Galaxies, Cosmology and Dark Matter

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Chapter 14

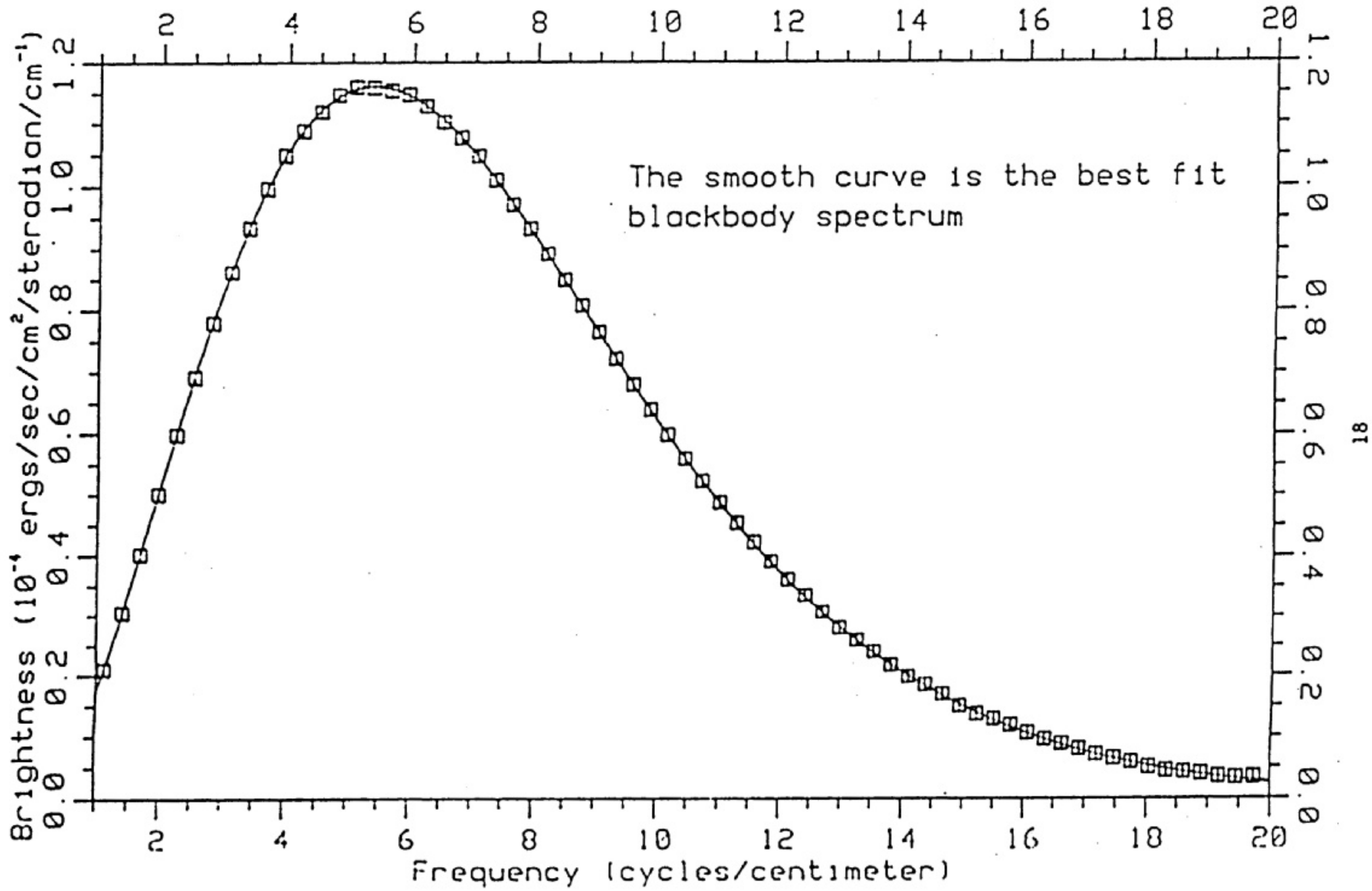
The Early Universe

14.1 The Cosmic Microwave Background

In 1964 Penzias and Wilson detected an isotropic radio emission at a wavelength of 7.35 cm, which was soon recognized to possibly correspond to the **cosmic microwave background (CMB)** predicted by the big bang model. In the following decades it was shown that the CMB shows an almost perfect black body spectrum leaving the cosmological interpretation as the only plausible one.

In 1992, the *COBE satellite* determined the temperature of the CMB very accurately to $T = 2.762$ K and, for the first time, detected temperature fluctuations of $\Delta T/T \sim 10^{-5}$ on angular scales of 10 degrees and larger (for implications see section below).

Cosmic Background Spectrum at the North Galactic Pole



Temperature and Spectral Evolution of the CMB

At earlier times the temperature T of the CMB must have been higher because of the redshift effect. The photon gas has a frequency-integrated energy density ε of

$$\varepsilon = \sum_{\nu} h\nu \cdot N(h\nu)$$

where the $N(h\nu)$ is the co-moving number density of photons. With $N = N_0(1+z)^3$ and $\nu = \nu_0(1+z)$ we obtain:

$$\varepsilon = \sum_{\nu_0} h\nu_0 N_0 (1+z)^4 \quad \Rightarrow \boxed{\varepsilon = \varepsilon_0 \cdot R^{-4}}$$

In case of black body radiation, the energy density is given by the Stefan Boltzmann Law: $\varepsilon = aT^4$ implying that the temperature should change like:

$$\Rightarrow \boxed{T_{Rad} = T_0(1+z)}$$

Inserting the last equation and $\nu = \nu_0(1+z)$ into the Planck function for the spectral energy distribution:

$$\varepsilon(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1} d\nu$$

we indeed find that the shape of the microwave spectrum is preserved, i.e. the CMB always keeps its black body spectrum.

$\Rightarrow$ In the past, the universe was hotter. As the CMB energy density goes with $(1+z)^4$ but matter density only with $(1+z)^3$, there must have been a time when the universe was radiation dominated.

Radiation dominated phase

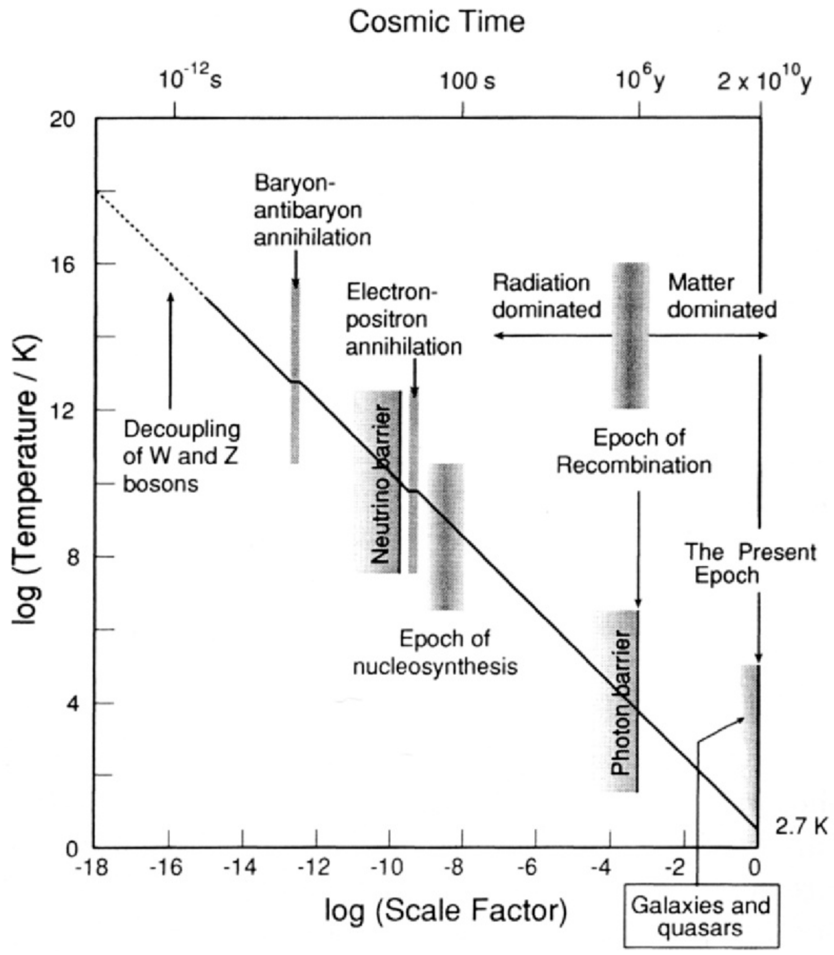
Solving the Friedmann equations for a mix of radiation and matter yields the scale factor, cosmic time and temperature for the moment of equality:

$$\begin{aligned} R_{EQ} &= 2.55 \cdot 10^{-5} h^{-2} \Omega^{-1} \qquad (\Omega = 1,\ \Lambda = 0) \\ t_{EQ} &\simeq 600\, yrs\, h^{-4} \\ z_{EQ} &\simeq \frac{1}{R_{EQ}} - 1 = 40000\, h^2\, \Omega \\ T_{EQ} &\simeq 10^5 K\ (10 eV) \end{aligned}$$

In the radiation-dominated early phases of the universe, more and more different particle species are present. Particles and their corresponding anti-particles are created and annihilated in equilibrium with the photon gas and other particles.

A particle species only disappears when the temperature falls below the rest mass energy of the particle. During the phases when particle species annihilate the temperature stays almost constant for short whiles. However not all particle species disappear completely because the number densities of particles and antiparticles are not exactly equal. The reason for this presumably lies in the so-called C- and CP-violation of unified field theories. Only because of this symmetry-breaking and non-equilibrium in the early universe, protons, neutrons and electrons can survive and form the baryonic matter which is now still present in the universe.

Thermal History of the Universe



14.2 The Decoupling of the Radiation

In the early universe, matter and photons are strongly coupled by Compton scattering. The typical rate for a photon to interact with an electron is given by:

$$\Gamma_\gamma = n_e \sigma_T c$$

with σ_T being the Thomson cross section of the electrons. The Thomson cross section of the nuclei is negligible and, therefore, the coupling between matter and photons is dominated by the electrons. The mean free path of the photons is:

$$\lambda_\gamma = c\Gamma_\gamma^{-1}$$

By cooling of the plasma through expansion, electrons and nuclei can 're'-combine. Then the photons decouple and move interaction-free. This happens when:

$$\lambda_\gamma > cH^{-1} \quad or \quad \Gamma_\gamma^{-1} > H^{-1}$$

i.e. or once the mean free path of the photons becomes comparable to the Hubble radius.

The equilibrium abundances for the electrons can be calculated from the Saha equation. In chemical equilibrium the following applies:

$$n_H = n_p n_e \left(\frac{mkT}{2\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{13.6eV}{kT}\right)$$

Defining the fractional abundances $X_H = \frac{n_H}{n_B} = 1 - X_e$ and $n_B = \eta n_\gamma$, with the abundance of photons n_γ, and the abundance of baryons n_B yields the Saha equation for the ionisation equilibrium:

$$\frac{1 - X_e}{x_e^2} \simeq 3.84\eta \left(\frac{kT}{m_e c^2}\right)^{3/2} \exp\left(\frac{13.6eV}{kT}\right)$$

The temperature can be calculated from $T = T_0(1+z)$ with $T_0 = 2.7K$. $\Rightarrow X_e(z)$ is a function of Ωh^2. So, for $\Omega h^2 = 0.1$: $X_e < 0.1$ for $(1+z) < 1300$.

Defining the recombination as the time when 90% of the electrons recombined, yields for $1 + z_{rec} = 1200 - 1400$:

$$\begin{aligned}
\Rightarrow T_{rec} &= T_0(1+z_{rec}) \simeq 3600K \hat{=} 0.3eV \\
t_{rec} &= \frac{2}{3} H_0^{-1} \Omega_0^{-1/2} (1+z_{rec})^{-3/2} \\
&\simeq 4.4 \cdot 10^{12}\, sec(\Omega h^2)^{-1/2} \simeq 2 \cdot 10^5\, yrs
\end{aligned}$$

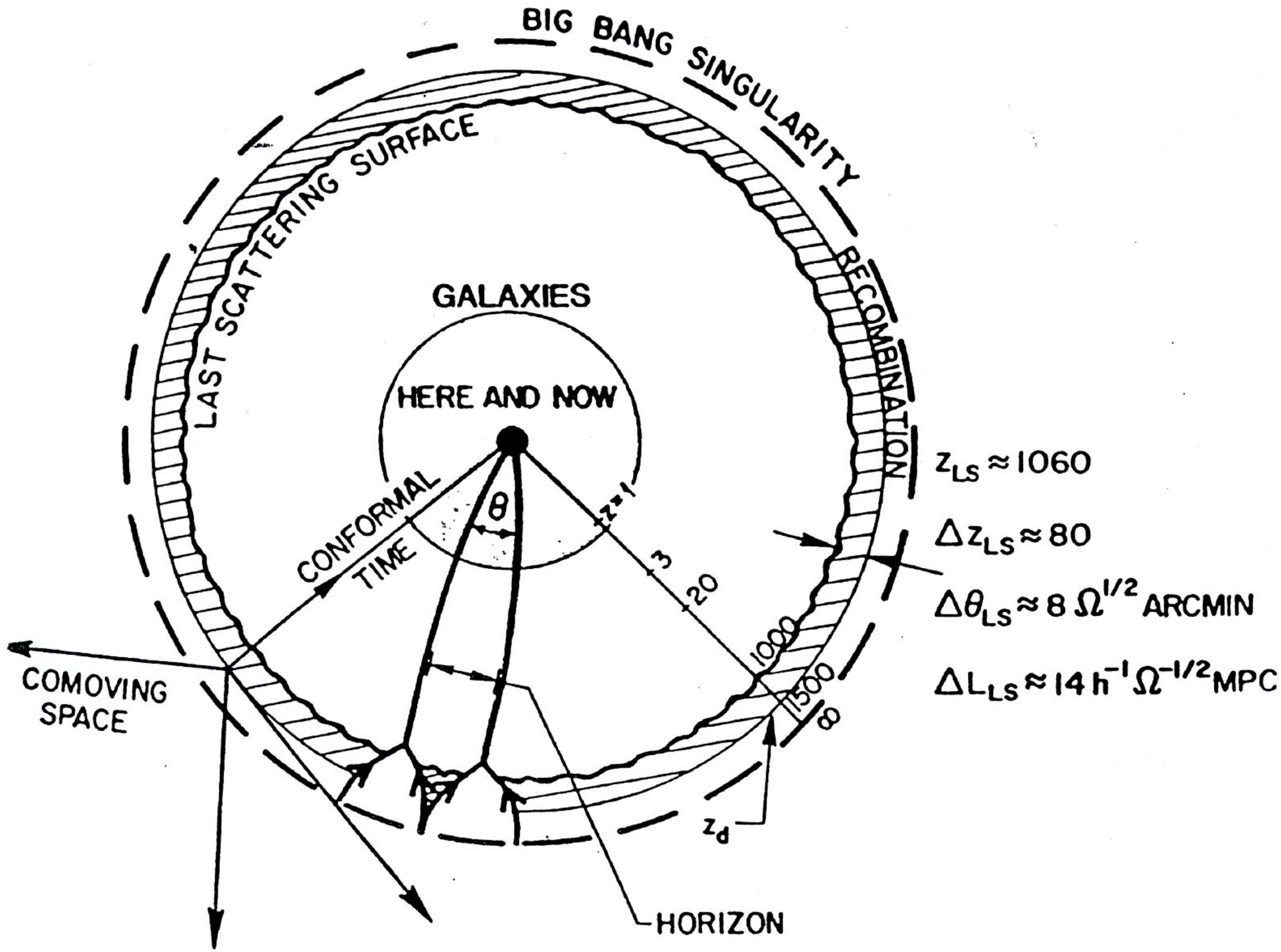
Thus, recombination does not occur at $13.6\,eV$ but at $0.3\,eV$ because even at this low temperature there are enough photons in the high energy tail of the Planck function to ionize hydrogen (because of the high photon to proton ratio).

At redshifts $z > 1400$ hydrogen is fully ionized. Helium is fully ionized at $z > 6000$.

The universe is optically thick against Thomson scattering at $z > 1400$

$\Rightarrow$ The universe becomes unobservable at redshifts $z \simeq 10^3$ which is the last scattering surface for the photons.

$\Rightarrow$ If fluctuations in the photon temperature are present, they will be preserved and should be visible in the cosmic background radiation (except if the universe was completely re-ionized at later times but when it was still quite homogeneous).


BIG BANG SINGULARITY
LAST SCATTERING SURFACE
RECOMBINATION
GALAXIES
HERE AND NOW
CONFORMAL TIME
θ
z=1
3
20
1000
1500
∞
COMOVING SPACE
HORIZON
z_d
z_LS ≈ 1060
Δz_LS ≈ 80
Δθ_LS ≈ 8 Ω^1/2 ARCMIN
ΔL_LS ≈ 14 h^-1 Ω^-1/2 MPC

14.3 Primordial Nucleosynthesis

At a temperature of $kT = 10MeV$ ($T = 10^{10}K$), the universe is radiation-dominated and the weak interaction rates are faster than the expansion of the universe. The following particles are in equilibrium: p, n, e^-, e^+, ν, $\bar{\nu}$. The ratio of neutrons to protons is set by:

$$\frac{N_n}{N_p} = \exp\left(-\frac{m_n - m_p}{kT}\right) = \exp\left(-\frac{1.293\,MeV}{kT}\right)$$

via the reactions:

$$\begin{aligned} e^+ + n &\longleftrightarrow p + \bar{\nu}_e \\ \nu_e + n &\longleftrightarrow p + e^- \end{aligned}$$

i.e. at 10 MeV: $\frac{n}{p} \sim 1$. The transition rates are determined by weak interaction. E.g. the rate with which a neutron interacts with a neutrino to produce a proton is given by:

$$\tau_{weak} \simeq (\sigma_{weak} N_\nu c)^{-1}$$

where

$$\sigma_{weak} \propto E^2 \propto (kT)^2$$

As $N_\nu \propto R^{-3} \propto T^3$ it follows:

$$\tau_{weak} \propto T^{-5}$$

which implies a very steep decline in the interaction efficiency with expansion. Around $1s$, $T \simeq 10^{10} K$, $\rho \simeq 10^{-2} g/cm^3$, the transformation rate between n and p becomes smaller than $1s^{-1}$ and therefore falls below the expansion rate of the universe (which then is $\approx 1s^{-1}$!). Weak interaction is not able to maintain the equilibrium between protons and neutrons and the actual ratio of protons to neutrons freezes out at this time. Only via neutron decay, the number of neutrons continues to decrease. However, when the neutron-to-proton ratio has dropped to about 1/7 (about $300s$ after Big Bang), nucleosynthesis sets in:

$$\begin{aligned} p + n &\longleftrightarrow d + \gamma \\ d + d &\longleftrightarrow {}^3He + n \longleftrightarrow t + p \\ {}^3He + d &\longleftrightarrow {}^4He + n \end{aligned}$$

i.e., the light elements d, t, ${}^3He$, ${}^4He$ and ${}^7Li$ are created.

Heavier elements cannot be synthesized because:

(1) At the time when ^4He is synthesized, the Coulomb wall has become rather difficult to overcome.
(2) There are no stable isotopes with mass numbers 5 or 8.
(3) The low density prevents the triple-α process ($2^4He \rightarrow^8 Be^*$, $^8Be^* + ^4He \rightarrow^{12} C + 2\gamma$) to form ^{12}C (as it is possible in red giants).

The primordial abundances of the light elements mostly depend on the baryon-to-photon ratio and in turn on the baryon density of the universe. **The higher the density, the more complete is the transformation of D into ^4He.** Further parameters are the number of neutrino species (families) and the neutron decay time.

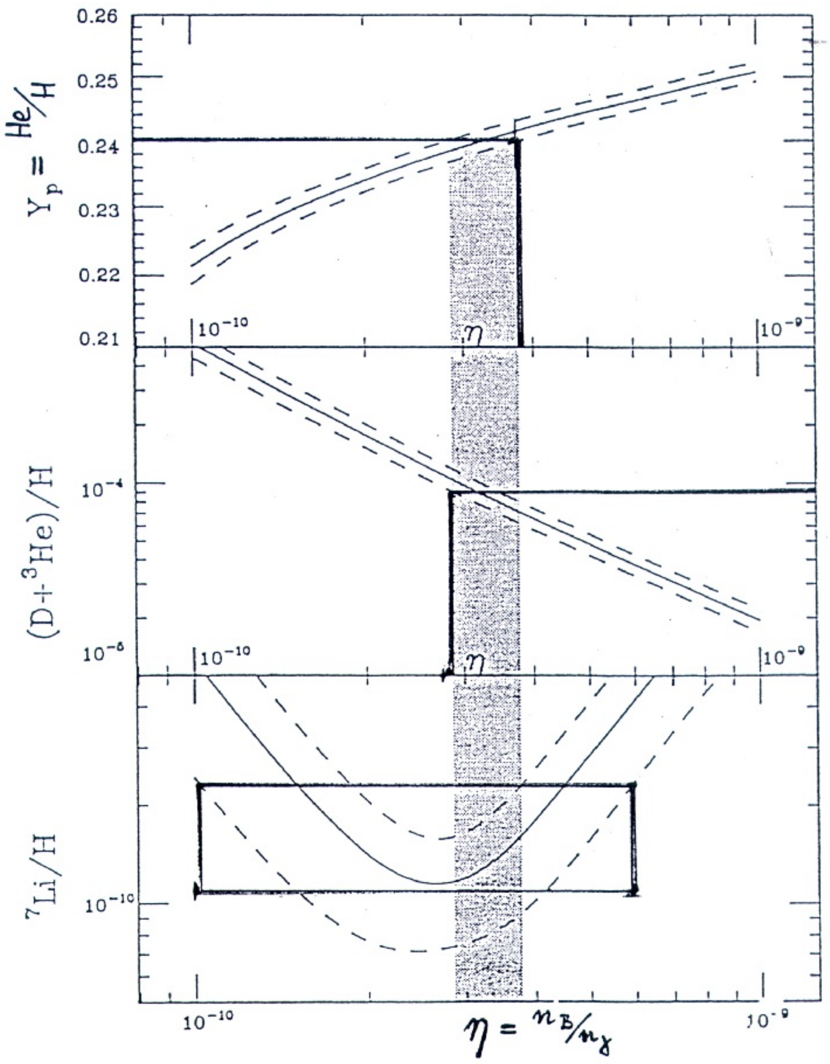
The current limits on the baryon density of the universe are:

$$\boxed{0.011 \leq \Omega_B h^2 \leq 0.037}$$

With $0.6 \leq h \leq 0.8$ one obtains:

$$\boxed{0.02 \leq \Omega_B \leq 0.1}$$

Therefore, if $\Omega_m > 0.1$, non-baryonic dark matter must exist which has fallen out of equilibrium before primordial nucleosynthesis occurred.


0.26
0.25
0.24
0.23
0.22
0.21
$Y_p = He/H$
10^{-10}
η
10^{-9}
$(D+^3He)/H$
10^{-4}
10^{-5}
$^7Li/H$
10^{-10}
$\eta = n_B/n_\gamma$