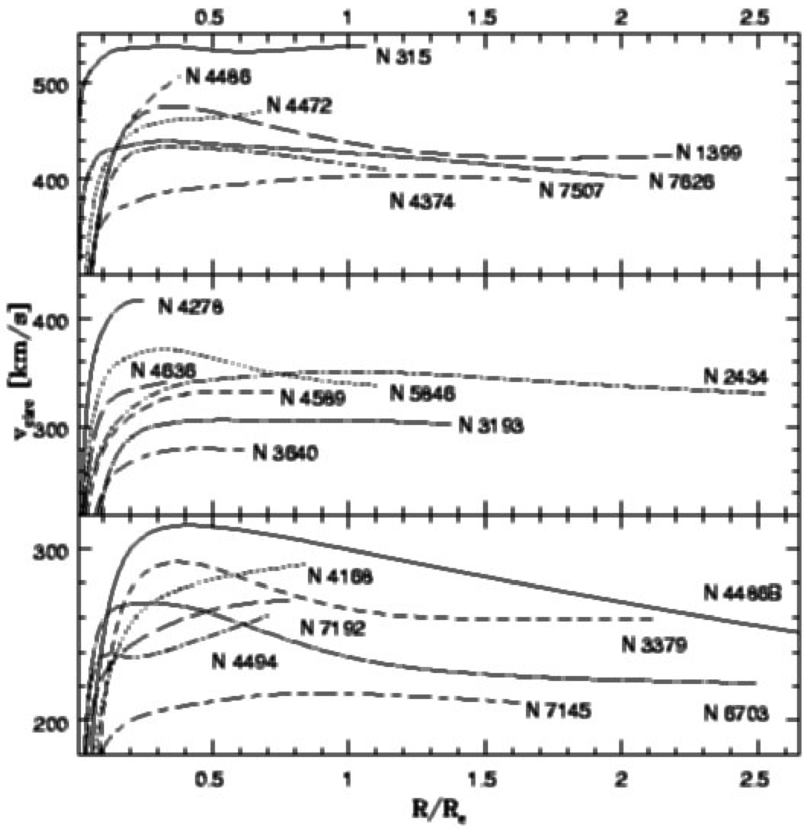
This technique has only recently become available with the invention of new analytic methods and improved instruments.



Influence of dominantly radial or tangential orbits on the line-of-sight velocity distribution in elliptical galaxies.



Profiles of velocity dispersion σ and velocity profile shape h_4 for round elliptical galaxies. The inferred anisotropy is given by β ($\beta > 0$ corresponds to radial anisotropy), from Kronawitter et al. (2000).



Profiles of circular velocities in elliptical galaxies as derived from velocity dispersion profiles and higher moment profiles, see Gerhard et al. (2000).

Using **planetary nebulae** and **globular clusters** the mass of elliptical galaxies can be measured at radii where the stellar density is too low for spectroscopy.

At very large radii the anisotropy is much less important than at small radii ($\frac{M}{L}$ becomes so high, that a variation of β is negligible).

An increase in the velocity dispersion indicates the existence of dark matter. So far, only very few galaxies have been analyzed in this way.

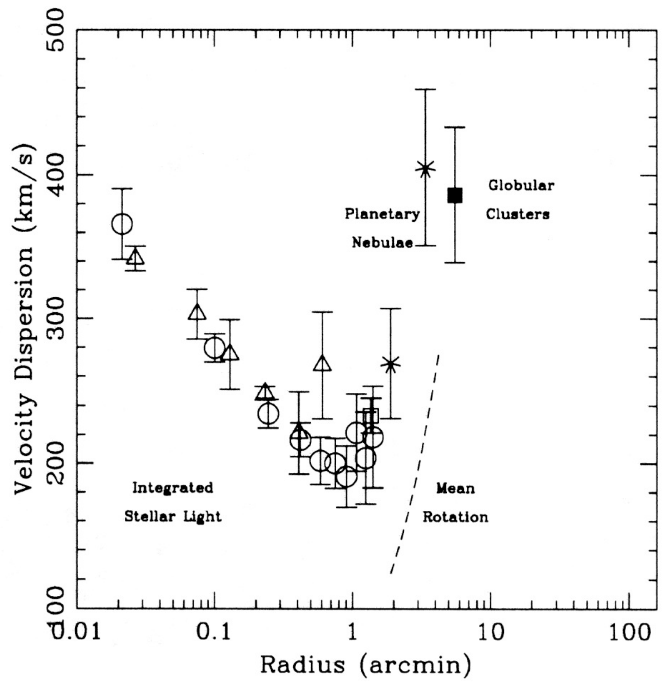


Figure 2. Velocity dispersion σ vs $\log r$ for (i) integrated light of NGC 1399 (open symbols), (ii) the globular cluster in NGC 1399 (filled square) and (iii) our PNs velocity measurements (stars). The dotted curve shows the rotational velocity for our solid body rotation fit to the velocity of the PNs. For comparison, the surface brightness of NGC 1399 at a radius of 3 arcmin is $\mu_B = 24.5$ mag arcsec^{-2}.

see: Arnaboldi, Freeman (1997) *ASP Conf.*, **116**, 54

7.7 Fundamental Plane of Elliptical Galaxies

A comprehensive set of global parameters of elliptical galaxies is:

- The **half light (or effective) radius** r_e
- The **mean surface brightness** Σ_e (or I_e) within r_e
- The **central velocity dispersion** σ_0
- The **luminosity** L
- The **mass** M of the visible matter

The following two relations relate these quantities:

$$\boxed{\Sigma_e = \frac{L/2}{\pi r_e^2}}$$ (Definition of mean surface brightness)

$$\boxed{\frac{M}{r_e} = c\sigma_0^2}$$ (Virial equilibrium)

with the **structure parameter** c which contains all unknown details about the galaxies' structure.

Multiplication yields an expected relation for these parameters:

$$r_e = \left(\frac{c}{2\pi}\right)\left(\frac{M}{L}\right)^{-1}\sigma_0^2\Sigma_e^{-1}$$

Because neither M/L nor c are expected to vary very much, the brackets are nearly constant and imply that ellipticals should define a **plane-like distribution** in the 3-space of their global parameters (r_e, Σ_e, σ_0^2).

Astonishingly, this plane is much better defined than naively expected, with very low dispersion perpendicular to the plane (implying a variance in the product of the brackets less than 10%) and a small but significant tilt (implying small but significant changes in the structure of ellipticals as a function of their luminosity or mass), see Djorgovski & Davis (1987), Dressler et al. (1987).

The observed so-called "**fundamental plane**" relation reads:

$$\boxed{r_e \propto \sigma_0^{1.4}\Sigma_e^{-0.85}}$$

This is consistent with the theoretical expectation, if

$$\left(\frac{2\pi}{c}\right)\left(\frac{M}{L}\right) \propto M^{0.2} \propto L^{0.25}$$

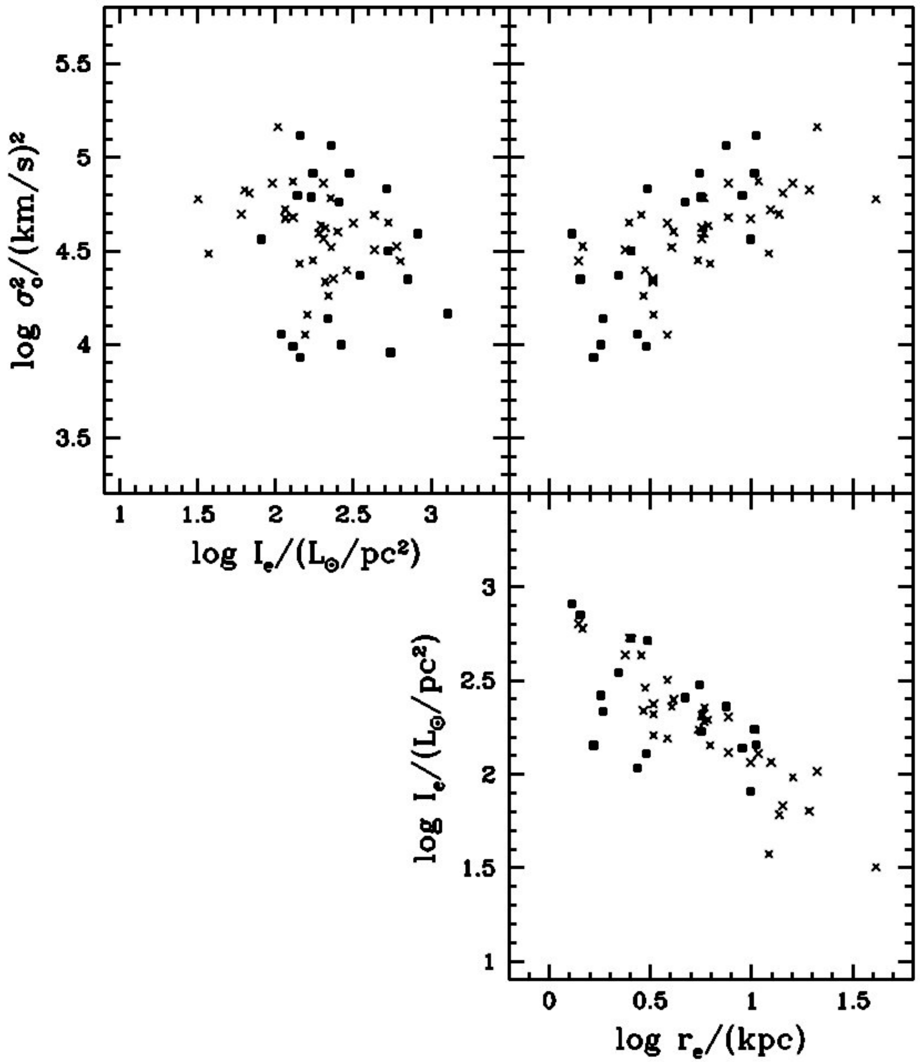
This implies a limitation of the variation of the dynamical structure (σ_0), of the $\frac{M}{L}$ of the stellar population, of the amount of dark matter within r_e, of the slope of the stellar initial mass function, and all other possibly varying parameters.

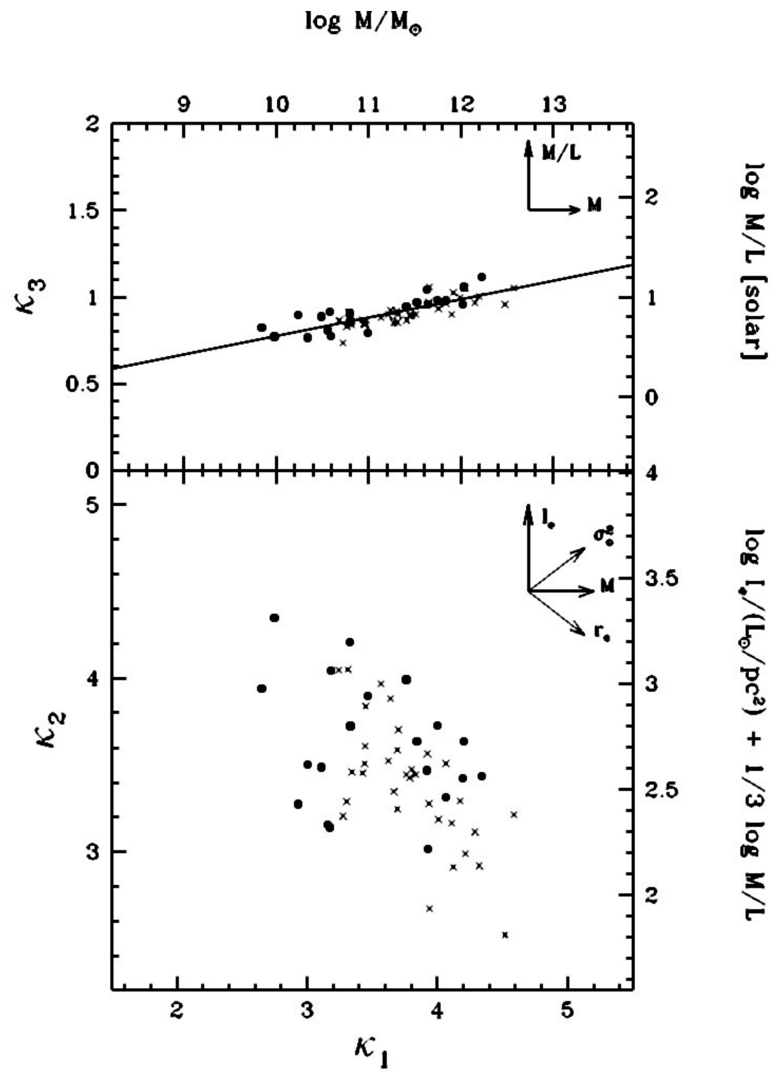
$\Rightarrow$ Even though the kinematics of ellipticals can appear to be highly complicated in detail, the objects must in fact be rather similar with respect to their global structure and their stellar $\frac{M}{L}$!

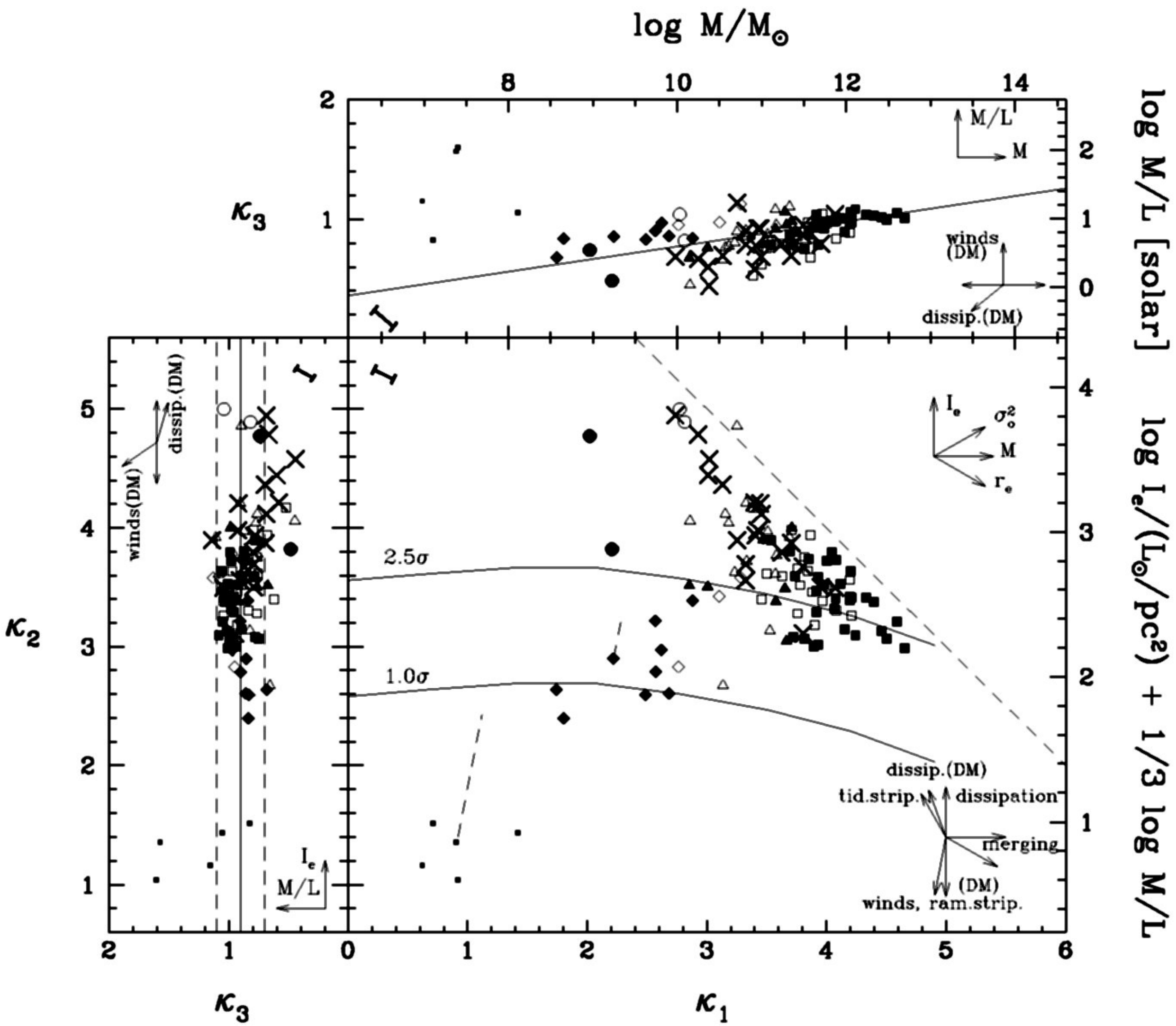
Note: The fundamental plane is an important distance indicator for elliptical galaxies, like the Tully-Fisher relation for spirals. At higher redshifts it is a useful indicator for the evolution of elliptical galaxies.

The fundamental plane can conveniently be visualized in the κ**-parameter space**, using the parameters (Bender, Burstein & Faber (1992, 1993, 1994)):

$$\kappa_1 = \frac{\log(\sigma^2 r_e)}{\sqrt{2}} \propto \log M, \quad \kappa_2 = \frac{\log(\sigma^2 \Sigma_e^2 / r_e)}{\sqrt{6}} \propto \log \Sigma_e \left(\frac{M}{L}\right)^{1/3}, \quad \kappa_3 = \frac{\log(\sigma^2 / \Sigma_e / r_e)}{\sqrt{3}} \propto \log\left(\frac{M}{L}\right)$$


log σ$_o^2$/(km/s)^2
log I$_e$/(L$_\odot$/pc^2)
log I$_e$/(L$_\odot$/pc^2)
log r$_e$/(kpc)


log M/M$_\odot$
M/L
M
log M/L [solar]
I$_e$
σ$_o^2$
M
r$_e$
log I$_e$/(L$_\odot$/pc^2) + 1/3 log M/L
κ$_1$
κ$_2$
κ$_3$



E: squares
S0: crosses
dE: diamonds
dSph: small squares
compact E: circles

curved lines indicate surface density of cold dark matter halos if collapsed by a factor 10

arrows indicate major physical processes acting on galaxies

Bender, Burstein, Faber (1992,93,94)

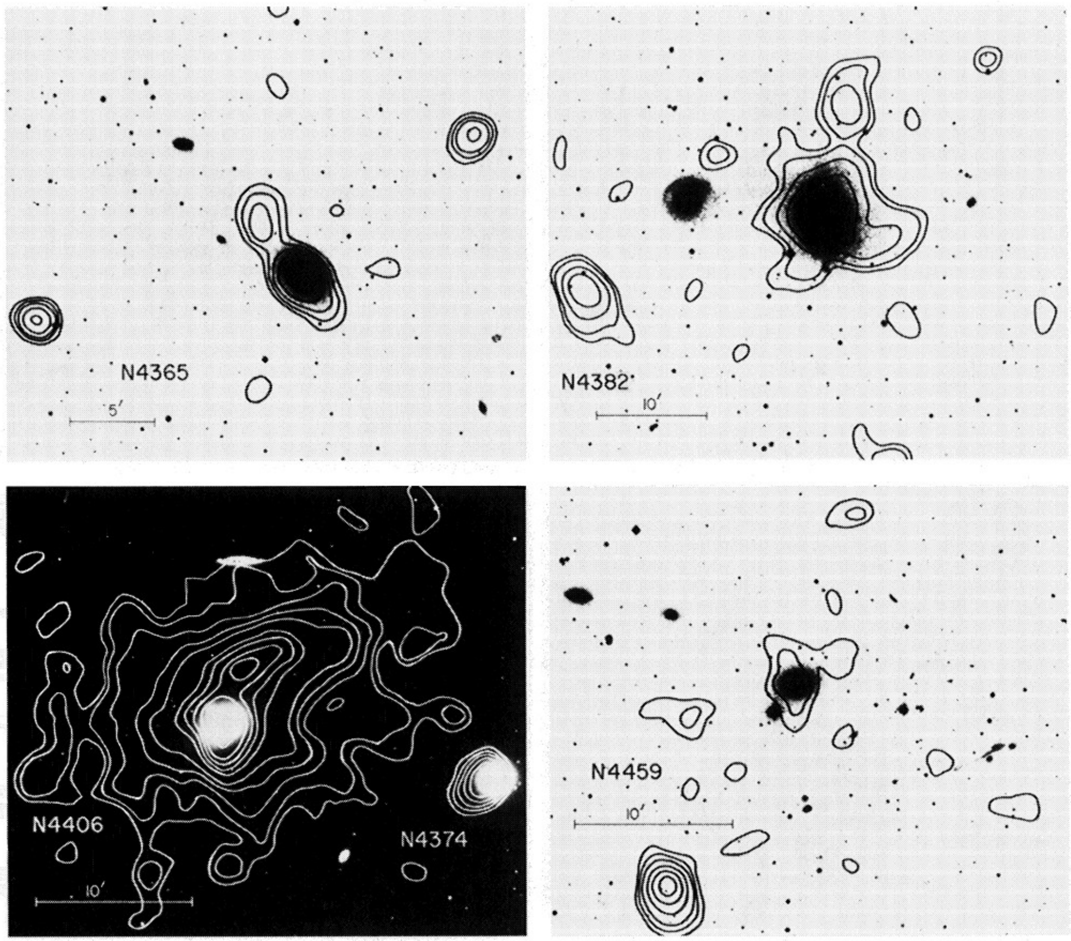
7.8 Hot X-Ray Gas around Elliptical Galaxies

7.8.1 Properties of the Gas Haloes

Massive elliptical galaxies are often surrounded by **X-ray coronae**. The spectra show that the emission can be explained by **thermal Bremsstrahlung** of a **thin hot gas** with the following properties:

$$
\begin{aligned}
T &\simeq 10^7\text{K} \\
n_e &\simeq 10^{-1}\ldots 10^{-4}\text{cm}^{-3} \\
M_\text{gas} &\simeq 10^8\ldots 10^{10}M_\odot \\
\frac{dL_x}{dV} &\simeq 2.4\cdot 10^{-27}T^{1/2}n_e^2\frac{\text{erg}}{\text{s cm}^3} \\
L_x &\simeq 10^{40}\ldots 10^{42}\frac{\text{erg}}{\text{s}} \\
\tau_\text{cool} &\simeq 10^8\ldots 10^9\text{yrs}
\end{aligned}
$$

- Since $\tau_{cool} \gg t_{gal}$ heating of the gas is needed because of the short cooling times, SN Ia are likely to provide the energy. Gas may flow out of the galaxy but is replaced by the mass loss of the stars.
- The dispersion of the X-ray luminosity may be as high as a factor of ~ 50 for a given optical luminosity (not yet understood).
- X-ray haloes might not be stationary phenomena, yet stationarity seems plausible due to the frequency and the measured temperatures of these objects.



see: Forman, Jones et al. (1985) *ApJ*, **293**, 102

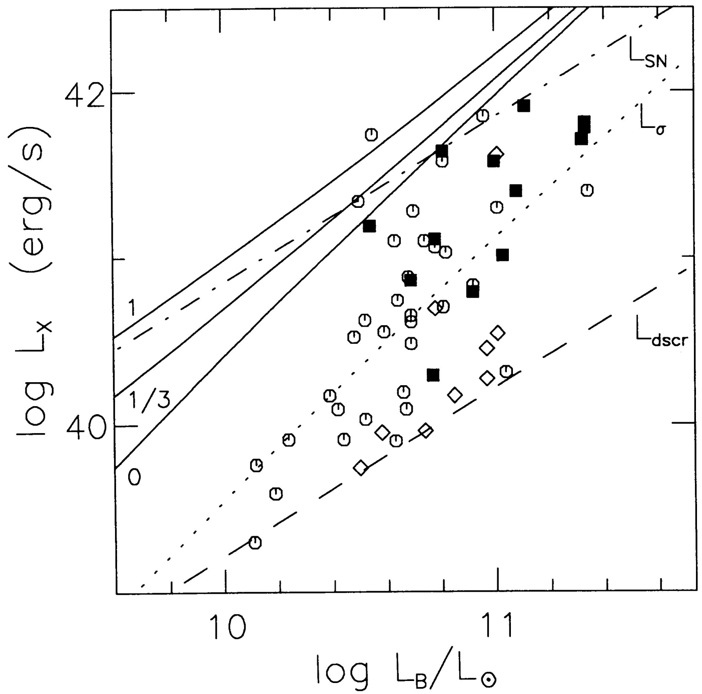


FIG. 1.—The X-ray vs. blue luminosity for early-type galaxies from Canizares, Fabbiano, & Trinchieri (1987). Filled squares and open diamonds refer to galaxies with boxy and disklike isophotal shapes, respectively. Open circles represent galaxies for which this morphological detail is unknown. The dot-dashed line shows the power L_{SN} generated by the SN I heating at the standard rate ($\vartheta_{SN} = 1$). The heating due to stellar motions L_σ is shown by the dotted line, while L_{dscr} represents the expected contribution from discrete stellar X-ray sources (*dashed line*). Finally, the solid lines represent the expected X-ray luminosity of steady state cooling flow models ($L_{inflow} = L_{SN} + L_\sigma + L^+_{grav}$), for the indicated values of the SN parameter ϑ_{SN}.

from Ciotti et al. (1991) *ApJ*, **376**, 380

7.8.2 Mass Determination using X-Ray Gas

Assuming the gas to be **spherically distributed** and in **dynamical equilibrium**, we obtain from pressure equilibrium of a small volume element at radius r:

$$dp \cdot dA = -\frac{GM(<r)}{r^2}\rho \cdot dA \cdot dr$$
$$\Rightarrow \frac{dp}{dr} = -\frac{GM(<r)}{r^2}\rho$$

Inserting the ideal gas equation $p = \frac{\rho}{\mu} k_B T$ yields:

$$\boxed{M(<r) = r\frac{k_B T}{G\mu}\left(-\frac{d\ln\rho}{d\ln r} - \frac{d\ln T}{d\ln r}\right)}$$

This is equivalent to the equation we derived for spherical stellar systems (except that the latter contain an extra anisotropy term):

$$M(< r) = \frac{\overline{v_r^2}}{G} r \left(-\frac{d \ln n}{d \ln r} - \frac{d \ln \overline{v_r^2}}{d \ln r} - 2\beta \right)$$

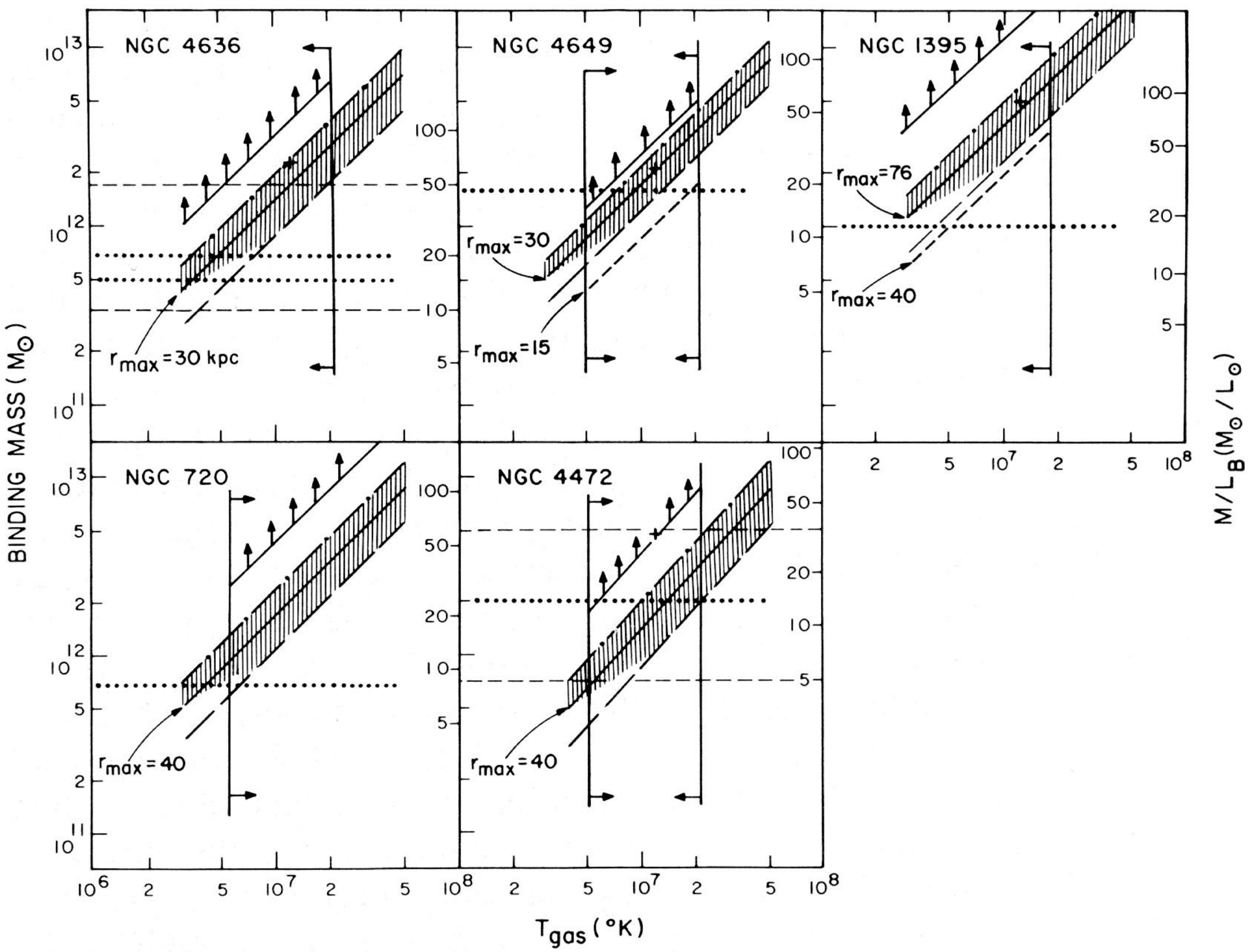
with:

$$T = \frac{\mu \overline{v_r^2}}{k_B} \simeq 40 \left(\frac{\overline{v_r^2}}{(km/s)^2} \right) K$$

To estimate the mass which is needed to keep the hot X-ray gas in dynamical equilibrium one can use that $\left(-\frac{d \ln \rho}{d \ln r} - \frac{d \ln T}{d \ln r} \right) \simeq 2$ in most cases and thus obtains:

$$M(< r) \simeq 4 \cdot 10^{11} M_\odot \cdot \left(\frac{T}{10^7 \text{K}} \right) \cdot \left(\frac{r}{10 \text{kpc}} \right)$$

This yields similar total masses as the virial approximation using planetary nebulae and globular cluster dynamics: $\Rightarrow$ dark matter also exists in ellipticals.

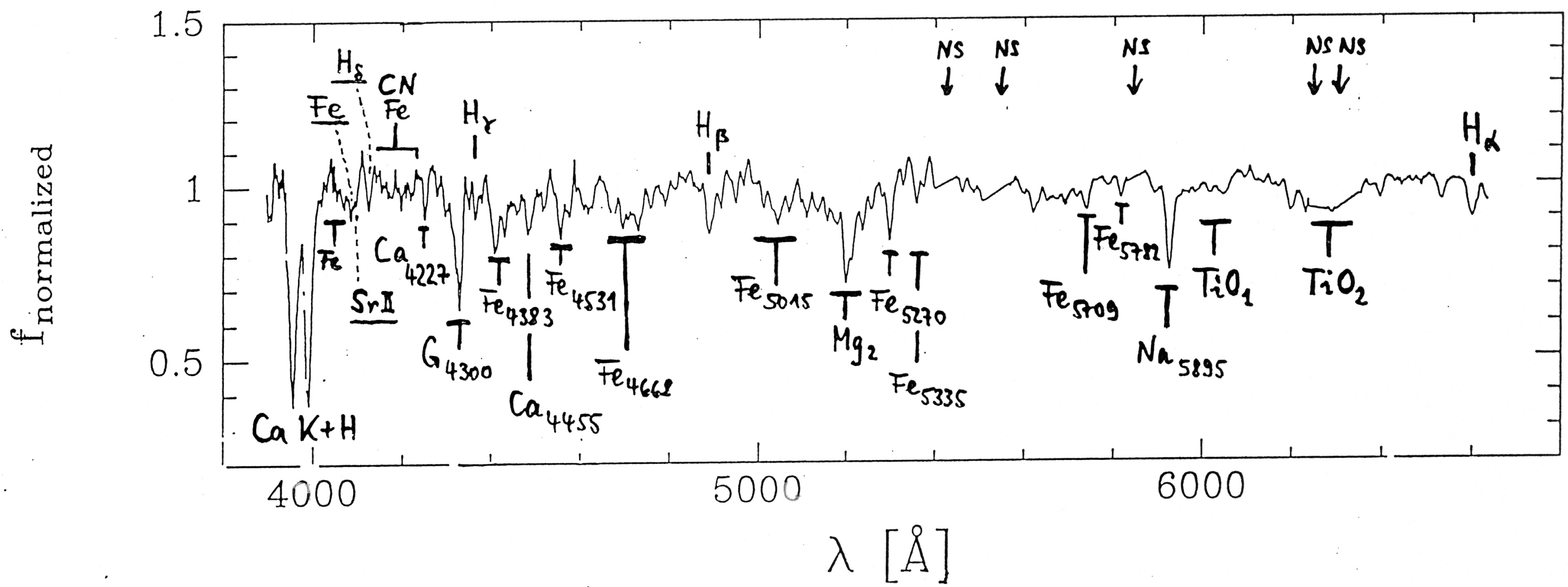


see: Fabbiano *ARAA*, **27**, 87

7.9 Stellar Populations of Elliptical Galaxies

- Elliptical galaxies and bulges appear red and show spectra that are dominated by red giants. $\Rightarrow$ No massive young stars $\Rightarrow$ ellipticals are > 5 Gyrs old.
- A determination of the exact age and metallicity distribution is difficult because of the so called **age-metallicity degeneracy**.
- Detailed analyses indicate that the differences between individual ellipticals are dominated by metallicity differences. These are followed by age differences and variations in the element ratios (but this is still much under debate!)
- Colour- and metallicity indices are usually tightly correlated.
- Strong global correlations between the Mg-absorption strength at 5180Å (Mg_2), colours and velocity dispersions of ellipticals exist.
- Mg and other α elements are likely over-abundant with respect to Fe. ($\Rightarrow$ short star formation timescales?).

Typical spectrum of an elliptical in the optical (continuum removed!):



Spectral appearance similar to K0 giants.

HRD of Elliptical Galaxies: Age-Metallicity Degeneracy (Worthey 1992)

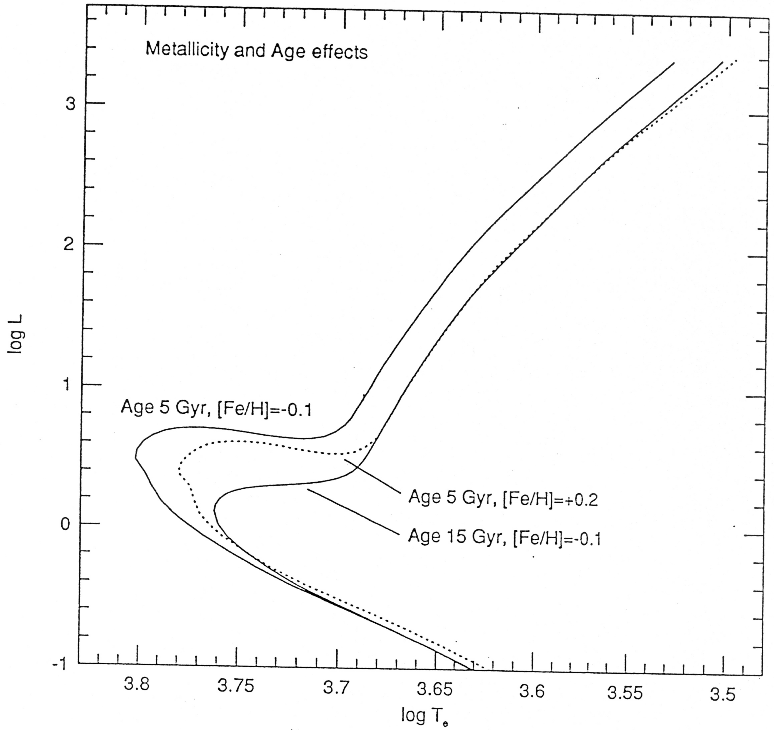


Fig. 1.1 — Three isochrones in log L, log T_e space. The hottest isochrone has age = 5 Gyr, and [Fe/H]= −0.1. The other two are variations on that theme. The dashed isochrone has a factor of two more heavy elements, the other solid isochrone has a factor of three greater age. Note that the RGBs of these two isochrones are nearly identical. Only evolution through the tip of the RGB is shown here for clarity. All evolutionary stages are included in the fluxes shown in Figure 1.2.

Note: A factor 3 in age can be compensated by a factor 2 in metalicity!

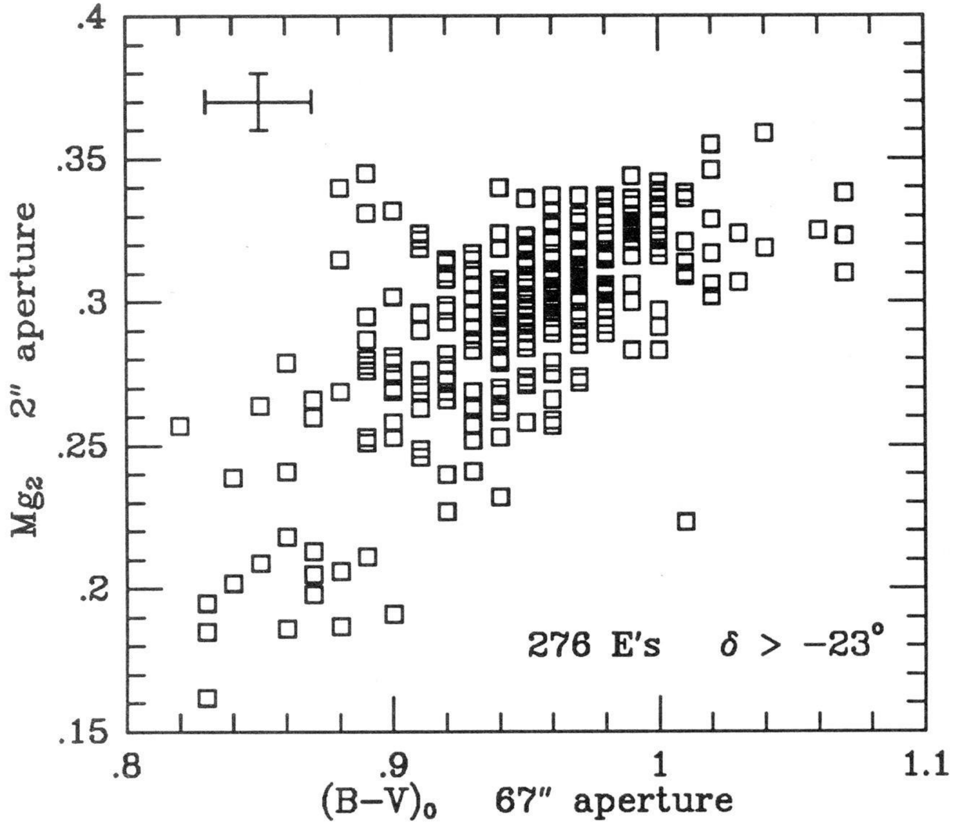
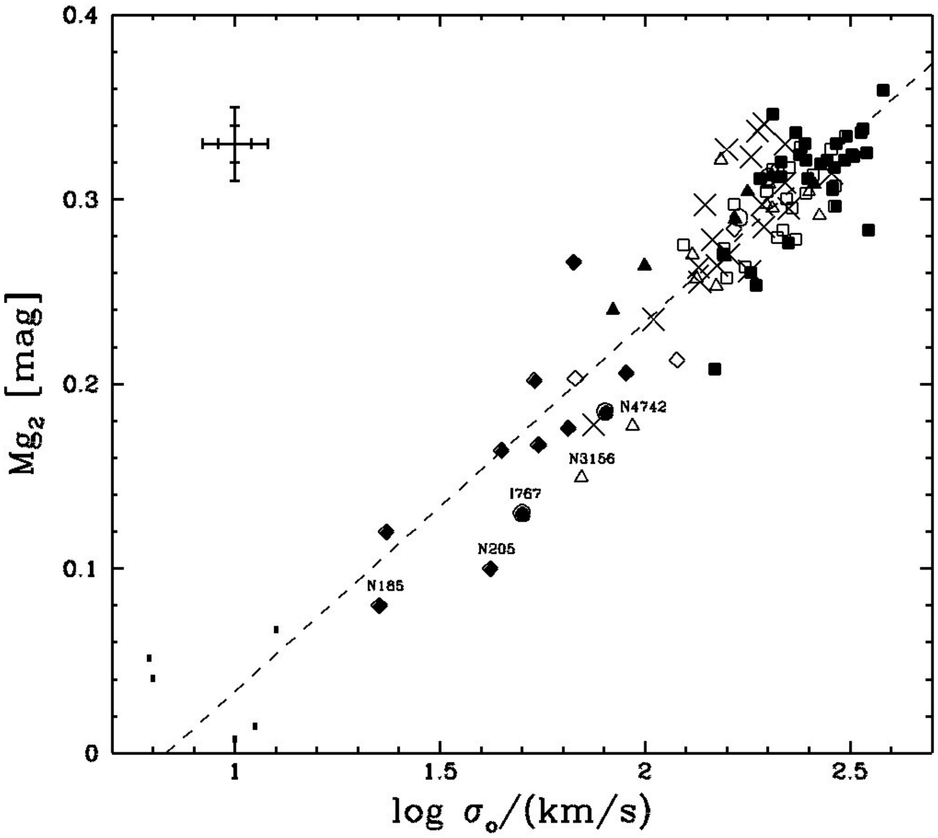


Fig. 2. (top): The relationship between $(B-V)_0$ color and the Mg_2 index for 276 ellipticals north of -23° declination.
Fig. 2. (bottom): The histogram of $(B-V)_0$ residuals from the $(B-V)_0$–Mg_2 relationship, compared to a normal distribution of points with a standard deviation (sd) of 0.023 mag (=0.02 mag for $(B-V)_0$). Observational errors alone appear to account for the scatter in this relationship.

from Burstein (1988) in Kron, Renzini *Towards understanding galaxies at high redshift*



Relationship between Mg_2 and σ_o for ellipticals (squares), S0 bulges (crosses), dwarf ellipticals (diamonds and small squares), special objects (open squares).
(Bender, IAU Symp 149)

Possible origin of the Mg-σ Relation of Elliptical Galaxies

The velocity dispersion σ is an indicator for potential depth $\sigma^2 \sim \frac{GM}{r_e}$, while the Mg absorption may indicate the mean metallicity of the stars (this is still under debate!).

If the potential is deeper, a larger fraction of the gas can be processed into stars before it may be lost from the galaxy in a Supernova-driven wind. The higher the ratio of stellar mass to gas mass is, the higher is the final metallicity of the gas and the average metallicity of the stars (see Section on Chemical Evolution).

The age-metallicity degeneracy can be resolved by the combined observation of an **age-dependent** absorption line (e.g. H_β) and a **metallicity-dependent** absorption line (e.g. Mg or Fe).

7.10 Why do Elliptical Galaxies exist?

In the most simple picture of galaxy formation, the major difference between the formation of ellipticals and spirals is depending on the relative time scales of **collapse** vs. **star formation**:

- State of equilibrium is reached before star formation starts:
  $\longrightarrow$ a gas cloud collapses until it forms a rotating disk
  $\longrightarrow$ star formation sets in
  $\longrightarrow$ a rotating stellar disk is produced
- Star formation starts and is completed during the collapse:
  $\longrightarrow$ a clumpy gas cloud starts to collapse
  $\longrightarrow$ star formation starts in gas clumps (also triggered by clump-clump collisions)
  $\longrightarrow$ the gas clumps are transformed into star clumps (the remaining gas is expelled)
  $\longrightarrow$ due to the clumpy collapse strong fluctuations in the potential are created
  $\longrightarrow$ the stellar clumps are dissolved and stars scattered at these fluctuations
  $\longrightarrow$ the energy of the stars changes and is redistributed in a stochastic way
  $\longrightarrow$ the galaxy becomes nearly spherical.

Two-body relaxation does not play an important role in bulges and elliptical galaxies. Nevertheless as a result of the clumpy collapse a **collective relaxation process** does occur.

$\Rightarrow$ This is called violent relaxation (Lynden-Bell (1967)).

The energy E of stars changes according to:

$$\frac{\partial E}{\partial t} = \frac{\partial \Phi}{\partial t}$$

as the potential Φ changes on a typical timescale $t_{fluc} \sim (G\rho)^{-1/2}$, which corresponds to the typical timescale in which, e.g. two clumps or two galaxies merge. This means that violent relaxation occurs within a few collapse timescales. It also implies that it is faster at small radii and takes longer at large radii.

Complete violent relaxation would result in an **isothermal Maxwell distribution**. Observations of galaxies and galaxy clusters, as well as numerical simulations show that violent relaxation is not complete (i.e. the observed systems do not have an isothermal Maxwell distribution). Therefore the velocity distribution in galaxies and galaxy clusters **are unlikely to be isotropic** (see above).

The actual formation of ellipticals is very likely more complicated than in the simple picture of a clumpy collapse of a gas cloud. Galaxies and galaxy clusters form out of collapsing **primordial density fluctuations**. But similar to the simple picture, the collapse must have been very clumpy and the physics of relaxation is very likely the same.

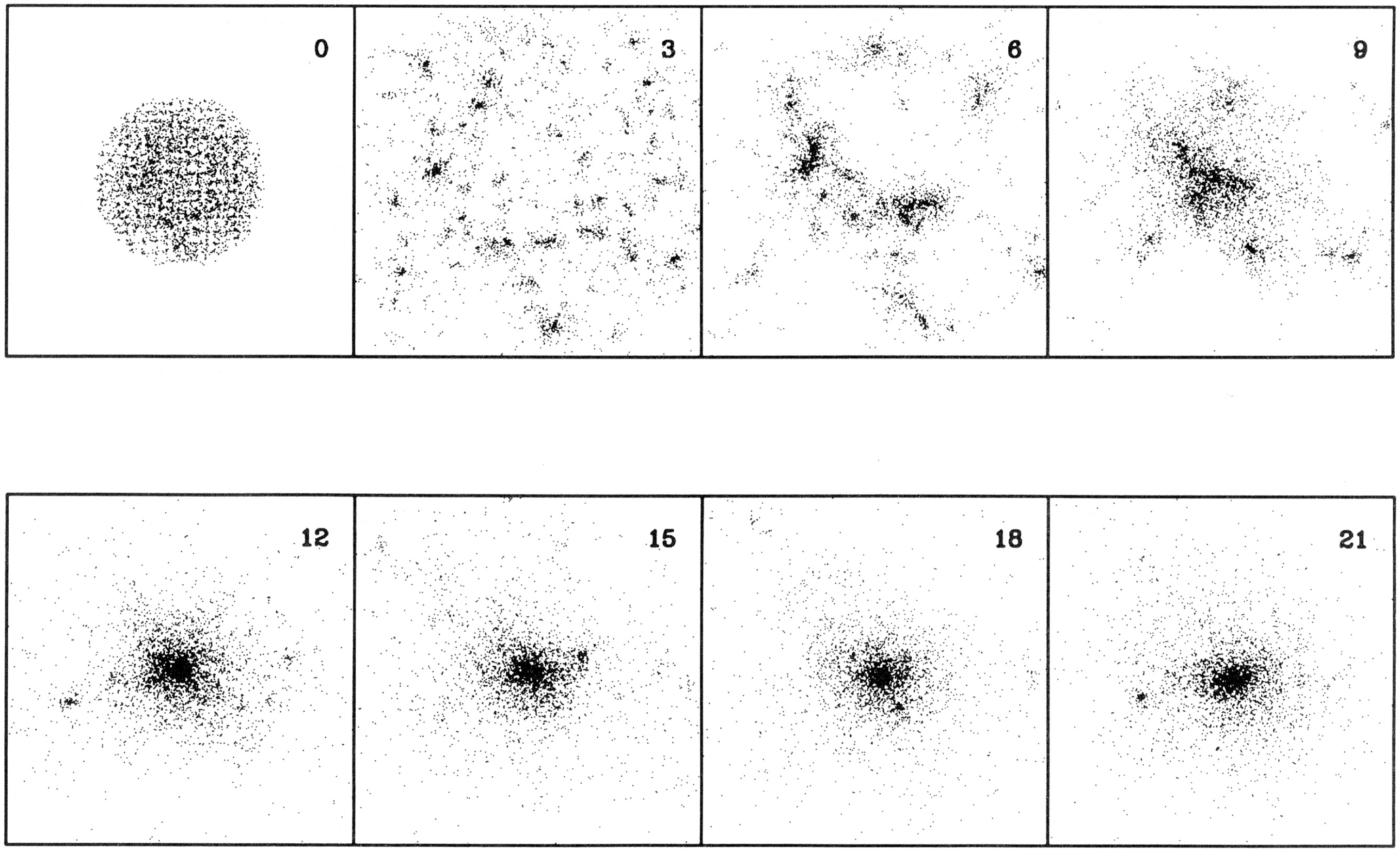
Violent Relaxation of a Growing Density Fluctuation

The following simulations, which are both valid for dark matter and/or stars, have been carried out by N. Katz ((1991) *ApJ*, **368**, 325).

The numbers indicate the time in units of the crossing time of the final state of the collapse.

The two simulations were computed for different spectra of density fluctuations, described by the form of the **power spectrum** $P(k)$ ($k \equiv 2\pi/\lambda$).
(See the chapter on Structure Formation in Cosmology).

$$P(k) \propto k^0 \propto \lambda^0$$



$$P(k) \propto k^{-2.5} \propto \lambda^{2.5}$$