Galaxies, Cosmology and Dark Matter

Lecture given by
Ralf Bender
USM

Script by:
Christine Botzler, Armin Gabasch,
Georg Feulner, Jan Snigula

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Chapter 15

Structure Formation in the Universe

15.1 The Linear Growth of Structure

We begin by writing down the equations of hydrodynamics in an expanding universe (the *equation of continuity*, *Euler's equation* and *Poisson's equation*, respectively)

$$\begin{aligned} \frac{\partial \varrho}{\partial t} + \nabla_x \cdot (\varrho \boldsymbol{v}) &= 0 \\ \frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla_x)\boldsymbol{v} &= -\frac{1}{\varrho}\nabla_x p - \nabla_x \Phi \\ \nabla_x^2 \Phi &= 4\pi G \varrho - \Lambda \end{aligned} \tag{15.1}$$

$\varrho(\boldsymbol{x}, t)$: density
$\boldsymbol{v}(\boldsymbol{x}, t)$: velocity field
$p(\boldsymbol{x}, t)$: pressure
$\Phi(\boldsymbol{x}, t)$: gravitational potential
Λ: cosmological constant

In the following we will neglect the cosmological constant.

Now we will consider small perturbations around the homogeneous solution. The density perturbation is usually written in terms of the **density contrast**

$$\boxed{\varrho\,(\boldsymbol{x}, t) \;\equiv \bar{\varrho}\,(t)\left(1 \;+\; \delta\,(\boldsymbol{x}, t)\right)} \tag{15.2}$$

Furthermore, we will change to co-moving coordinates using $\boldsymbol{x} = R(t)\boldsymbol{r}$. Then the velocity field can be written as

$$\boldsymbol{v} \;=\; \dot{R}\boldsymbol{r} \;+\; R\dot{\boldsymbol{r}} \;\equiv\; \dot{R}\boldsymbol{r} \;+\; \boldsymbol{u} \tag{15.3}$$

where $\dot{R}\boldsymbol{r}$ is the uniform Hubble flow, $\boldsymbol{u}$ is the peculiar velocity field.

The gravitational potential can be written as

$$\Phi \;=\; \Phi_0 \;+\; \phi \tag{15.4}$$

with the unperturbed potential Φ_0 and its perturbation ϕ.

Then the fluid equations (15.1) can be written as

$$\begin{aligned} \frac{\partial \delta}{\partial t} + \frac{1}{R}\nabla_r \cdot \boldsymbol{u} + \frac{1}{R}\nabla_r \cdot \left(\delta \boldsymbol{u}\right) &= 0 \\ \frac{\partial \boldsymbol{u}}{\partial t} + \frac{1}{R}\left(\boldsymbol{u} \cdot \nabla_r\right)\boldsymbol{u} + \frac{\dot{R}}{R}\boldsymbol{u} &= -\frac{1}{R}\frac{1}{\varrho}\nabla_r\, p - \frac{1}{R}\nabla_r\,\phi \\ \nabla_r^2\,\phi &= 4\,\pi\,G\,R^2\,\bar{\varrho}\,\delta \end{aligned} \qquad (15.5)$$

where δ, $\boldsymbol{u}$, and ϕ are functions of the co-moving coordinate $\boldsymbol{r}$ now. Neglecting terms of second order in the perturbed variables simplifies the set of equations:

$$\boxed{\begin{aligned} \frac{\partial \delta}{\partial t} + \frac{1}{R}\nabla_r \cdot \boldsymbol{u} &= 0 \\ \frac{\partial \boldsymbol{u}}{\partial t} + \frac{\dot{R}}{R}\boldsymbol{u} &= -\frac{1}{R}\frac{1}{\bar{\varrho}}\nabla_r\, p - \frac{1}{R}\nabla_r\,\phi \\ \nabla_r^2\,\phi &= 4\,\pi\,G\,R^2\,\bar{\varrho}\,\delta \end{aligned}} \qquad (15.6)$$

These equations govern the dynamics of small density fluctuations in a perfect fluid for an expanding background cosmology without cosmological constant.

In the linear regime, density fluctuations on different scales evolve independently. Thus it is useful to define Fourier–transformed quantities in the following way:

$$\begin{aligned}\delta\left(\boldsymbol{x}, t\right) &= \sum_{\boldsymbol{k}} \delta_{\boldsymbol{k}}\left(t\right) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \\ \delta_{\boldsymbol{k}}\left(t\right) &= \frac{1}{V} \int \delta\left(\boldsymbol{x}, t\right) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \, d^3x\end{aligned}$$

If the pressure p is a function of the density ϱ alone, it can be written as

$$p = p\left(\bar{\varrho}\right) + \frac{dp}{du} \bar{\varrho}\, \delta = p\left(\bar{\varrho}\right) + c_s^2 \, \bar{\varrho}\, \delta$$

with the sound speed c_s.

Then the linearized equations (15.6) can be combined to give the **linear perturbation equation**

$$\boxed{\frac{d^2\delta_{\boldsymbol{k}}}{dt^2} + 2\,\frac{\dot{R}}{R}\,\frac{d\delta_{\boldsymbol{k}}}{dt} - \left(4\pi G\bar{\varrho} - \left(\frac{c_s^2 k}{R}\right)^2\right)\delta_{\boldsymbol{k}} = 0} \qquad (15.7)$$

which is a wave equation for the perturbation $\delta_{\boldsymbol{k}}$.

The time evolution of the scale factor $R(t)$ is governed by the Friedmann equations, e.g. the solutions for an Einstein–de-Sitter universe and an empty universe are

$$\begin{aligned} \Omega_m &= 1 \quad \Lambda = 0\text{:} \quad R = \left(\tfrac{3}{2}\,H_0\,t\right)^{2/3} \\ \Omega_m &\to 0 \quad \Lambda = 0\text{:} \quad R = H_0\,t \end{aligned}$$

We consider growing fluctuations for which the gravitational attraction is much stronger than the pressure force, i.e.

$$4\pi G\bar{\varrho} \gg \left(\frac{c_s^2 k}{R}\right)^2$$

In these cases the linear perturbation equation can be written as

$$\frac{d^2\delta_{\boldsymbol{k}}}{dt^2} + \frac{4}{3\,t}\frac{d\delta_{\boldsymbol{k}}}{dt} - \frac{2}{3\,t^2}\delta_{\boldsymbol{k}} = 0 \qquad (\Omega = 1, \Lambda = 0)$$
$$\frac{d^2\delta_{\boldsymbol{k}}}{dt^2} + \frac{2}{t}\frac{d\delta_{\boldsymbol{k}}}{dt} = 0 \qquad (\Omega \to 0, \Lambda = 0)$$

with the non-decaying solutions

$$\delta_{\boldsymbol{k}}^{+} \propto t^{2/3} \propto R \propto (1+z)^{-1} \quad (\Omega = 1, \Lambda = 0)$$
$$\delta_{\boldsymbol{k}}^{+} = \text{const.} \quad (\Omega \to 0, \Lambda = 0)$$

These solutions are valid in the matter–dominated epoch (i.e. $z < 10^4$).

For adiabatic fluctuations and strong coupling between photons and baryons ($z > 1500$), the perturbations of the **baryonic density** obey the relation

$$\delta_B \;=\; \frac{\Delta\varrho_B}{\bar{\varrho}_B} \;\simeq\; 3\,\frac{\Delta T}{T}$$

Thus at the time of recombination ($z \simeq 1500$), we have

$$\boxed{\frac{\Delta T}{T} \;\simeq\; 10^{-5} \quad\Rightarrow\quad \delta_B \;<\; 5 \cdot 10^{-5}}$$

and thus we would expect

$$\boxed{\delta_B\,(t = t_0) \;<\; 0.1}$$

for the amplitude of baryonic fluctuations today, **in contrast to the large inhomogeneities observed in the local universe!**

We need dark matter to form the structures observed today!

Structure Formation with Dark Matter

In this case we have to consider the wave equations for the fluctuations in the baryonic and the dark matter component. If the dark matter is non-baryonic, then the fluctuations in the two components are described by two wave equations, coupled by the gravitational interaction only:

$$\frac{d^2\delta_B}{dt^2} + 2\,\frac{\dot{R}}{R}\,\frac{d\delta_B}{dt} = A\,\bar{\varrho}_B\,\delta_B + A\,\bar{\varrho}_{DM}\,\delta_{DM} \qquad (15.8)$$

$$\frac{d^2\delta_{DM}}{dt^2} + 2\,\frac{\dot{R}}{R}\,\frac{d\delta_{DM}}{dt} = A\,\bar{\varrho}_{DM}\,\delta_{DM} + A\,\bar{\varrho}_B\,\delta_B \qquad (15.9)$$

with $A = 4\pi G$ for the matter–dominated case, and $A = 32\pi G/3$ for the radiation–dominated case. In the approximation

$$\Omega_B \ll \Omega_{DM} \simeq 1$$

the second equation (15.9) reduces to a simple wave equation with the solution

$$\delta_{DM}(z) \;=\; \delta_{DM}(0)\,(1+z)^{-1} \;=\; \delta_{DM}(0)\,R$$

Inserting this solution into the wave equation (15.8) for the baryonic component yields

$$\frac{d^2\delta_B}{dt^2} \;+\; 2\,\frac{\dot{R}}{R}\,\frac{d\delta_B}{dt} \;=\; A\,\bar{\varrho}_{DM}\,\delta_{DM}(0)\,R$$

(using $\bar{\varrho}_B \ll \bar{\varrho}_{DM}$). Furthermore, for a flat universe we have

$$1 \;\simeq\; \Omega_{DM} \;=\; \frac{8\pi G\bar{\varrho}_{DM}}{3H_0^2} \quad \text{and} \quad R \;=\; \left(\frac{3}{2}\,H_0\,t\right)^{2/3}$$

and thus the differential equation for the baryonic fluctuations can be written as

$$R^{3/2}\,\frac{d}{dR}\left(R^{-1/2}\,\frac{d\delta_B}{dR}\right) \;+\; 2\,\frac{d\delta_B}{dR} \;=\; \frac{3}{2}\,\delta_{DM}(0)$$

with the solution

$$\begin{aligned}\delta_B &= \delta_{DM}(0)\,(R + \kappa) \\ &= \delta_{DM}(z)\,(1+z)\left(\frac{1}{1+z} + \kappa\right)\end{aligned}$$

($\kappa = \mathrm{const.}$). One interesting solution can be obtained by setting

$$\kappa \equiv -\frac{1}{1+z_N},$$

yielding

$$\boxed{\delta_B(z) = \delta_{DM}(z)\left(1 - \frac{1+z}{1+z_N}\right)}$$

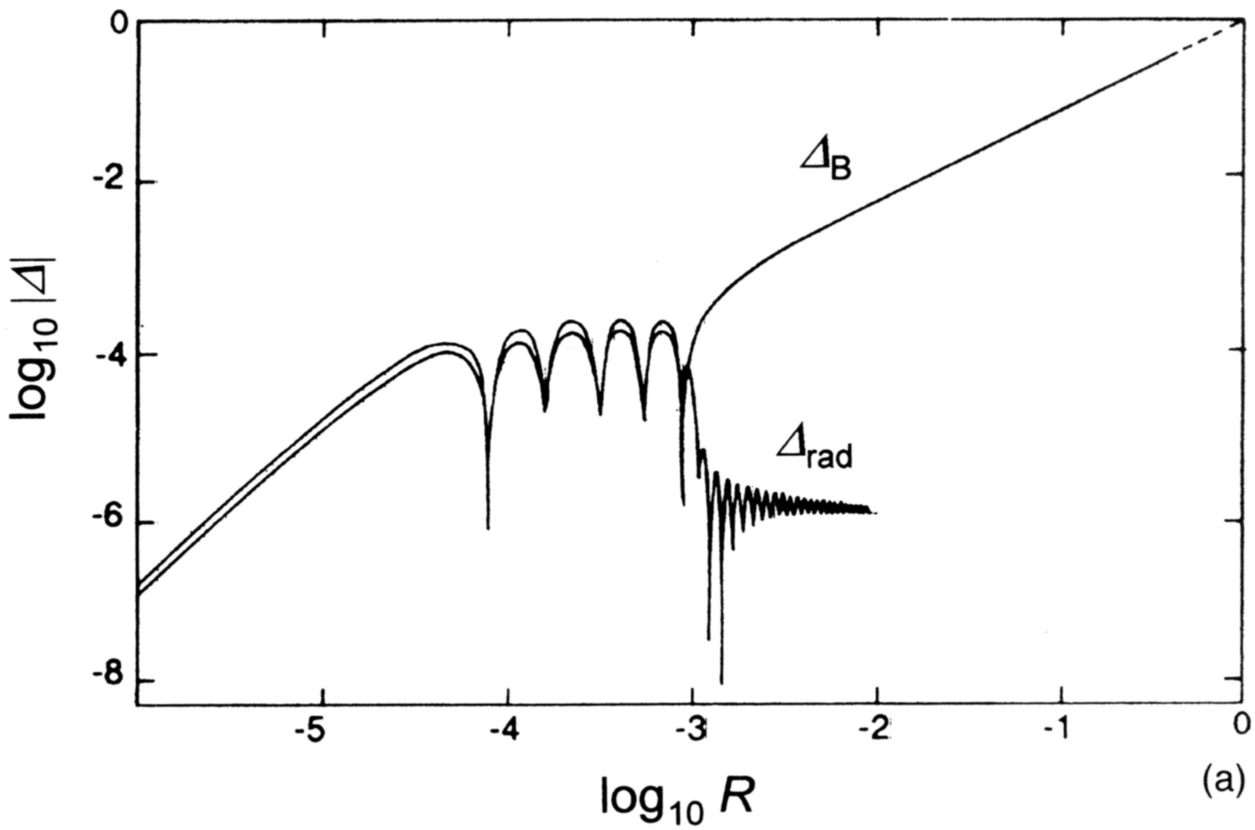
This means that

$$
\begin{aligned}
z \;\rightarrow\; z_N \quad &\Rightarrow \quad \delta_B \;\rightarrow\; 0 \\
z \;\ll\; z_N \quad &\Rightarrow \quad \delta_B \;\simeq\; \delta_{DM}
\end{aligned}
$$

Thus the baryonic fluctuations can be very small at a redshift of, say, $z_N \approx 1000$, whereas the fluctuations in the dark matter component have a finite amplitude at this time. At $z \ll z_N$, the baryonic fluctuations have the same amplitude as the dark matter perturbations.

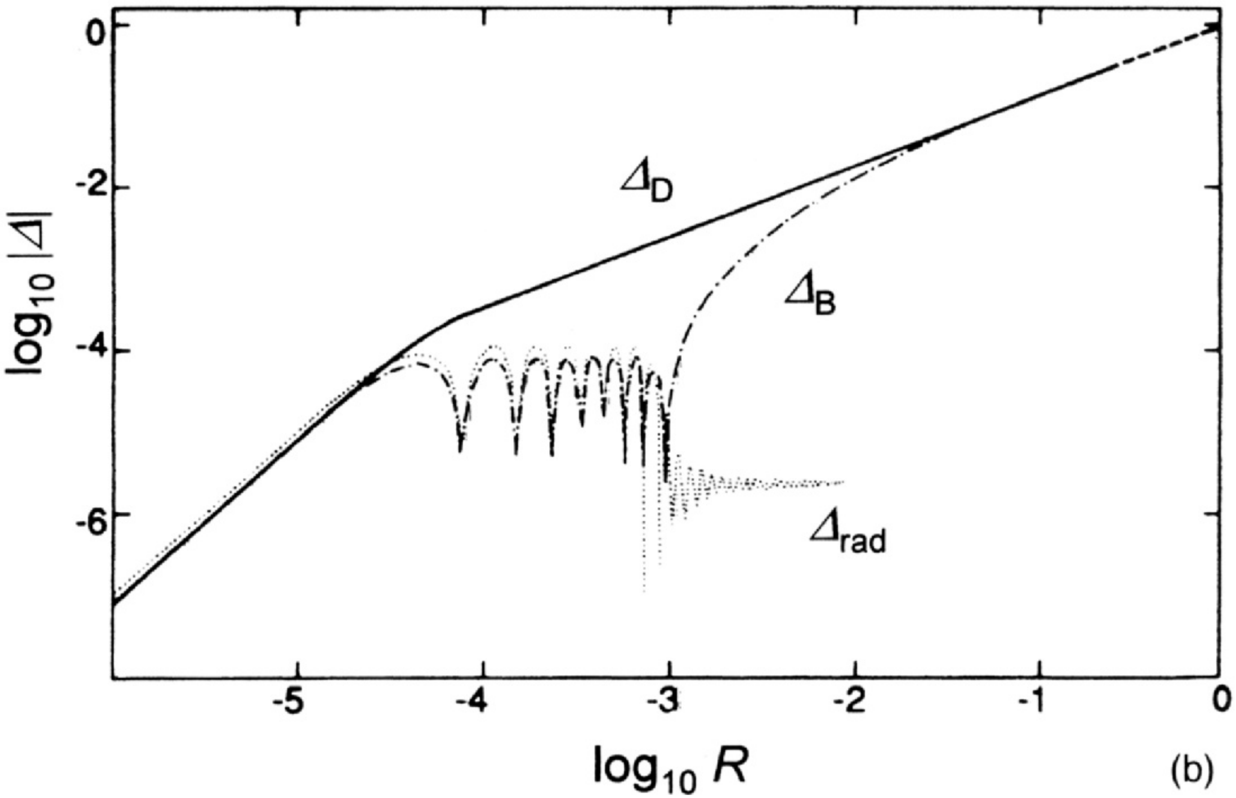
Dark matter makes galaxy formation possible!

The evolution of baryonic and radiative perturbations



(From Longair, *Galaxy Formation*, Springer-Verlag 1998)

The evolution of baryonic and dark matter perturbations



(From Longair, *Galaxy Formation*, Springer-Verlag 1998)

15.2 Non-linear Growth: The Spherical Collapse Model

So far, we have only discussed the linear regime of structure formation in which the density contrast δ is very small: $\delta \ll 1$.

To get some insight into the non-linear stages of the collapse of over-dense regions, we consider the simplified case of a homogeneous over-dense region with spherical geometry. As explained above, the equation of motion for the sphere is identical to the equation of motion for the universe, i.e. Friedmann's equation

$$\dot{R}_s^2 \;=\; \frac{8\pi G \varrho_{s,0}}{3}\,\frac{1}{R_s} \;-\; \frac{c^2}{R_{c,s,0}^2}$$

Using $\Omega_{s,0} = (8\pi G \varrho_{s,0})/(3H_0^2)$ and $H_0^2(\Omega_{s,0} - 1) = c^2/R_{c,s,0}^2$ this can be re-written:

$$\boxed{\dot{R}_s^2 \;=\; \Omega_{s,0}\, H_0^2\,\frac{1}{R_s} \;-\; H_0^2\left(\Omega_{s,0} - 1\right)} \tag{15.10}$$

The solution of this equation can be parametrized as follows

$$\boxed{R_s = a\left(1 - \cos\theta\right) \qquad t = b\left(\theta - \sin\theta\right)}$$

with the constants a and b given by

$$a = \frac{\Omega_{s,0}}{2(\Omega_{s,0} - 1)} \qquad b = \frac{\Omega_{s,0}}{2H_0(\Omega_{s,0} - 1)^{3/2}}$$

For $\Omega_{s,0} = 2$ we thus have

$$a = 1 \qquad \text{and} \qquad b = \frac{1}{H_0}$$

and the radius $R_{s,0} = 1$ is reached at $\theta = \pi/2$, $t_0 = 0.57H_0^{-1}$.

The solution of the spherical collapse model has the following interesting properties:

- The expansion ceases at $\theta = \theta_{max} = \pi$. At this **turn around** we have

$$R_{s,max} = \frac{\Omega_{s,0}}{\Omega_{s,0} - 1} = 2 \quad (\Omega_{s,0} = 2)$$
$$t_{max} = \frac{\pi\Omega_{s,0}}{2H_0(\Omega_{s,0} - 1)^{3/2}} = \frac{\pi}{H_0} \quad (\Omega_{s,0} = 2)$$

Furthermore, the ratio of the sphere's density ϱ_s at t_{max} and the density of a background universe (taken to be $\Omega_m = 1$) can be calculated according to

$$\left(\frac{\varrho_s}{\bar\varrho}\right)_{t_{max}} = \frac{\bar\varrho\Omega_{s,0}}{\bar\varrho}\left(\frac{R_s\left(t_{max}\right)}{R\left(t_{max}\right)}\right)^3 = \frac{\Omega_{s,0}^{-2}\left(\Omega_{s,0} - 1\right)^3}{\left[\left(\frac{3}{2}H_0 t_{max}\right)^{2/3}\right]^{-3}}$$

Inserting the expression for t_{max} yields

$$\boxed{\left(\frac{\varrho_s}{\bar\varrho}\right)_{t_{max}} = \left(\frac{3}{4}\pi\right)^2 \simeq 5.55}$$

Thus, by the time the perturbed sphere has stopped expanding, its density is already 5.55 times greater than the background density.

Note that this argument is valid *both* for a whole universe with $\Omega_m > 1$ *and* for an over-dense region within a universe with $\Omega_m = 1$.

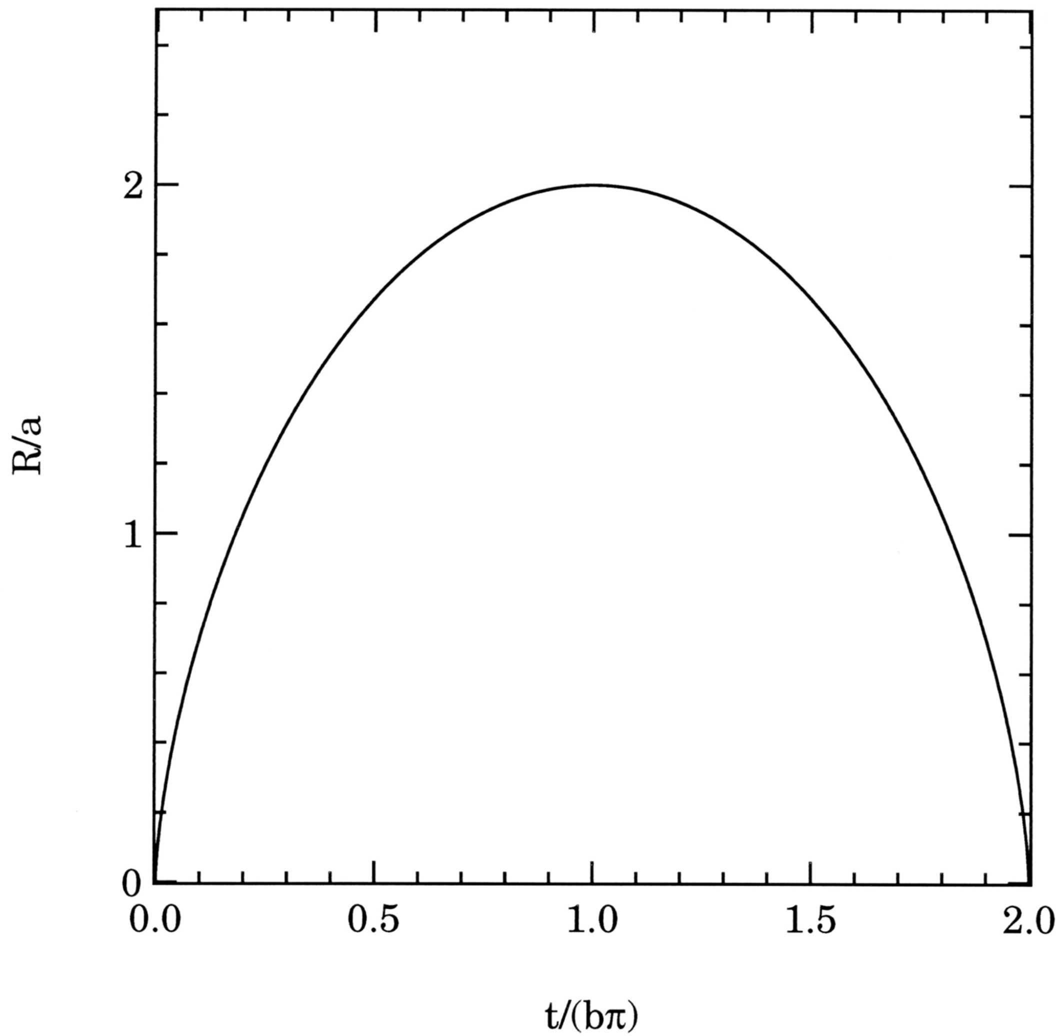
Note also that the time t_0 can be chosen arbitrarily, and the calculation is valid at all times. This means that an over-dense spherical region always has a density 5.55 times higher than the background at turn around.

- Neglecting the growing pressure, the perturbed region collapses to a point after $t = 2t_{max}$. If we assume that this is equal to the formation epoch for the virialized object (e.g. the galaxy), then the redshifts z_{max} of the turn around and z_{form} of the formation of the galaxy are related by

$$\frac{1 + z_{max}}{1 + z_{form}} = \frac{R\,(2t_{max})}{R\,(t_{max})} = 2^{2/3} \simeq 1.59 \qquad (\Omega_m = 1)$$

E.g.: $z_{max} \simeq 10 \quad \Rightarrow \quad z_{form} \simeq 6$

The evolution of the radius of the perturbation in the spherical collapse model



- In reality, density perturbations are not perfectly spherical, leading to condensations on smaller scales and thus violent relaxation during the collapse.

  The radius of the virialized region can be deduced from the virial theorem:

$$E = E(t_{max}) = T + \Phi = \Phi_{max} = -\frac{GM^2}{R_{max}}$$
$$E = E(t_{vir}) = \frac{1}{2}\Phi_{vir}$$

Thus we have

$$-\frac{GM^2}{R_{max}} = -\frac{1}{2}\frac{GM^2}{R_{vir}}$$

or, equivalently,

$$\boxed{R_{vir} = \frac{1}{2}R_{max} \quad \Rightarrow \quad \varrho_{vir} = 8\,\varrho_{max}} \tag{15.11}$$

The density at virialization is 8 times that at turn around!

Thus the density of a virialized region is at least 44 times higher than the mean background density at turn around.
Note: Dissipation in the baryonic component can lead to an enhanced collapse relative to the dark matter component.

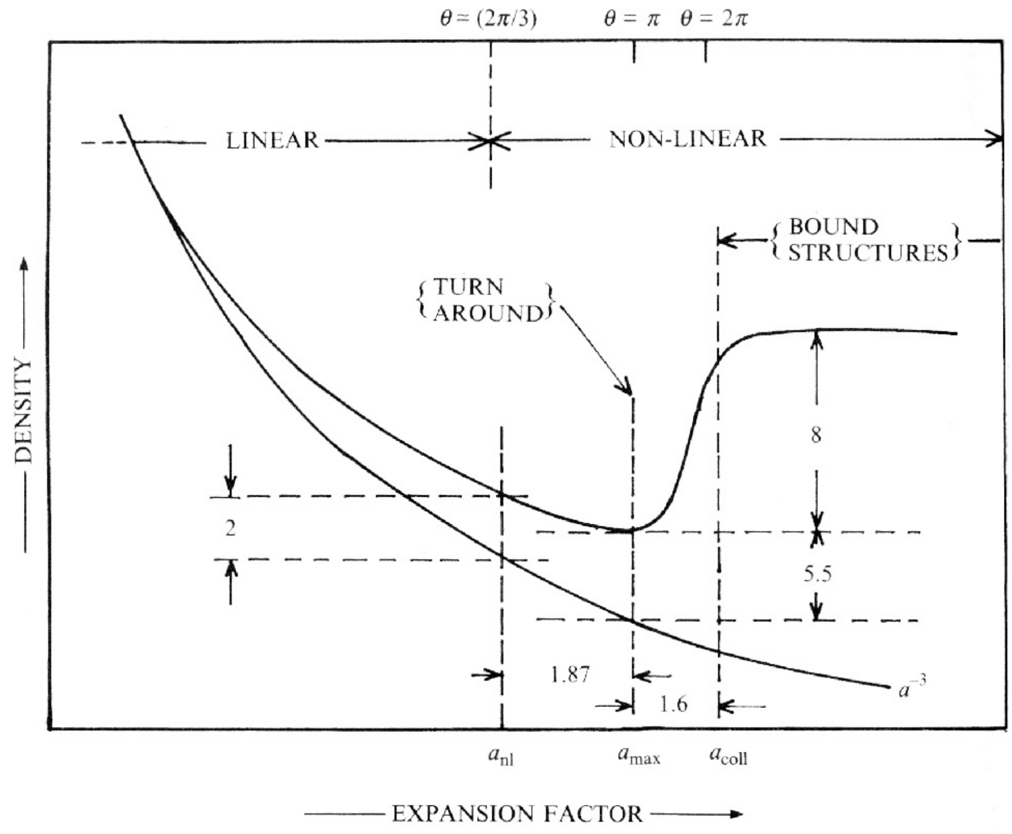
From this we can derive a limit on the redshift of formation of galaxies and clusters:

$$\boxed{\varrho_{vir} \ > \ 5.55 \cdot 8 \cdot \bar{\varrho}_0 \cdot \left(1 \ + \ z_{maz}\right)^3}$$

According to the considerations above, the density ϱ_{vir} is the density of the dark matter component. Using $(1 + z_{max}) \simeq 1.59(1 + z_{form})$ we can conclude

$$\boxed{\varrho_{vir} \ > \ 180\, \bar{\varrho}_0 \left(1 \ + \ z_{form}\right)^3}$$

Non-linear evolution of density perturbations



(From Padmanabhan, *Structure formation in the universe*, Cambridge 1993)

Examples

1. **The formation redshift of the Milky Way:**
   For the Milky Way's dark matter halo we have

$$\left.\begin{array}{lcl} M_{DM} & \approx & 3\cdot 10^{11}\,M_\odot \\ R_{DM} & \approx & 50\,\mathrm{kpc} \end{array}\right\} \quad \Rightarrow \quad \varrho_{DM} \approx 4\cdot 10^{-26}\,\mathrm{g\;cm}^{-3} \qquad \text{(Milky Way)}$$

   The background density of the universe is $\bar{\varrho}_0 = 1.88\cdot 10^{-29}\,h^2\,\Omega_m\,\mathrm{g\;cm}^{-3}$. Thus:

$$\boxed{\begin{array}{lcl} z_{max} & < & 5 \\ z_{form} & < & 2.5 \end{array}}$$

2. **The formation redshift of a typical cluster of galaxies:**
   For a typical cluster of galaxies, the density of the dark matter halo is

$$\varrho_{DM} \simeq \frac{M_{DM}}{4\pi/3\,R_{DM}^3} \simeq \frac{\sigma^2 R/G}{4\pi/3\,R^3} = \frac{4\pi}{3G}\,\frac{\sigma^2}{r^2}$$

   (virial theorem; $R_{DM} \sim R$).

In more appropriate units:

$$\varrho_{DM} \approx 7 \cdot 10^{-26}\,\mathrm{g\ cm^{-3}} \left(\frac{\sigma}{1000\,\mathrm{km/s}}\right)^{2} \left(\frac{R}{\mathrm{Mpc}}\right)^{-2}$$

Virgo cluster: $R \simeq 1.5\,\mathrm{Mpc}$, and $\sigma \simeq 600\,\mathrm{km\ s^{-1}}$. Thus:

$$\varrho_{DM} \approx 10^{-26}\,\mathrm{g\ cm^{-3}} \qquad \text{(Virgo cluster)}$$

which leads to

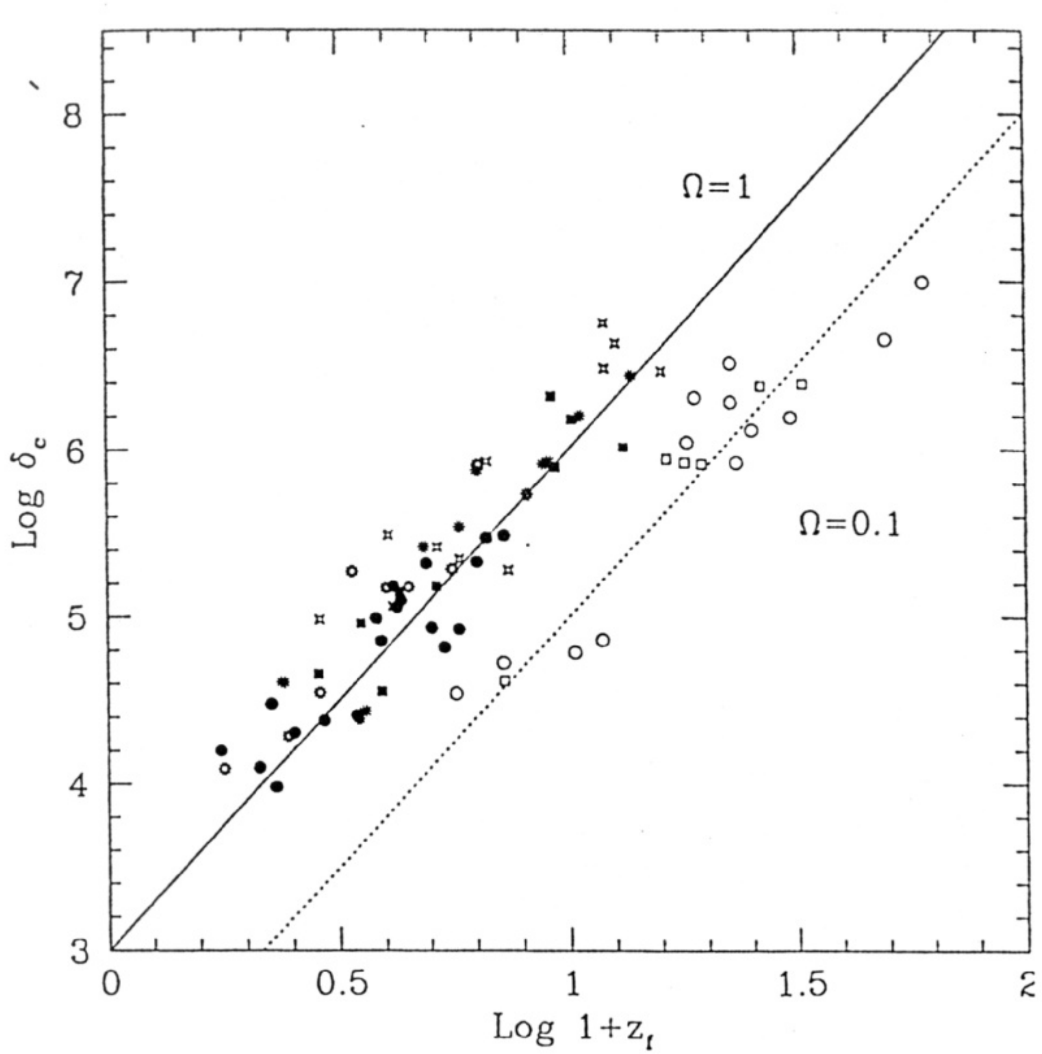
$$\boxed{\begin{aligned} z_{max} &< 2.5 \\ z_{form} &< 1 \end{aligned}}$$

The general equations for the radius r and the velocity dispersion σ in the spherical collapse model are

$$r \simeq 258\,\mathrm{kpc}\,(1 + z_{form})^{-1} \left(\frac{M}{10^{12}\,M_\odot}\right)^{1/3} h_{50}^{-2/3}$$

$$\sigma \simeq 100\,\frac{\mathrm{km}}{\mathrm{s}}\,(1 + z_{form})^{1/2} \left(\frac{M}{10^{12}\,M_\odot}\right)^{1/3} h_{50}^{1/3}$$

Remember that all considerations developed above are only valid, if perturbations on a given scale *exist*. We will discuss the influence of the *type* of dark matter on the structures formed in the next section.



Relation between halo over-density and formation redshift from numerical simulations

(White, 1996, in: *Gravitational dynamics*, Cambridge University Press)

15.3 Different Types of Dark Matter

The spatial distribution of matter in the universe depends strongly on the nature of the dark matter particles.

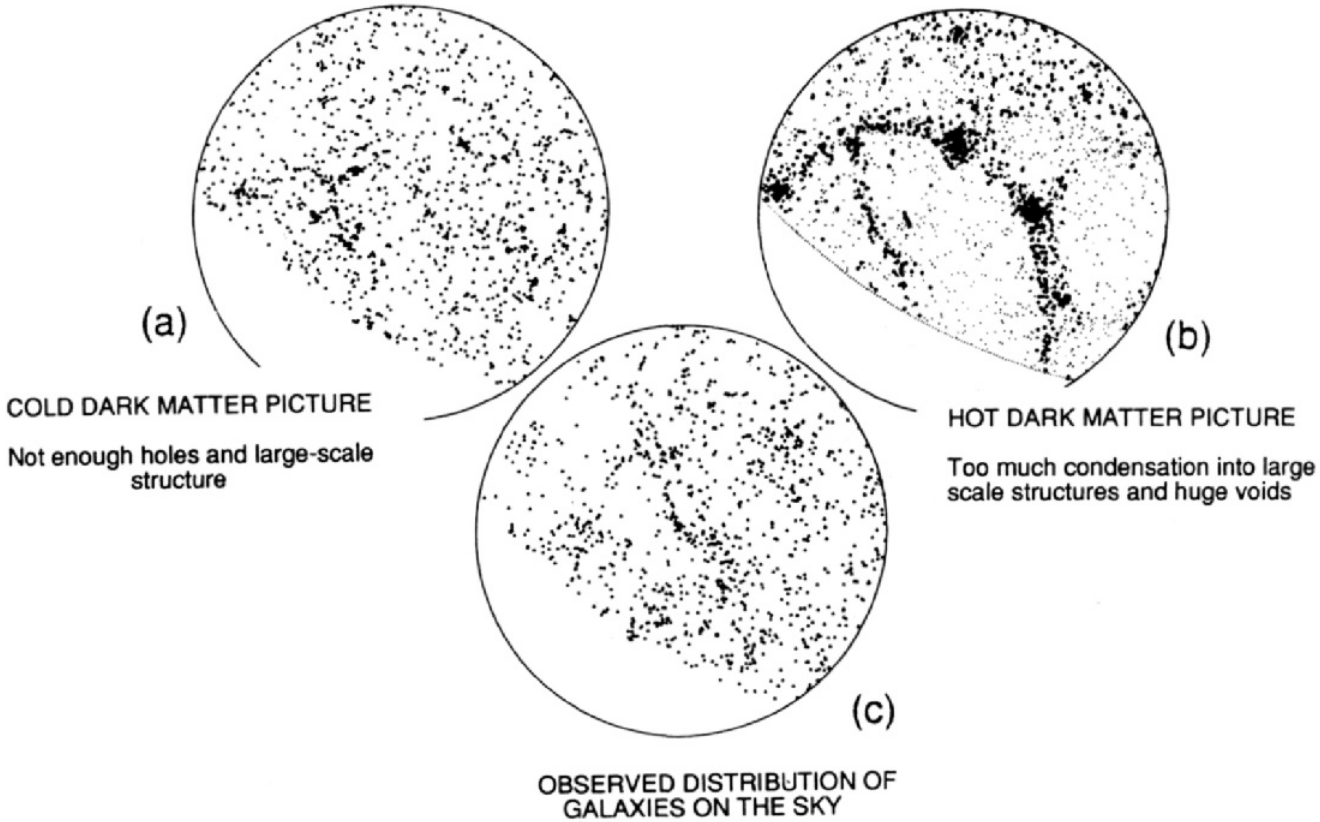
Hot Dark Matter (e.g. low-mass neutrinos) do not form small structures, since a large mass ($> 10^{14} M_{\odot}$) is needed to keep these particles gravitationally bound.

Cold Dark Matter on the other hand preferably forms structure on small scales. Larger objects are formed by merging of smaller sub-units (a process called **hierarchical clustering**).

The amount of structure on different scales is measured by the **power spectrum** $P(k)$ (remember that we have defined the Fourier transform $\delta_{\boldsymbol{k}}$ of the density contrast in terms of the co-moving wave number $k \equiv 2\pi/\lambda$):

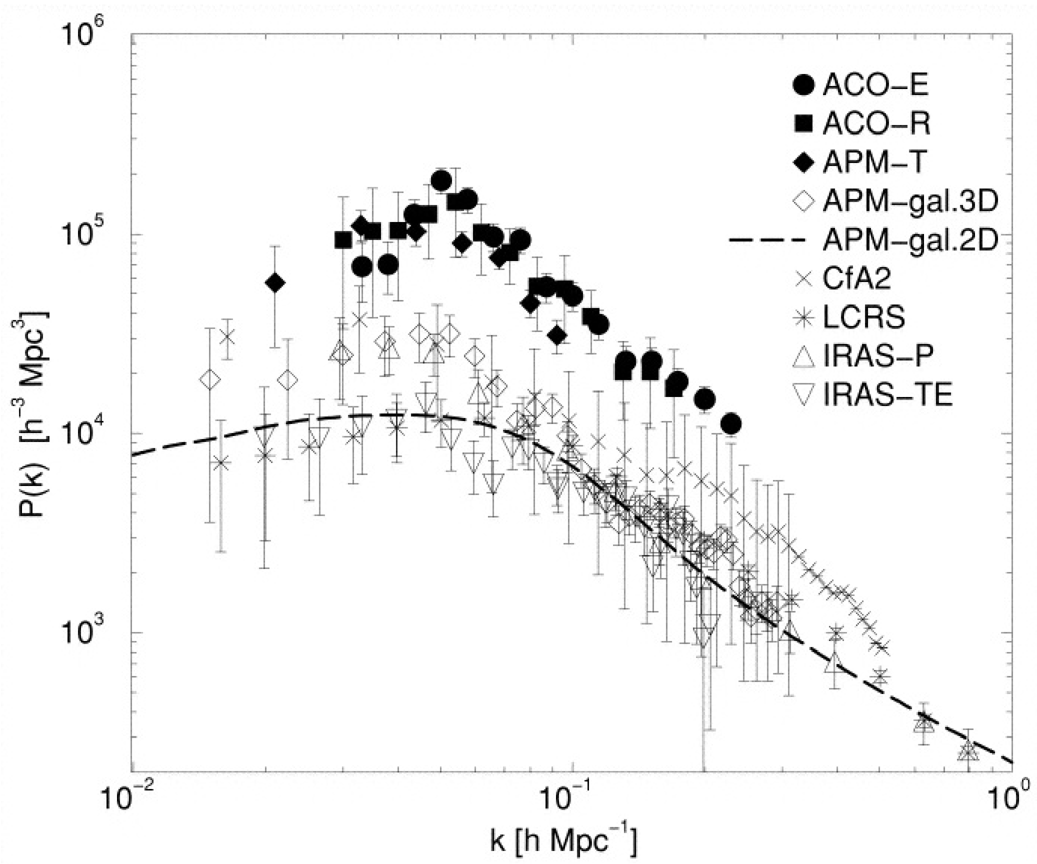
$$\boxed{P(k,t) \equiv |\delta_{\boldsymbol{k}}(t)|^2} \tag{15.12}$$

Measurements of the power spectrum disfavour a neutrino–dominated universe.



(From Longair, *Galaxy Formation*, Springer-Verlag 1998)

The power spectrum derived from several galaxy surveys



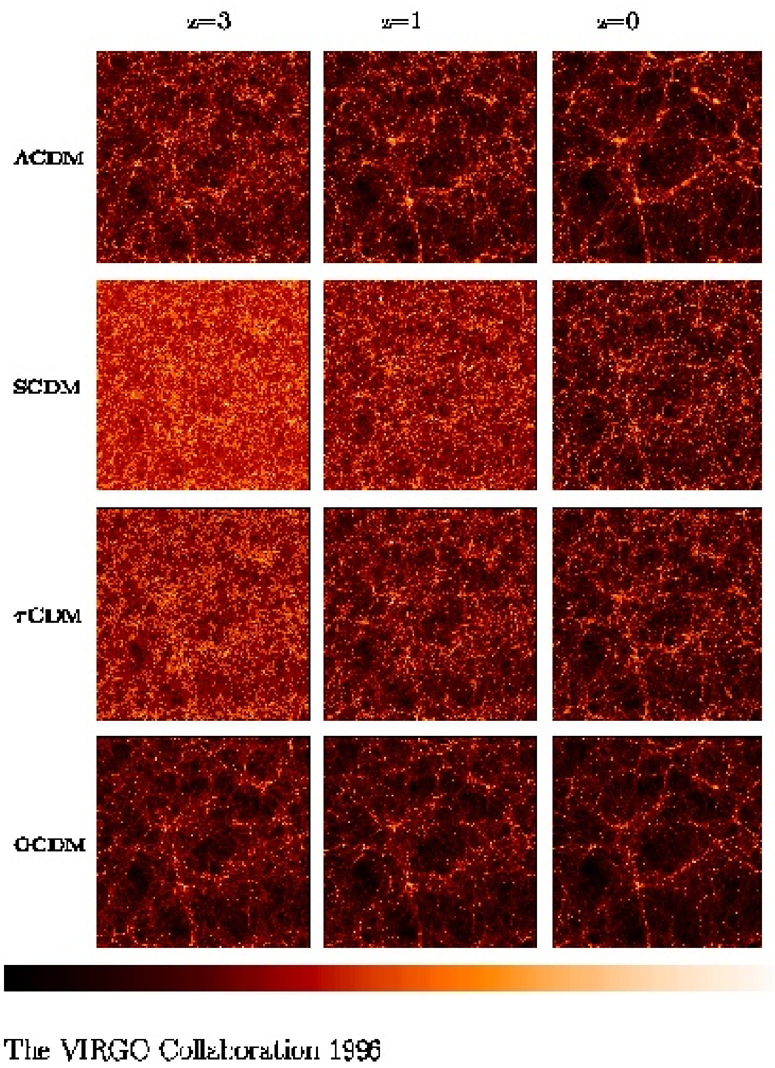
(From Einasto *et al.*, 1999, *ApJ*, **519**, 441)

15.4 N-Body Simulations

Analytic approximations of the spherical collapse yield rough estimates of the time scales and the density of the dissipation-less dark matter component. To understand the *structure* and the *flattening* of collapsing regions, and the *tidal forces* acting on nearby regions, one has to do **numerical simulations**.

- The dark matter component is relatively easy to model, since it is governed by gravitational interaction only.
- Simulations of the behaviour of the baryonic component (including cooling, star formation, heating, . . .) is much more complicated.

The following simulations were done by the Virgo Consortium.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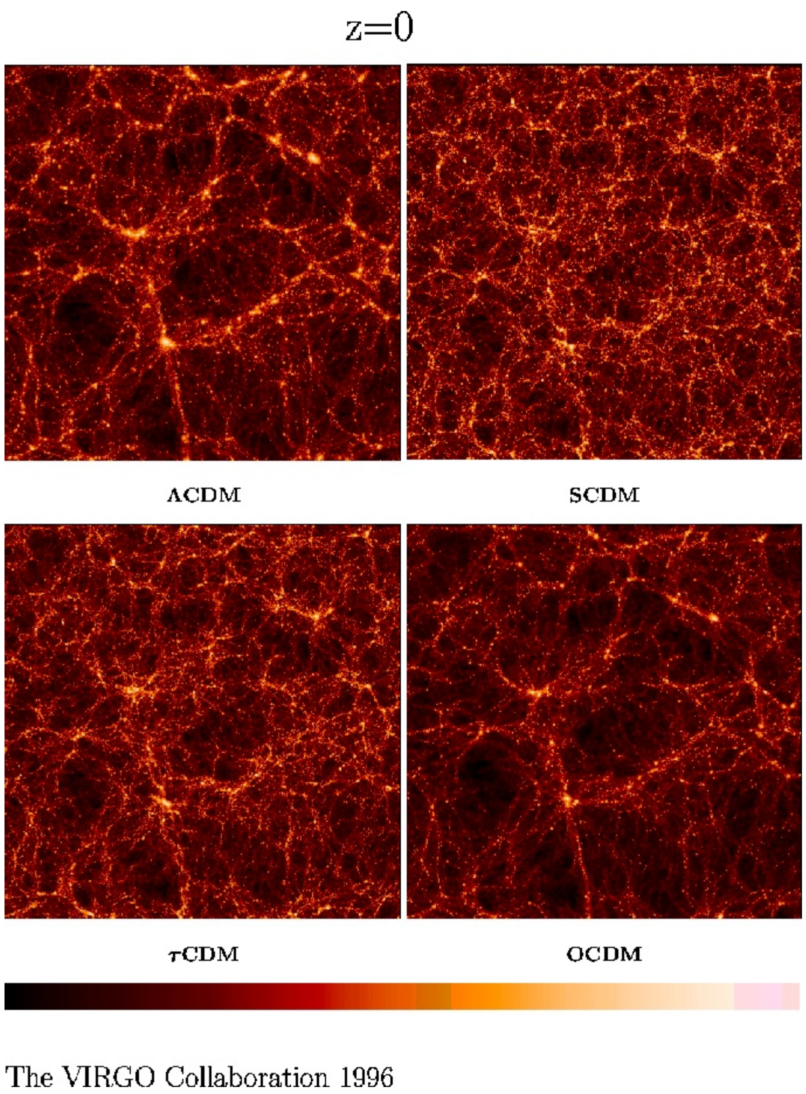
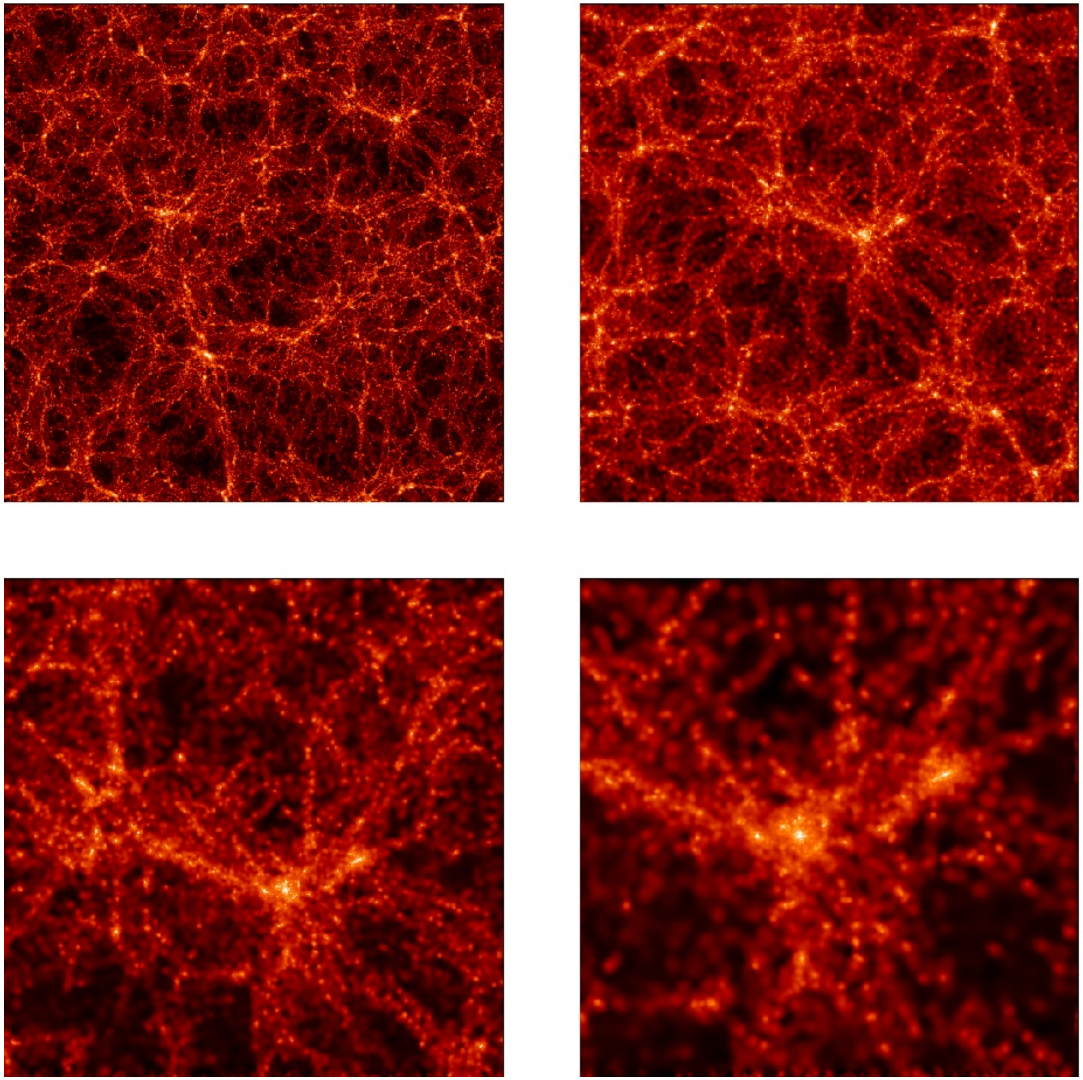
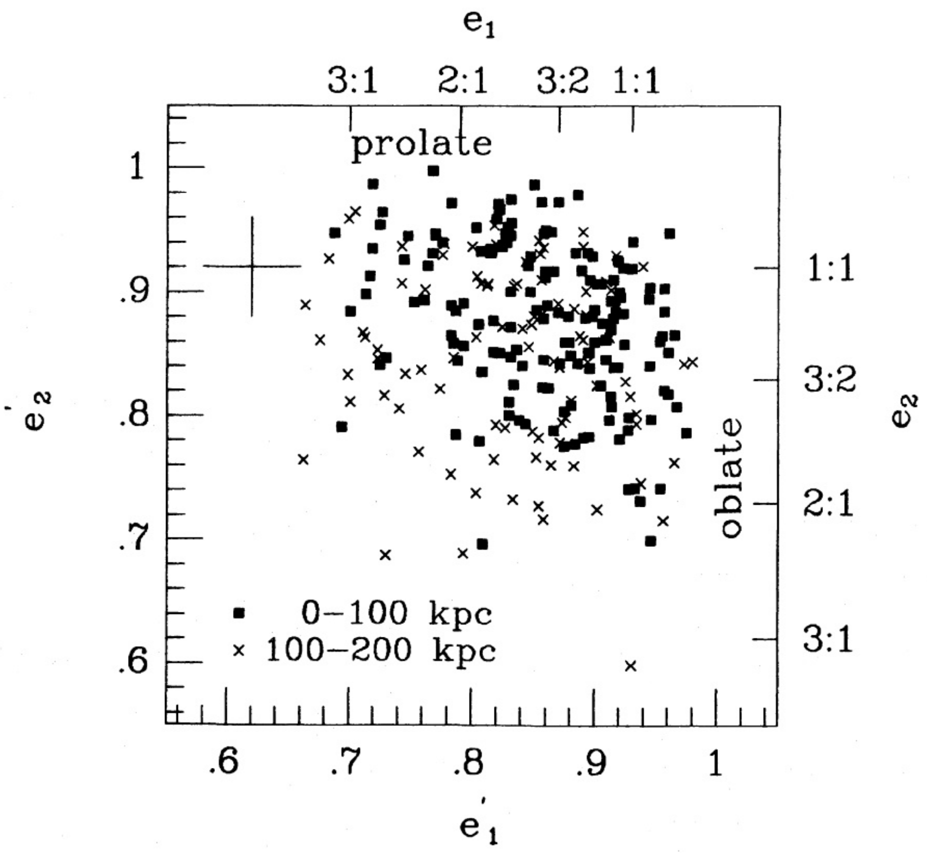
z=1
z=0
ΛCDM
SCDM
τCDM
OCDM
The VIRGO Collaboration 1996


z=0
ΛCDM
SCDM
τCDM
OCDM
The VIRGO Collaboration 1996

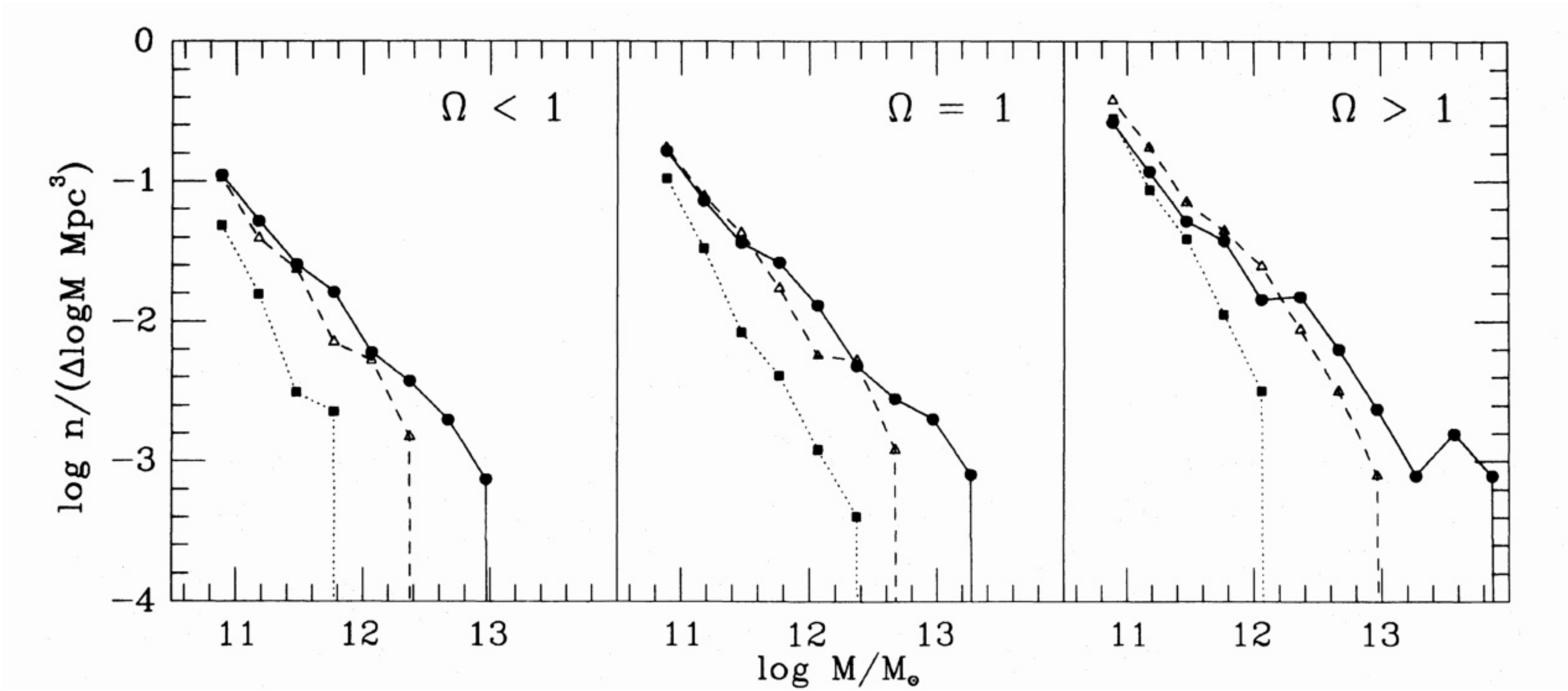


Axial ratios of haloes in CDM simulations



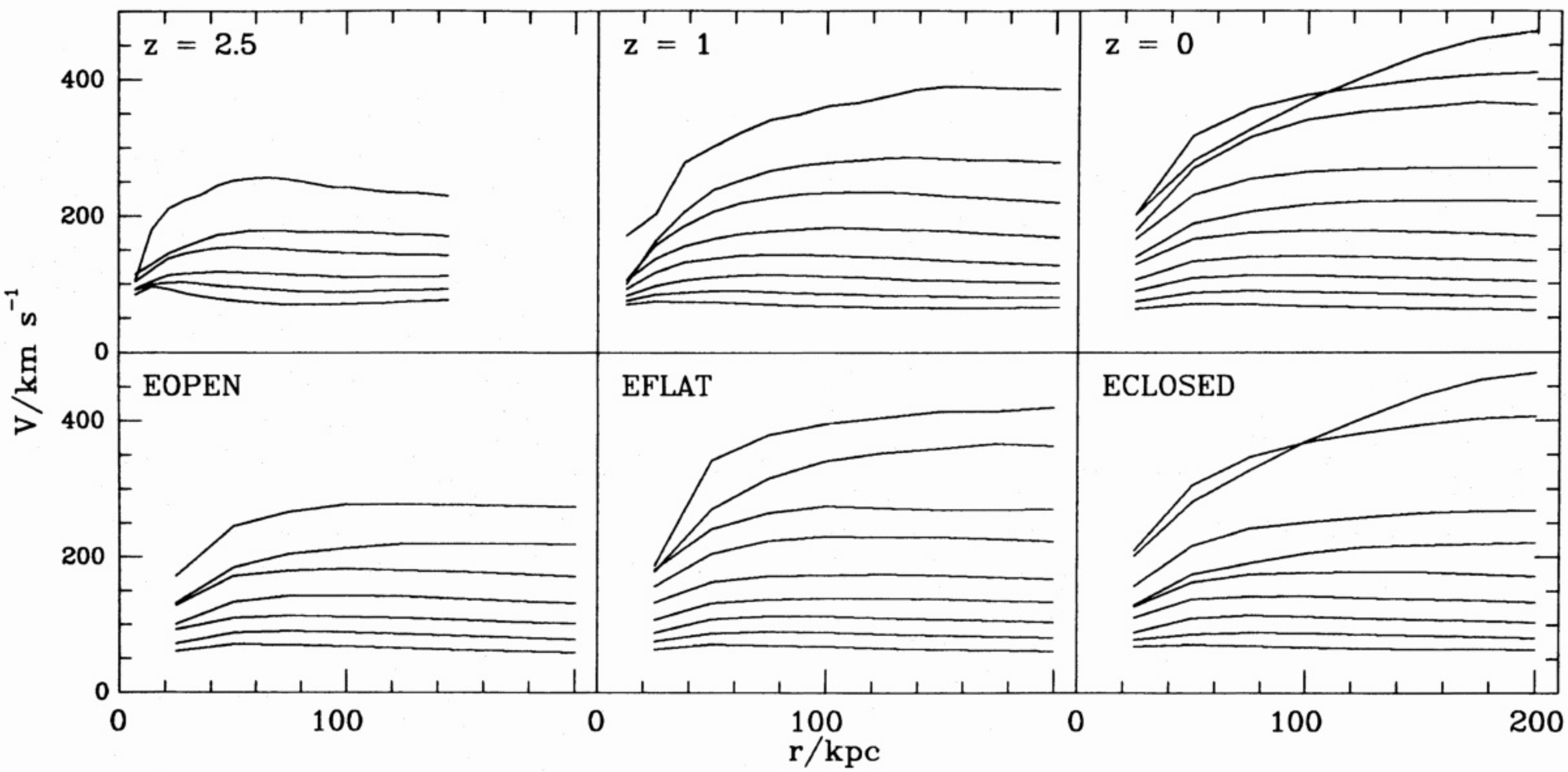
(From Frenk *et al.*, 1988, *ApJ*, **327**, 507)

Number of haloes per unit volume and per unit logarithmic mass interval



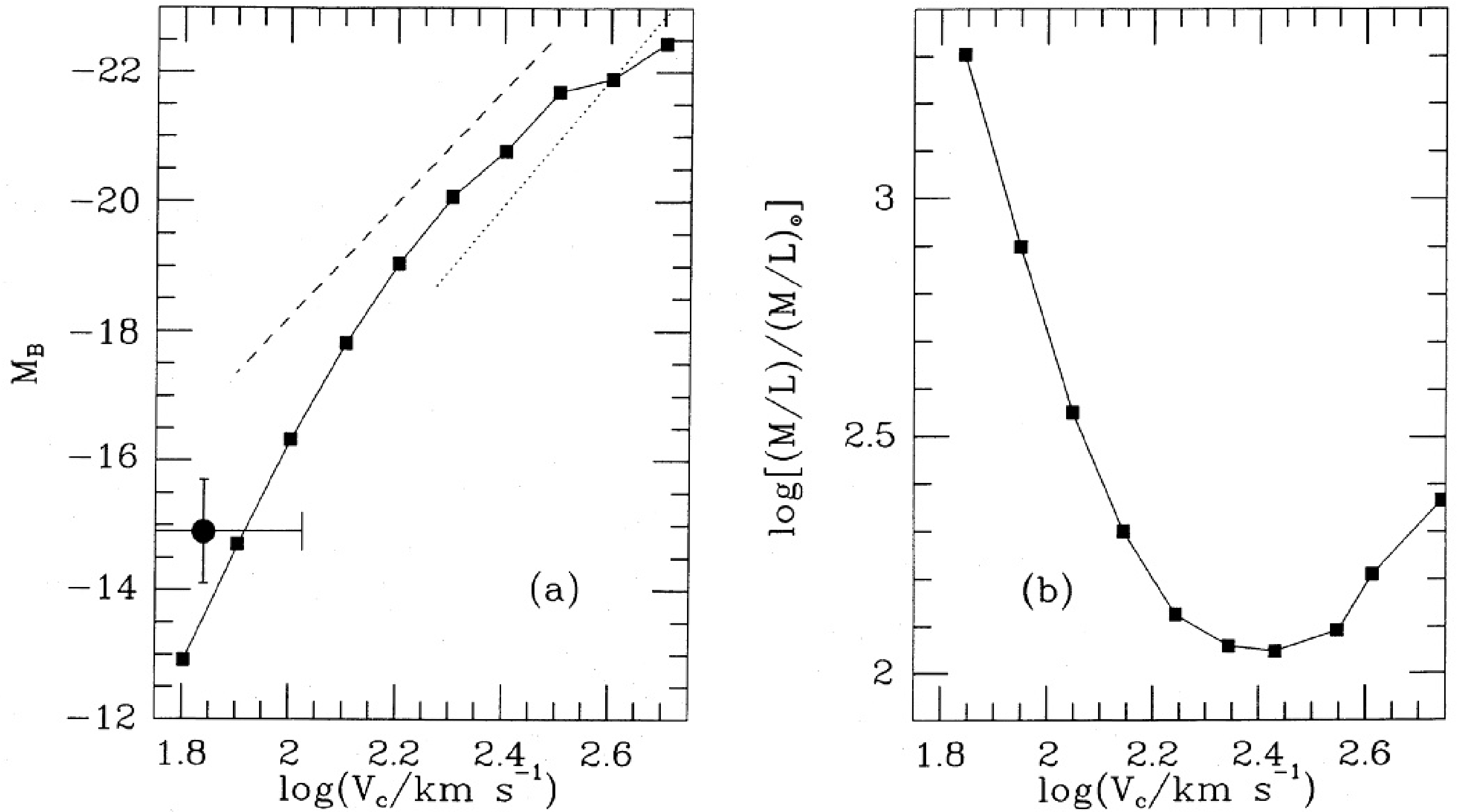
(From Frenk *et al.*, 1988, *ApJ*, **327**, 507)

Flat circular velocity curves in CDM simulations



(From Frenk *et al.*, 1988, *ApJ*, **327**, 507)

Luminosity–circular velocity relation and mass-to-light ratios



(From Frenk *et al.*, 1988, *ApJ*, **327**, 507)

15.5 Angular Power Spectrum of CMB Fluctuations

The COBE (Cosmic Background Explorer) satellite detected temperature fluctuations in the Cosmic Microwave Background (CMB), shown on page 557. These fluctuations, measured on angular scales of $\theta \sim 7°$, have typical amplitudes of

$$\frac{\Delta T}{T} \simeq 10^{-5}$$

The temperature fluctuations in the CMB are connected to density fluctuations at the epoch of recombination by three physical processes:

- On *large angular scales* ($\theta \sim 10°$) the dominant source of fluctuations is the **Sachs–Wolfe effect**, simply describing the fact that photons lose (or gain) energy when they escape from over-dense (or under-dense) regions (gravitational redshift).
- On *intermediate scales* ($\theta \sim 1°$) the baryonic perturbations oscillate, which can be observed as **acoustic peaks** in the angular spectrum of CMB fluctuations.

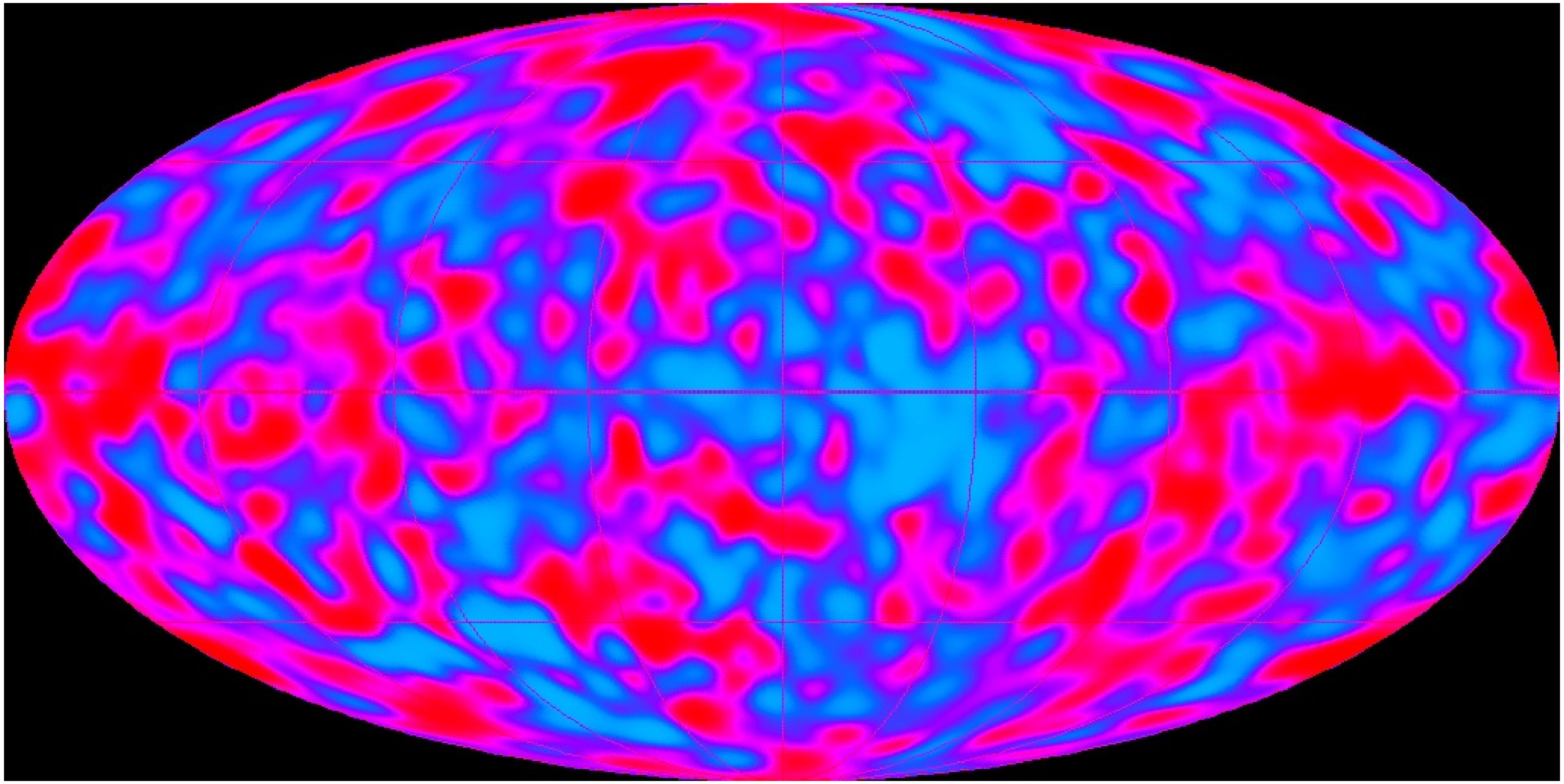
- On *small angular scales* ($\theta < 1^\circ$) the oscillations are damped, mainly by the process called **Silk damping** (photon diffusion suppresses small-scale perturbation).

It turns out that the form of the angular power spectrum is strongly dependent on cosmological parameters. The CMB fluctuations is usually expressed in terms of spherical harmonics

$$\frac{\Delta T}{T}(\theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} a_{lm} Y_{lm}(\theta, \varphi)$$

The angular power spectrum is then defined as follows:

$$C_l = \frac{1}{2l+1} \sum_m a_{lm} a_{lm}^* = \langle |a_{lm}|^2 \rangle$$

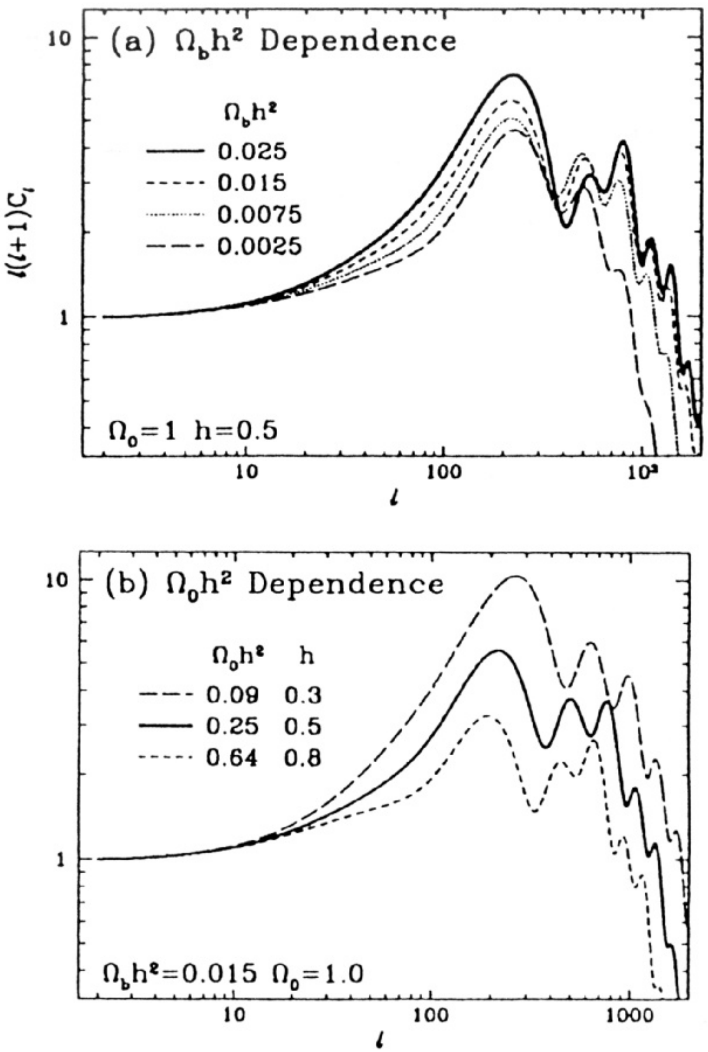


Temperature fluctuations in the Cosmic Microwave Background as measured by the COBE satellite.

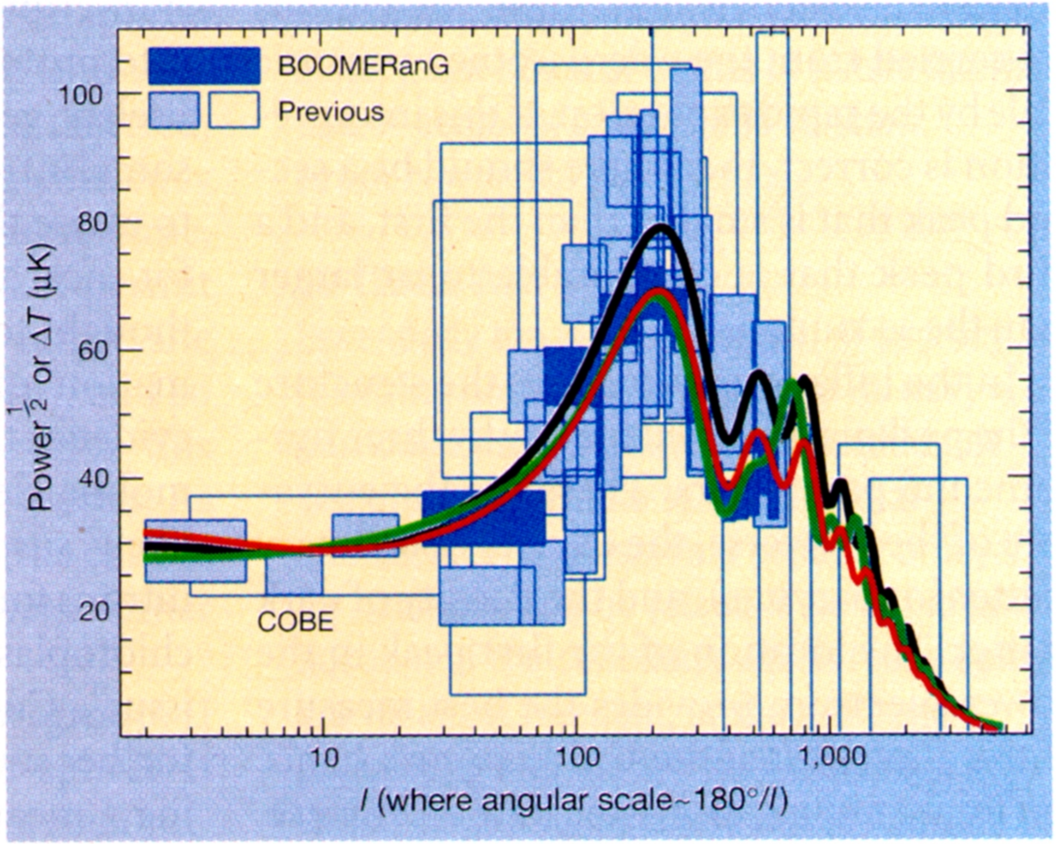
As already explained, COBE could measure the amplitude on large angular scales.

When the importance of the angular spectrum of CMB fluctuations for the measurement of cosmological parameters was realized, new satellite missions like Planck or MAP.

Furthermore, balloon experiment are carried out. Recently published results of the **BOOMERANG** (Balloon Observations Of Millimetric Extragalactic Radiation And Geophysics) suggest a flat universe.



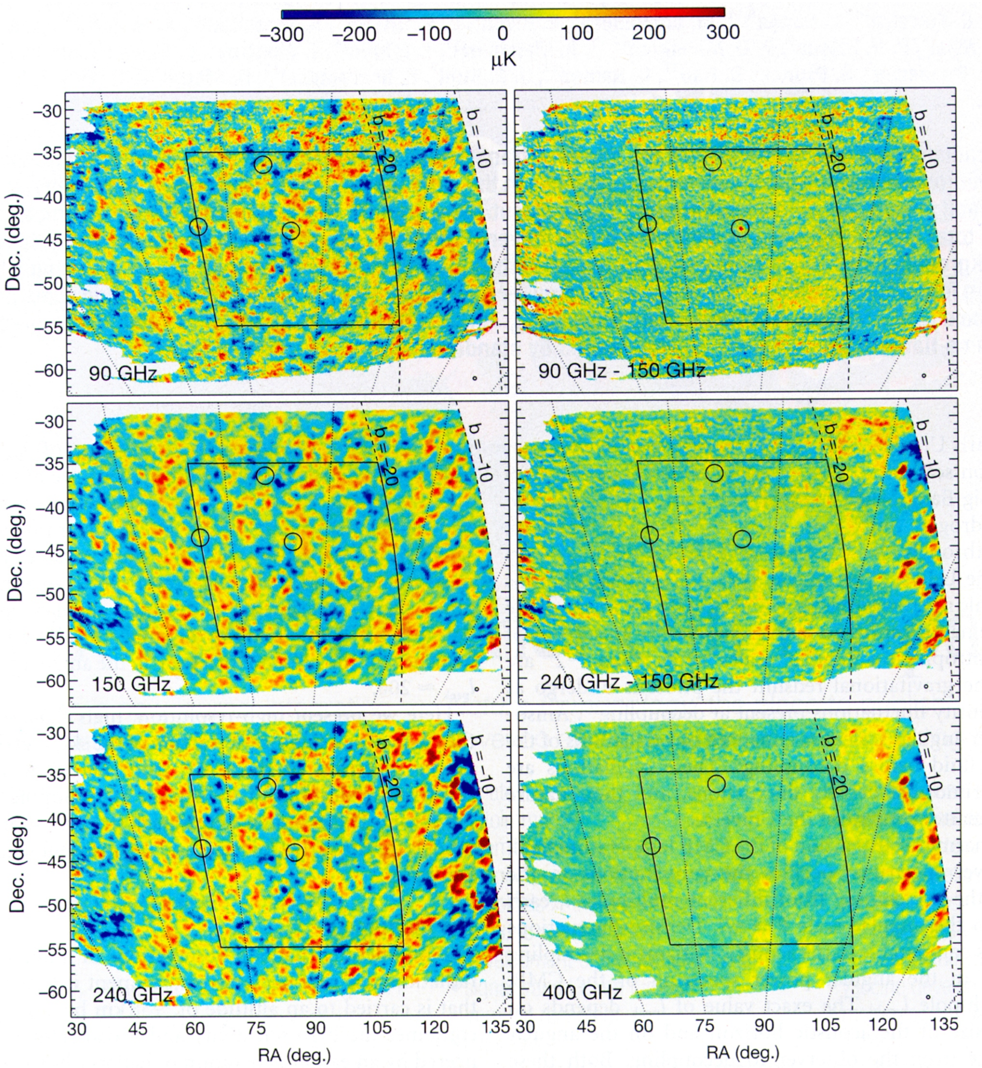
(From Longair, *Galaxy Formation*, Cambridge 1993)



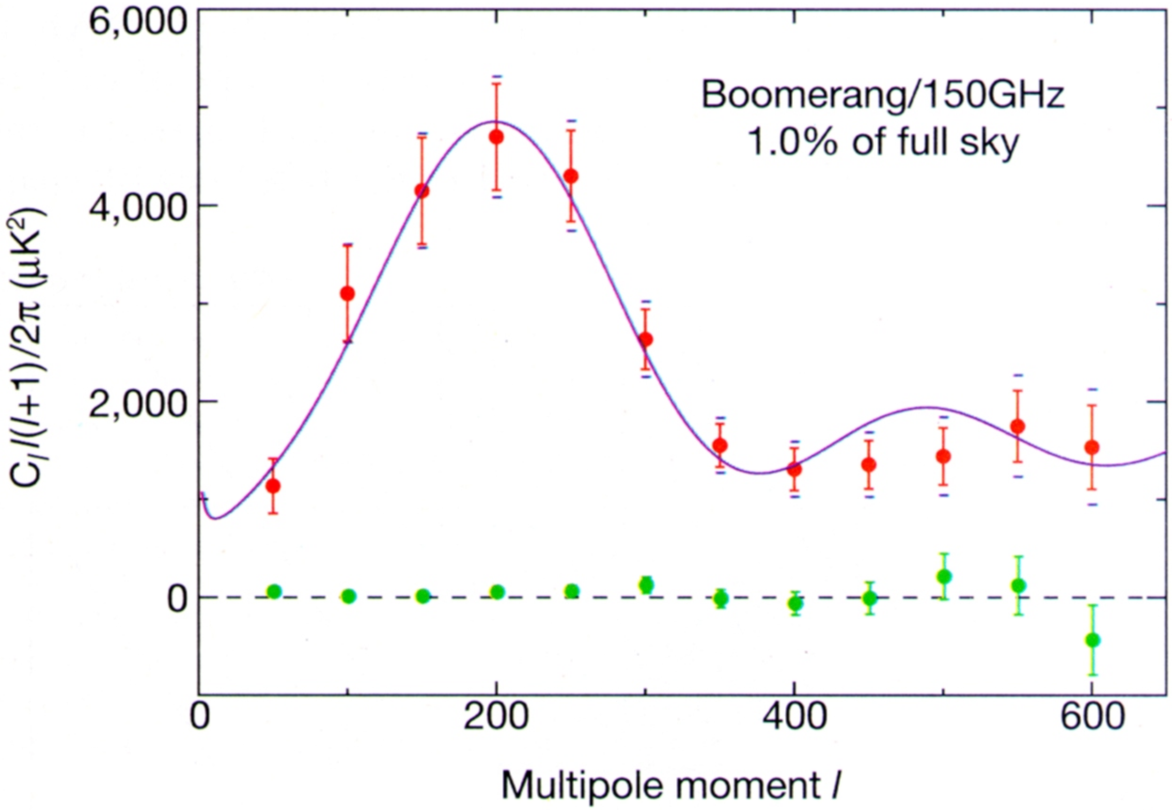
(From Hu, 2000, *Nature*, **404**, 939)

<u>The Boomerang map of the CMB</u>

(de Bernardis *et al.*, 2000, *Nature*, **404**, 955)

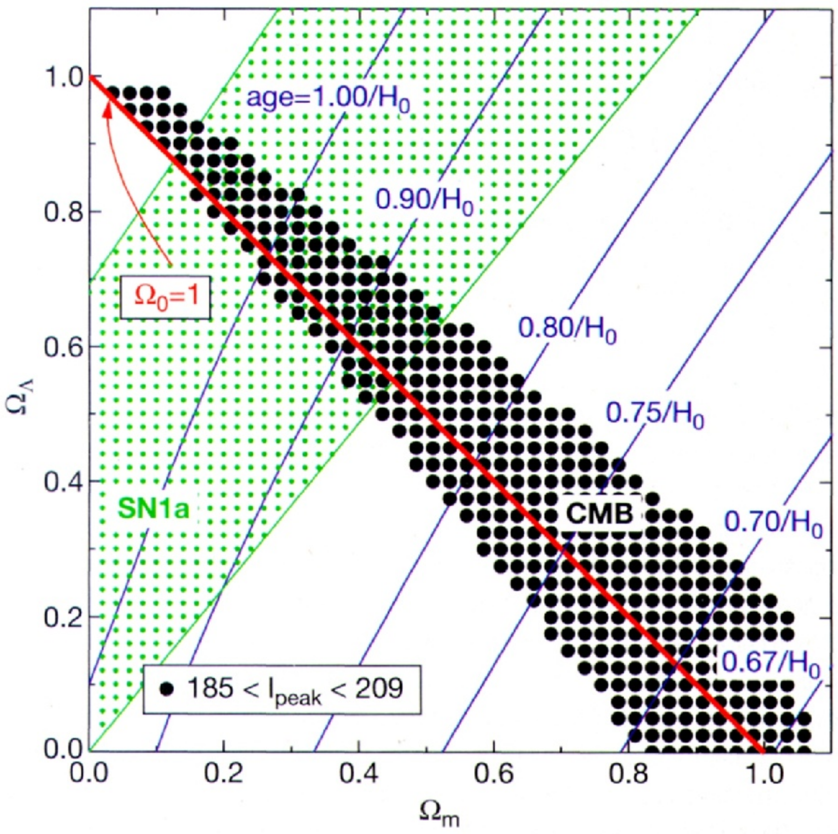


The angular power spectrum of CMB fluctuations



(From de Bernardis *et al.*, 2000, *Nature*, **404**, 955)

Constraints on Ω_m and Ω_Λ from CMB fluctuations



(From de Bernardis *et al.*, 2000, *Nature*, **404**, 955)